RECOGNIIING ACHIEVEMENT

## OXFORD CAMBRIDGE AND RSA EXAMINATIONS

## Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

## 4761

Mechanics 1
Friday 14 JANUARY $2005 \quad$ Morning 1 hour 30 minutes
Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{g} \mathrm{m} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $\mathrm{g}=9.8$.
- The total number of marks for this paper is 72 .

1 The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time $t$ is given by

$$
\mathbf{r}=t^{2} \mathbf{i}+\left(5 t-2 t^{2}\right) \mathbf{j}
$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.
(i) Find an expression for the acceleration of the particle.

The particle is subject to a force F and a force 12 jN .
(ii) Find $\mathbf{F}$.

2 Particles of mass 2 kg and 4 kg are attached to the ends $X$ and $Y$ of a light, inextensible string. The string passes round fixed, smooth pulleys at P, Q and R, as shown in Fig. 2. The system is released from rest with the string taut.


Fig. 2
(i) State what information in the question tells you that
(A) the tension is the same throughout the string,
(B) the magnitudes of the accelerations of the particles at X and Y are the same.

The tension in the string is $T \mathrm{~N}$ and the magnitude of the acceleration of the particles is $a \mathrm{~m} \mathrm{~s}^{-2}$.
(ii) Draw a diagram showing the forces acting at X and a diagram showing the forces acting at Y .
(iii) Write down equations of motion for the particles at X and at Y . Hence calculate the values of $T$ and $a$.

3 A particle is in equilibrium when acted on by the forces $\left(\begin{array}{r}x \\ -7 \\ z\end{array}\right),\left(\begin{array}{r}4 \\ y \\ -5\end{array}\right)$ and $\left(\begin{array}{r}5 \\ 4 \\ -7\end{array}\right)$, where the units are newtons.
(i) Find the values of $x, y$ and $z$.
(ii) Calculate the magnitude of $\left(\begin{array}{r}5 \\ 4 \\ -7\end{array}\right)$.

4 A particle is projected vertically upwards from a point O at $21 \mathrm{~ms}^{-1}$.
(i) Calculate the greatest height reached by the particle.

When this particle is at its highest point, a second particle is projected vertically upwards from O at $15 \mathrm{~ms}^{-1}$.
(ii) Show that the particles collide 1.5 seconds later and determine the height above O at which the collision takes place.

5 A small box B of weight 400 N is held in equilibrium by two light strings AB and BC . The string BC is fixed at C . The end A of string AB is fixed so that AB is at an angle $\alpha$ to the vertical where $\alpha<60^{\circ}$. String BC is at $60^{\circ}$ to the vertical. This information is shown in Fig. 5.


Fig. 5
(i) Draw a labelled diagram showing all the forces acting on the box.
(ii) In one situation string AB is fixed so that $\alpha=30^{\circ}$.

By drawing a triangle of forces, or otherwise, calculate the tension in the string BC and the tension in the string AB .
(iii) Show carefully, but briefly, that the box cannot be in equilibrium if $\alpha=60^{\circ}$ and BC remains at $60^{\circ}$ to the vertical.

7 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 7. AD is horizontal and BC is parallel to AD .

The stone is projected from A with speed $40 \mathrm{~ms}^{-1}$ at $50^{\circ}$ to the horizontal.


Fig. 7
(i) Write down an expression for the horizontal displacement from A of the stone $t$ seconds after projection. Write down also an expression for the vertical displacement at time $t$.
(ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. Calculate the range of the stone.

You are given that $X=30$.
(iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C .
(iv) Calculate the direction of the motion of the stone at C .

## 6 In this question take $g$ as $10 \mathrm{~m} \mathrm{~s}^{-2}$.

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.


Fig. 6
For this model,
(i) calculate the distance fallen from $t=0$ to $t=7$,
(ii) find the acceleration of the ball from $t=2$ to $t=6$, specifying the direction,
(iii) obtain an expression in terms of $t$ for the downward speed of the ball from $t=2$ to $t=6$,
(iv) state the assumption that has been made about the resistance to motion from $t=0$ to $t=2$.

The part of the motion from $t=2$ to $t=7$ is now modelled by $v=-\frac{3}{2} t^{2}+\frac{19}{2} t+7$.
(v) Verify that $v$ agrees with the values given in Fig. 6 at $t=2, t=6$ and $t=7$.
(vi) Calculate the distance fallen from $t=2$ to $t=7$ according to this model.

## Solutions and mark scheme

| Q1 |  | mark |  |  |
| :--- | :--- | :--- | :--- | :--- |
| (i) | Differentiate  <br> $\mathbf{v}=2 t \mathbf{i}+(5-4 t) \mathbf{j}$  <br>  Differentiate <br> $\mathbf{a}=2 \mathbf{i}-4 \mathbf{j}$ <br> (ii) M1 <br> A1  | At least 1 cpt correct <br> Award for RHS seen <br> M1 <br> F1 | Do not award if $\mathbf{i}$ and $\mathbf{j}$ lost in $\mathbf{v .}$. At least 1 cpt correct. FT <br> FT from their 2 component $\mathbf{v}$ |  |


| Q2 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { (i) } \\ & \text { (A) } \\ & \text { (B) } \end{aligned}$ | the pulleys are smooth and the string is light the string is inextensible | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \end{aligned}$ | Accept only 'the pulley is smooth'. | 2 |
| (ii) | Diagrams | B1 | All forces present with labels and arrows. Acc not reqd. | 1 |
|  | For X, N2L upwards $T-2 g=2 a$ <br> For Y, N2L downwards $4 g-T=4 a$ <br> Solve for $a$ and $T$ $\begin{aligned} a & =\frac{g}{3} \quad(3.27(3 \text { s. f. })) \\ T & \left.=\frac{8}{3} g \quad(26.1 \text { (3 s. f. })\right) \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { F1 } \end{aligned}$ | N2L. Allow $F=m g a$. All forces present Award for equation for X or Y or combined Any form <br> Any form <br> FT second answer | 5 |
|  | total | 8 |  |  |

Solutions and mark scheme

| Q3 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) | $\left(\begin{array}{c} x \\ -7 \\ z \end{array}\right)+\left(\begin{array}{c} 4 \\ y \\ -5 \end{array}\right)+\left(\begin{array}{c} 5 \\ 4 \\ -7 \end{array}\right)=\left(\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right)$ <br> Equating components gives $x=-9, y=3, z=12$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | [Allow SC $2 / 4$ if $9,-3,-12$ obtained] | 4 |
| (ii) | We need $\sqrt{5^{2}+4^{2}+(-7)^{2}}$ $=\sqrt{90}$ or $9.48683 \ldots$ so 9.49 (3 s. f.) | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | Any reasonable accuracy | 2 |
|  | total | 6 |  |  |


| Q4 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) | Height reached by first particle is given by $\begin{aligned} & 0=21^{2}-2 \times 9.8 \times s \\ & \text { so } s=22.5 \text { so } 22.5 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | Other methods must be complete. Allow $g= \pm 9.8, \pm 10$ <br> Accept with consistent signs | 2 |
| (ii) | Sol (1) <br> $t$ seconds after second particle projected its height is $15 t-4.9 t^{2}$ and the first particle has height $22.5-4.9 t^{2}$ ( or $21 t-4.9 t^{2}$ ) <br> either <br> Sub $t=1.5$ to show both have same value State height as 11.475 m <br> or $15 t-4.9 t^{2}=22.5-4.9 t^{2}$ <br> giving $t=1.5$ and height as 11.475 m | M1 <br> A1 <br> M1 <br> A1 <br> E1 <br> A1 <br> M1 <br> A1 | Allow $g= \pm 9.8, \pm 10$ <br> Allow $g= \pm 9.8, \pm 10$ <br> Award only if used correctly <br> (or sub $t=3.64$ into $21 t-4.9 t^{2}$ for $1^{\text {st }} \& t=1.5$ for $2^{\text {nd }}$ ) cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained. <br> Both. $t$ shown. Ht cao (to any reasonable accuracy) |  |
|  | Sol (2) $t$ seconds after second particle projected its height is $15 t-4.9 t^{2}$ and the first particle has fallen $4.9 t^{2}$ <br> Collide when $15 T-4.9 T^{2}+4.9 T^{2}=22.5$ so $T=1.5$ $H=22.5-4.9 \times 1.5^{2}=11.475 \mathrm{~m}$ | M1 <br> A1 <br> B1 <br> M1 <br> E1 <br> A1 | Allow $g= \pm 9.8, \pm 10$ <br> Or other correct method <br> cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained. | 6 |
|  | total | 8 |  |  |

## Solutions and mark scheme

| Q 5 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) |  | B1 | Different labels. All forces present with arrows in correct directions. Condone no angles. | 1 |
| (ii) | Using triangle of forces <br> Triangle isosceles so tension in BC is 400 N Tension in $B A$ is $2 \times 400 \times \cos 30=400 \sqrt{3} \mathrm{~N}$ (693 N, (3 s. f.)) | M1 <br> B1 <br> A1 <br> F1 | Attempt at triangle of forces. Ignore angles and arrows. Accept 90, 60, 30 triangle. <br> Triangle, arrows, labels and angles correct <br> cao <br> FT BC only <br> [If resolution used, M1 for 1 equn; M1 for $2^{\text {nd }}$ equn + attempt to elim; A1; F1. For M marks all forces present but allow $s \leftrightarrow c$ and sign errors. No extra forces. If Lami used: M1 first pair of equations in correct format, condone wrong angles. A1. M1 second pair in correct format, with correct angles.F1 FT their first answer if necessary.] | 4 |
| (iii) | Resolve at $B$ perpendicular to the line ABC <br> Weight has unbalanced component in this direction | E1 <br> E1 | Attempt to argue unbalanced force <br> Complete, convincing argument. <br> [or Resolve horiz and establish tensions equal E1 Resolve vert to show inconsistency. E1] | 2 |
|  | total | 7 |  |  |

Solutions and mark scheme

| Q6 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) | Area under curve $\begin{aligned} & 0.5 \times 2 \times 20+0.5 \times(20+10) \times 4+0.5 \times 10 \times 1 \\ & =85 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { B1 } \\ & \text { A1 } \end{aligned}$ | Attempt to find any area under curve or use const accn results <br> Any area correct (Accept 20 or 60 or 5 without explanation) cao | 3 |
| (ii) | $\frac{20-10}{4}=2.5$ <br> upwards | $\begin{aligned} & \hline \hline \text { M1 } \\ & \text { A1 } \\ & \text { B1 } \end{aligned}$ | $\Delta v / \Delta t$ <br> accept $\pm 2.5$ <br> Accept -2.5 downwards (allow direction specified by diagram etc). Accept 'opposite direction to motion'. | 3 |
| (iii) | $\begin{aligned} & v=-2.5 t+c \\ & v=20 \text { when } t=2 \\ & v=-2.5 \mathrm{t}+25 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ | Allow their $a$ in the form $v= \pm a t+c$ or $v= \pm a(t-2)+c$ <br> cao [Allow $v=20-2.5(t-2)$ ] <br> [Allow $2 / 3$ for different variable to $t$ used, e.g. $x$. Allow any variable name for speed] | 3 |
| (iv) | Falling with negligible resistance | E1 | Accept 'zero resistance', or 'no resistance' seen. | 1 |
| (v) | $\begin{aligned} & -1.5 \times 4+9.5 \times 2+7=20 \\ & -1.5 \times 36+9.5 \times 6+7=10 \\ & -1.5 \times 49+9.5 \times 7+7=0 \end{aligned}$ | E1 <br> E1 | One of the results shown <br> All three shown. Be generous about the 'show'. | 2 |
| (vi) | $\begin{aligned} & \int_{2}^{7}\left(-1.5 t^{2}+9.5 t+7\right) d t \\ & =\left[-0.5 t^{3}+4.75 t^{2}+7 t\right]_{2}^{7} \\ & =\left(-\frac{343}{2}+\frac{19 \times 49}{4}+49\right)-(-4+19+14) \\ & =81.25 \mathrm{~m} \end{aligned}$ | M1 <br> A1 <br> A1 <br> A1 <br> M1 <br> A1 <br> A1 | Limits not required <br> A1 for each term. Limits not required. Condone $+c$ <br> Attempt to use both limits on an integrated expression <br> Correct substitution in their expression including subtraction ( may be left as an expression). cao. | 7 |
|  | total | 19 |  |  |

## Solutions and mark scheme

| Q 7 |  | mark |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) | Horiz $\quad(40 \cos 50) t$ <br> Vert $\quad(40 \sin 50) t-4.9 t^{2}$ | B1 <br> M1 <br> A1 | Use of $s=u t+0.5 a t^{2}$ with $a= \pm 9.8$ or $\pm 10$. <br> Allow $u=40$. Condone $\mathrm{s} \leftrightarrow \mathrm{c}$. <br> Any form | 3 |
| (ii) | Need $(40 \sin 50) t-4.9 t^{2}=0$ <br> so $t=\frac{40 \sin 50}{4.9}$ <br> $=6.2534 \ldots$ so $6.253 \mathrm{~s}(3 \mathrm{~d} . \mathrm{p}$. <br> Range is $(40 \cos 50) \times 6.2534 \ldots$ <br> $=160.78 \ldots$ so 161 m ( $3 \mathrm{~s} . \mathrm{f}$. ) | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { E1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ | Equating their $y$ to zero. Allow quadratic $y$ only <br> Dep on $1^{\text {st }} \mathrm{M} 1$. Attempt to solve. <br> Clearly shown <br> [or M1 (allow $u=40$ and $\mathrm{s} \leftrightarrow \mathrm{c}$ ) A1 time to greatest height; E1] <br> Use of their horiz expression <br> Any reasonable accuracy | 5 |
| (iii) | Time AB is given by <br> $(40 \cos 50) T=30$ so $T=1.16679 \ldots$ so 1.17 s <br> then <br> either <br> By symmetry, time AC is time AD - time AB <br> so time AC is $6.2534 \ldots-\frac{30}{40 \cos 50}$ $=5.086 \ldots$ so $5.09 \mathrm{~s}(3 \mathrm{~s} . \mathrm{f}$.) <br> or <br> height is $(40 \sin 50) T-4.9 T^{2}$ <br> and we need $(40 \sin 50) t-4.9 t^{2}=(40 \sin 50) T-4.9 T^{2}$ <br> solved for larger root <br> i.e. solve $4.9 t^{2}-(40 \sin 50) t+29.08712 \ldots=0$ for larger root giving 5.086... | M1 <br> A1 <br> M1 <br> A1 <br> M1 <br> A1 | Equating their linear $x$ to 30 . <br> Symmetry need not be explicit. Method may be implied. Any valid method using symmetry. <br> cao <br> Complete method to find time to second occasion at that height <br> cao | 4 |
| (iv) | $\begin{aligned} & \hat{x}=40 \cos 50 \\ & \hat{y}=40 \sin 50-9.8 \times 5.086 \ldots \end{aligned}$ <br> Need $\arctan \frac{\oint}{\mathfrak{x}}$ <br> So - $36.761 \ldots{ }^{\circ}$ <br> so $36.8^{\circ}$ below horizontal (3 s.f.) | B1 <br> M1 <br> A1 <br> M1 <br> A1 | Must be part of a method using velocities. <br> Use of vert cpt of vel Allow only sign error. <br> FT use of their 5.086.. <br> May be implied. Accept $\arctan \frac{\mathcal{\&}}{\& \&}$ but not use of $\&$. <br> Accept $\pm 36.8$ or equivalent. Condone direction not clear. | 5 |
|  | total | 17 |  |  |

