

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4768/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 A certain industrial process requires a supply of water. It has been found that, for best results, the mean water pressure in suitable units should be 7.8. The water pressure is monitored by taking measurements at regular intervals. On a particular day, a random sample of the measurements is as follows.

7.50 7.64 7.68 7.51 7.70 7.85 7.34 7.72 7.74

These data are to be used to carry out a hypothesis test concerning the mean water pressure.

- (i) Why is a test based on the Normal distribution not appropriate in this case? [2]
 - (ii) What distributional assumption is needed for a test based on the t distribution? [1]
 - (iii) Carry out a t test, with a 2% level of significance, to see whether it is reasonable to assume that the mean pressure is 7.8. [9]
 - (iv) Explain what is meant by a 95% confidence interval. [2]
 - (v) Find a 95% confidence interval for the actual mean water pressure. [4]
- 2 A particular species of reed that grows up to 2 metres in length is used for thatching. The lengths in metres of the reeds when harvested are modelled by the random variable X which has the following probability density function, $f(x)$.

$$f(x) = \begin{cases} \frac{3}{16} (4x - x^2) & \text{for } 0 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Sketch $f(x)$. [3]
 - (ii) Show that $E(X) = \frac{5}{4}$ and find the standard deviation of the lengths of the harvested reeds. [8]
 - (iii) Find the standard error of the mean length for a random sample of 100 reeds. [2]
- Once the harvested reeds have been collected, any that are shorter than 1 metre are discarded.
- (iv) Find the proportion of reeds that should be discarded according to the model. [2]
 - (v) Reeds are harvested from a large area which is divided into several reed beds. A sample of the harvested reeds is required for quality control. How might the method of cluster sampling be used to obtain it? [3]

- 3 In the manufacture of child car seats, a resin made up of three ingredients is used. The ingredients are two polymers and an impact modifier. The resin is prepared in batches. Each ingredient is supplied by a separate feeder and the amount supplied to each batch, in kg, is assumed to be Normally distributed with mean and standard deviation as shown in the table below. The three feeders are also assumed to operate independently of each other.

	Mean	Standard deviation
Polymer 1	2025	44.6
Polymer 2	1565	21.8
Impact modifier	1410	33.8

- (i) Find the probability that, in a randomly chosen batch of resin, there is no more than 2100 kg of polymer 1. [3]
- (ii) Find the probability that, in a randomly chosen batch of resin, the amount of polymer 1 exceeds the amount of polymer 2 by at least 400 kg. [4]
- (iii) Find the value of b such that the total amount of the ingredients in a randomly chosen batch exceeds b kg 95% of the time. [4]
- (iv) Polymer 1 costs £1.20 per kg, polymer 2 costs £1.30 per kg and the impact modifier costs £0.80 per kg. Find the mean and variance of the total cost of a batch of resin. [3]
- (v) Each batch of resin is used to make a large number of car seats from which a random sample of 50 seats is selected in order that the tensile strength (in suitable units) of the resin can be measured. From one such sample, the 99% confidence interval for the true mean tensile strength of the resin in that batch was calculated as (123.72, 127.38). Find the mean and standard deviation of the sample. [4]

[Question 4 is printed overleaf.]

- 4 (a) At a college, two examiners are responsible for marking, independently, the students' projects. Each examiner awards a mark out of 100 to each project. There is some concern that the examiners' marks do not agree, on average. Consequently a random sample of 12 projects is selected and the marks awarded to them are compared.

(i) Describe how a random sample of projects should be chosen. [2]

(ii) The marks given for the projects in the sample are as follows.

Project	1	2	3	4	5	6	7	8	9	10	11	12
Examiner A	58	37	72	78	67	77	62	41	80	60	65	70
Examiner B	73	47	74	71	78	96	54	27	97	73	60	66

Carry out a test at the 10% level of significance of the hypotheses $H_0: m = 0$, $H_1: m \neq 0$, where m is the population median difference. [8]

- (b) A calculator has a built-in random number function which can be used to generate a list of random digits. If it functions correctly then each digit is equally likely to be generated. When it was used to generate 100 random digits, the frequencies of the digits were as follows.

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	6	8	11	14	12	9	15	5	14	6

Use a goodness of fit test, with a significance level of 10%, to investigate whether the random number function is generating digits with equal probability. [8]

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Question			Answer	Marks	Guidance
1	(i)		A Normal test is not appropriate since the sample is small and ... the population variance is not known (it must be estimated from the data).	E1 E1 [2]	Allow use of “ σ ”, otherwise insist on “population”.
1	(ii)		The sample is taken from a Normal population.	B1 [1]	
1	(iii)		$H_0: \mu = 7.8$ $H_1: \mu \neq 7.8$ where μ is the mean water pressure. $\bar{x} = 7.631 \quad s = 0.1547$ Test statistic is $\frac{7.631 - 7.8}{\frac{0.1547}{\sqrt{9}}}$ $= -3.27(7).$ Refer to t_8 . Double-tailed 2% point is ± 2.896 . Significant. Sufficient evidence to suggest that the mean water pressure has changed.	B1 B1 B1 M1 A1 M1 A1 A1 A1 [9]	Both hypotheses. Hypotheses in words only must include “population”. Do NOT allow “ $\bar{X} = \dots$ ” or similar unless \bar{X} is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of “population” if correct notation μ is used. $s_n = 0.1459$ but do <u>NOT</u> allow this here or in construction of test statistic, but ft from there. Allow c’s \bar{x} and/or s_{n-1} . Allow alternative: $7.8 + (c's - 2.896) \times 0.1547/\sqrt{9}$ (= 7.65...) for subsequent comparison with \bar{x} . (Or $\bar{x} - (c's - 2.896) \times 0.1547/\sqrt{9}$ (= 7.78...) for comparison with 7.8.) c.a.o. but ft from here in any case if wrong. Use of $\mu - \bar{x}$ scores M1A0. No ft from here if wrong. Must compare test statistic with <u>minus</u> 2.896 unless absolute values are being compared. No ft from here if wrong. Allow $P(t < -3.27(7) \text{ or } t > 3.27(7)) = 0.0113$ for M1A1. ft only c’s test statistic if both M’s scored. ft only c’s test statistic if both M’s scored. Conclusion in context to include “average” o.e.

Question			Answer	Marks	Guidance
1	(iv)		In repeated sampling, 95% of all confidence intervals constructed in this way will contain the true mean.	E1 E1 [2]	
1	(v)		CI is given by $7.631 \pm$ 2.306 $\times \frac{0.1547}{\sqrt{9}}$ $= 7.631 \pm 0.118(9) = (7.512, 7.750)$	M1 B1 M1 A1 [4]	ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_8 is OK. Allow c's \bar{x} . 2.306 seen. Allow c's s_{n-1} . c.a.o. Must be expressed as an interval.
2	(i)			G1 G1 G1 [3]	Curve with positive gradient, through the origin and in the first quadrant only. Correct shape for an inverted parabola ending at maximum point. End point (2, 3/4) labelled.

Question		Answer	Marks	Guidance
2	(ii)		M1	Correct integral for $E(X)$ with limits (which may appear later).
		$E(X) = \frac{3}{16} \int_0^2 (4x^2 - x^3) dx$		
		$= \frac{3}{16} \left[\frac{4x^3}{3} - \frac{x^4}{4} \right]_0^2$	M1	Correctly integrated. Dep on previous M1.
		$= \frac{3}{16} \left\{ \left(\frac{32}{3} - \frac{16}{4} \right) - 0 \right\}$		
		$= \frac{5}{4}$	A1	Limits used correctly to obtain PRINTED ANSWER (BEWARE) convincingly. Condone absence of “-0”.
		$E(X^2) = \frac{3}{16} \int_0^2 (4x^3 - x^4) dx$	M1	Correct integral for $E(X)$ with limits (which may appear later).
		$= \frac{3}{16} \left[x^4 - \frac{x^5}{5} \right]_0^2$	M1	Correctly integrated. Dep on previous M1.
		$= \frac{3}{16} \left\{ \left(16 - \frac{32}{5} \right) - 0 \right\}$		
2	(iii)	$= \frac{9}{5}$	A1	Limits used correctly to obtain result. Condone absence of “-0”.
		$\text{Var}(X) = \frac{9}{5} - \left(\frac{5}{4} \right)^2 = \frac{19}{80}$	M1	Use of $\text{Var}(X) = E(X^2) - E(X)^2$.
		$\text{sd} = \sqrt{\frac{19}{80}} = 0.487(3)$	A1	cao
			[8]	
			M1	
		$\text{SE}(\bar{X}) = \frac{0.487}{\sqrt{100}}$		
		$= 0.0487$	A1	ft c's $\sigma/10$.
			[2]	

Question			Answer	Marks	Guidance
3	(ii)		Require $P(P1 - P2 > 400)$ $P1 - P2 \sim (2025 - 1565 = 460,$ $44.6^2 + 21.8^2 = 2464.4)$ $P(\text{this} > 400) =$ $P\left(Z > \frac{400 - 460}{\sqrt{2464.4}} = -1.208(6)\right) = 0.8864/5$	M1 B1 B1 A1 [4]	Mean. Variance. Accept sd (= 49.64). cao
3	(iii)		$T = P1 + P2 + I \sim N(5000,$ $\sigma^2 = 44.6^2 + 21.8^2 + 33.8^2 = 3606.84)$ Require b s.t. $P(T > b) = 0.95$ $\therefore \frac{b - 5000}{\sqrt{3606.84}} = -1.645$ $\therefore b = 5000 - 1.645 \times \sqrt{3606.84} = 4901.2..$	B1 B1 B1 A1 [4]	Mean. Variance. Accept sd (= 60.056...). -1.645 seen. c.a.o.
3	(iv)		Mean = $(1.2 \times 2025) + (1.3 \times 1565) +$ $(0.8 \times 1410) = \pounds 5592.50$ Var = $(1.2^2 \times 44.6^2) + (1.3^2 \times 21.8^2) +$ $(0.8^2 \times 33.8^2) = 4398.7076 \approx \pounds^2 4399$	B1 M1 A1 [3]	Condone absence of £. Use of at least one of $(1.2^2 \times 44.6^2)$ etc... Condone absence of £ ² .
3	(v)		Mean = $(123.72 + 127.38)/2 = 125.55$ $s = \frac{127.38 - 125.55}{2.576/\sqrt{50}} = 5.02(3)$	B1 B1 M1 A1 [4]	Cao Sight of 2.576. Or equivalent. cao

Question			Answer	Marks	Guidance
4	(b)		H ₀ : The random number function is performing as it should. H ₁ : The random number function is not performing as it should.	B1	Both hypotheses. Must be the right way round. Allow use of the uniform distribution/model. Do not accept “data fit model” oe.
			All expected frequencies are 10	B1	
			$X^2 = 1.6 + 0.4 + 0.1 + 1.6 + 0.4 + 0.1 + 2.5 + 2.5 + 1.6 + 1.6$ $= 12.4$	M1 A1	Calculation of X^2 . c.a.o.
			Refer to χ^2_9 .	M1	Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. $P(X^2 > 12.4) = 0.1916$.
			Upper 10% point is 14.68.	A1	No ft from here if wrong.
			Not significant.	A1	ft only c’s test statistic.
			Insufficient evidence to suggest that the random number function is not performing as it should.	A1	ft only c’s test statistic. Conclusion in context. Allow in terms of the uniform distribution/model. Do not accept “data fit model” oe.
				[8]	

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General Comments

As might be expected on a paper at this level, the scripts indicated that most candidates knew what they were doing most of the time. In addition, there were very few scripts which showed evidence of candidates running out of time. That being the case, it is disappointing to report that a large number of scripts suffered from a lack of precision that manifested itself in a number of ways across the paper. Examples include final answers being given to more than 5 significant figures, not enough accuracy being used in calculations, hypotheses and conclusions being given without context, conclusions to hypothesis tests being too assertive, and other examples which will be commented on below. The cumulative effect of these errors was significant for many candidates.

Comments on Individual Questions

- 1** Water pressure - t test
- 1(i)(ii)** Well understood by most candidates, but a lack of precision meant that some candidates did not state that it was the *population* variance that was unknown, and others stated that the data had to be Normally distributed.
- (iii)** The hypotheses were usually well stated. A few candidates did not define μ , and a few gave a description without context. Of the small proportion who gave their hypotheses in words, the majority used mean rather than population mean. It was pleasing to see the overwhelming majority of candidates correctly opted to use s_{n-1} rather than s_n , but a number of candidates used a truncated value for \bar{x} in the calculation and lost accuracy as a result. Virtual all candidates correctly calculated the test statistic. The correct point of t_8 was usually used, although some candidates opted for t_9 . It was not always possible to see if this was through a misunderstanding or a misreading of the table. Most candidates correctly rejected the null hypothesis, but too many gave conclusions which were too assertive or lacked context.
- (iv)(v)** A majority of candidates knew the meaning of a 95% confidence interval, although some definitions were clearer than others. A minority of candidates gave definitions in terms of just one interval and others had a definition which included the capture of sample means. The calculation of the confidence interval was well done – a few candidates used 1.96 or 2.326 and some changed to t_9 .
- 2** Reed beds - probability density function
- (i)** A wide range of sketches was seen. A fully correct sketch was the most common outcome, but some sketches extended the parabola well beyond the defined range, others clearly did not have zero gradient at $x = 2$. Some sketches were unlabelled, and some had an increasing, rather than a decreasing gradient. A few sketches reached the maximum point at $x = 2$ and then continued with a horizontal line.
- (ii)** This was extremely well done by almost all candidates. Apart from a few arithmetic slips, the only errors which occurred were presenting $E(X^2)$ as the variance, and forgetting to take the square root for the final answer.

- (iii) A large number of candidates did not know the meaning of the term *standard error*. Many gave an interval as their response and a significant number did not respond at all.
- (iv) This was well done by most candidates, but occasionally the wrong limits were seen and there were some attempts at Normal approximations.
- (v) Only a minority of candidates gained full marks here. Defining the reed beds as clusters was rarely seen and many candidates talked about clusters of reeds. Many responses lacked context.

3 Child car seat - linear combinations of Normal distributions

- (i) This part was almost invariably correct. Virtually all of those who did not score full marks here had selected the wrong tail.
- (ii) This part was again almost invariably correct.
- (iii) Most candidates were able to calculate the mean and variance of the distribution correctly. Most of these candidates then correctly identified the correct z value of -1.645 . The most common error was the omission of the minus sign, but 1.96 was occasionally seen. Many candidates gave answers to 6 or more significant figures.
- (iv) Virtually all candidates correctly calculated the mean cost. Many candidates also knew how to calculate the variance, but a few used multipliers of 1.2, 1.3 and 0.8 instead of their squares. Many variances were given to 6 significant figures, and even 8 significant figures were regularly seen.
- 3(v) Many candidates were able to find the mean and standard deviation, but often not very efficiently. The correct value of 2.576 was usually used, but various other z values were also seen. A surprising number of candidates were unable to multiply both sides of their equation by $\sqrt{50}$ correctly.

4 Wilcoxon paired test and goodness of fit test

- (i) Some candidates simply gave a definition of a random sample and others described other sampling methods, but most candidates gave a correct description. It was surprising how many candidates wanted to pick numbers out of a hat as a method of random selection. Surely, at this level, a random number generator is a better choice.
- (ii) This question was done extremely well by most candidates. Very few errors were made in calculating the differences and the ranks. The Wilcoxon statistic was almost always correctly calculated. Virtually all candidates gave the correct critical value of 17. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.
- (b)(iii) This question was well done by most candidates. Most candidates gave acceptable hypotheses and were able to calculate X^2 correctly. A very few candidates confused expected and observed values or combined groups. The correct point of the chi squared distribution was usually quoted, although 8, 11 and 12 degrees of freedom were all seen. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.

- 6** The LP question was generally done reasonably well. The summer 2012 report on 4771 contained the following quotation: “Far too many candidates, if they remembered to define their variables, neglected that essential phrase “the number of ...”. The issue remains live! Again, in this examination, far too many candidates failed explicitly to define their variables. The phrase “x is hats”, and variants of it, scored zero.
- The insistence on the phrase “number of ...” secures the definition of units in the case of continuous variables, eg “Let x be the number of litres of ...”, and points to the need for integer values in other cases. In this question most candidates failed adequately to deal with that integer requirement in part (ii). The majority of candidates were happy to round the LP solution to (13, 18). Few looked at nearby lattice points, and only a handful found the optimal integer point, (12, 19).