OCR
Oxford Cambridge and RSA

Wednesday 15 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4767/01 Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4767/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A researcher believes that there may be negative association between the quantity of fertiliser used and the percentage of the population who live in rural areas in different countries. The data below show the percentage of the population who live in rural areas and the fertiliser use measured in kg per hectare, for a random sample of 11 countries.

Percentage of population	33	6	58	35	81	69	61	7	74	71	17
Fertiliser use	76	44	6	68	3	10	7	176	5	137	157

(i) Draw a scatter diagram to illustrate the data. **[3]**

(ii) Explain why it might not be valid to carry out a test based on the product moment correlation coefficient in this case. **[2]**

(iii) Calculate the value of Spearman's rank correlation coefficient. **[5]**

(iv) Carry out a hypothesis test at the 1% significance level to investigate the researcher's belief. **[6]**

(v) Explain the meaning of '1% significance level'. **[1]**

(vi) In order to carry out a test based on Spearman's rank correlation coefficient, what modelling assumptions, if any, are required about the underlying distribution? **[1]**

2 When a genetic sequence of plant DNA is given a dose of radiation, some of the genes may mutate. The probability that a gene mutates is 0.012. Mutations occur randomly and independently.

(i) Explain the meanings of the terms 'randomly' and 'independently' in this context. **[2]**

A short stretch of DNA containing 20 genes is given a dose of radiation.

(ii) Find the probability that exactly 1 out of the 20 genes mutates. **[2]**

A longer stretch of DNA containing 500 genes is given a dose of radiation.

(iii) Explain why a Poisson distribution is an appropriate approximating distribution for the number of genes that mutate. **[2]**

(iv) Use this Poisson distribution to find the probability that there are

(*A*) exactly two genes that mutate, **[3]**

(*B*) at least two genes that mutate. **[2]**

A third stretch of DNA containing 50 000 genes is given a dose of radiation.

(v) Use a suitable approximating distribution to find the probability that there are at least 650 genes that mutate. **[5]**

3 Many types of computer have cooling fans. The random variable X represents the lifetime in hours of a particular model of cooling fan. X is Normally distributed with mean 50 600 and standard deviation 3400.

(i) Find $P(50\,000 < X < 55\,000)$. **[3]**

(ii) The manufacturers claim that at least 95% of these fans last longer than 45 000 hours. Is this claim valid? **[3]**

(iii) Find the value of h for which 99.9% of these fans last h hours or more. **[3]**

(iv) The random variable Y represents the lifetime in hours of a different model of cooling fan. Y is Normally distributed with mean μ and standard deviation σ. It is known that $P(Y < 60\,000) = 0.6$ and $P(Y > 50\,000) = 0.9$. Find the values of μ and σ. **[5]**

(v) Sketch the distributions of lifetimes for both types of cooling fan on a single diagram. **[4]**

4 **(a)** A random sample of 80 GCSE students was selected to take part in an investigation into whether attitudes to mathematics differ between girls and boys. The students were asked if they agreed with the statement 'Mathematics is one of my favourite subjects'. They were given three options 'Agree', 'Disagree', 'Neither agree nor disagree'. The results, classified according to sex, are summarised in the table below.

	Agree	Disagree	Neither
Male	17	13	8
Female	12	11	19

The contributions to the test statistic for the usual χ^2 test are shown in the table below.

	Agree	Disagree	Neither
Male	0.7550	0.2246	1.8153
Female	0.6831	0.2032	1.6424

(i) Calculate the expected frequency for females who agree. Verify the corresponding contribution, 0.6831, to the test statistic. **[3]**

(ii) Carry out the test at the 5% level of significance. **[6]**

(b) The level of radioactivity in limpets (a type of shellfish) in the sea near to a nuclear power station is regularly monitored. Over a period of years it has been found that the level (measured in suitable units) is Normally distributed with mean 5.64. Following an incident at the power station, a researcher suspects that the mean level of radioactivity in limpets may have increased. The researcher selects a random sample of 60 limpets. Their levels of radioactivity, x (measured in the same units), are summarised as follows.

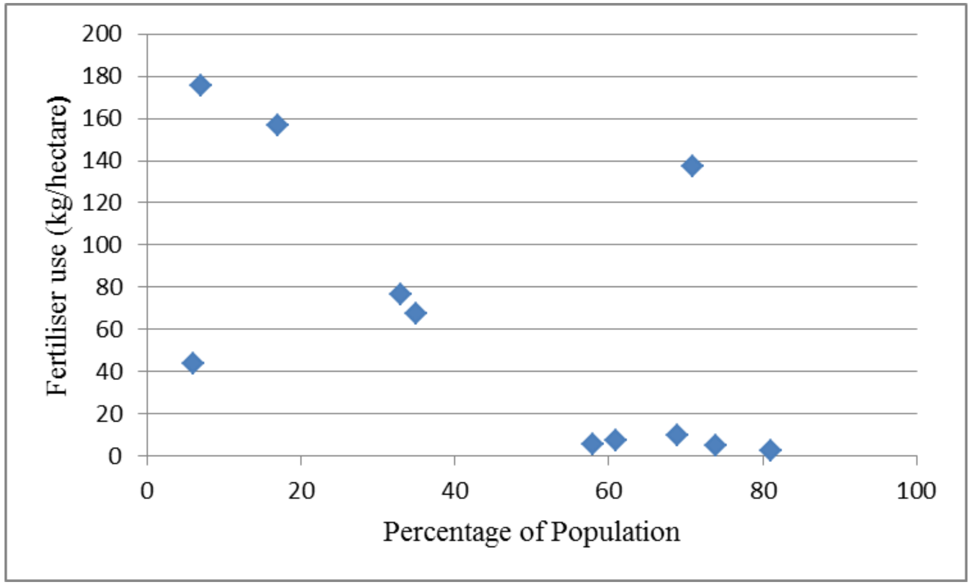
$$\sum x = 373 \qquad \sum x^2 = 2498$$

Carry out a test at the 5% significance level to investigate the researcher's belief. **[11]**

END OF QUESTION PAPER

1 (i)

1 (ii)

Question		Answer	Marks	Guidance
1	(i)		G1 G2,1,0 [3]	For suitably labelled axes. Condone absence of scale here. G2 for 11 points correctly plotted relative to a suitable **linear** scale. G1 if 9 or 10 correctly plotted. G0 if 3 or more incorrectly plotted/omitted Allow axes interchanged
1	(ii)	(The points in the scatter diagram) do not appear to be roughly elliptical. The **population** may not have a bivariate Normal distribution.	E1 E1 [2]	For "not elliptical". For not **underlying** bivariate Normal. Do not allow "the data" in place of population/underlying. Allow "data is not **from** a bivariate Normal distribution". Do not allow Normal bivariate ….
1	(iii)	See table below	M1 M1	For ranking (allow ranks reversed) **NB No ranking scores 0/5** For d^2

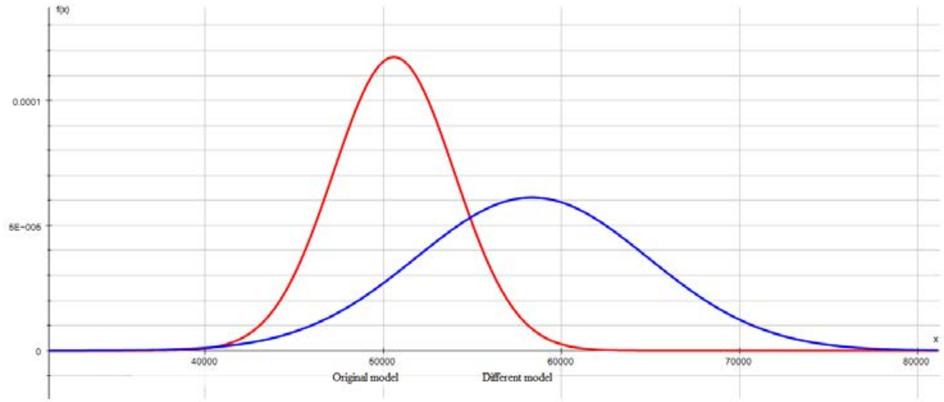
Percentage	33	6	58	35	81	69	61	7	74	71	17
Fertiliser use	76	44	6	68	3	10	7	176	5	137	157
Rank percentage	4	1	6	5	11	8	7	2	10	9	3
Rank Fertiliser	8	6	3	7	1	5	4	11	2	9	10
d	4	5	-3	2	-10	-3	-3	9	-8	0	7
d^2	16	25	9	4	100	9	9	81	64	0	49

Question			Answer	Marks	Guidance	
			$\Sigma d^2 = 366$	A1	For Σd^2 (May be embedded in the calculation)	
			$r_s = 1 - \dfrac{6\sum d^2}{n(n^2-1)} = 1 - \dfrac{6\times366}{11\times120} = 1 - \dfrac{2196}{1320} = 1 - 1.6636$	M1	For method for r_s	
			= - 0.664 (to 3 s.f.) [*allow* -0.66 to 2 s.f. or -73/110]	A1 **[5]**	FT their Σd^2 provided $-1 < r_s < 0$, and ranking used. **NB No ranking scores 0/5**	
1	**(iv)**		H_0: no association between percentage of population living in rural areas and fertiliser use (in the population of countries)	B1	For null hypothesis in context **NB** H_0 H_1 <u>not</u> ito ρ.	
			H_1: **negative** association between percentage of population living in rural areas and fertiliser use (in the population of countries)	B1	For alternative hypothesis in context. Context needed in at least one of the hypotheses.	
				B1	For **population of countries** or **underlying population**.	
			One tail test critical value at 1% level is -0.7091	B1	For ±0.7091 **No further marks from here if incorrect**.	
			Since - 0.664 > -0.7091 [or 0.664 < 0.7901] there is …	M1	For sensible comparison of their "- 0.664" with ±0.7091 **seen**, leading to conclusion, only if $-1 <$ their $r_s < 0$.	
			…insufficient evidence to reject H_0. There is insufficient evidence to suggest that there is **negative** association between percentage of population living in rural areas and fertiliser use (in the population of countries)	A1 **[6]**	for not significant, oe, and correct conclusion in context. FT their r_s with correct cv.	

Question			Answer	Marks	Guidance	
1	(v)		It means that the probability of rejecting H_0 given that it is correct is 1% o.e.	E1 **[1]**	Allow "the probability of a false positive is 1%", "the probability of a Type I Error is 1%". Do not allow "It means that the probability rejecting H_0 when it should have been accepted is 1%"	
1	(vi)		None	E1 **[1]**		
2	(i)		'Randomly' means that **mutations** occur with no (predictable) pattern. 'Independently' means that the occurrence of one **mutation** does not affect the **probability** of another mutation occurring.	E1 E1 **[2]**	In context. Allow "not predictable" Must include 'probability' and context. Allow "chance". If not indicated, assume first comment relates to randomness.	
2	(ii)		P(Exactly one) = $\binom{30}{29} \times 0.85^{29} \times 0.15^{1}$ $^{20}C_1 \times 0.012^1 \times 0.988^{19}$ = 0.1908	M1 A1 **[2]**	For correct structure i.e. $20p(1-p)^{19}$ Allow 0.191. Allow 0.19 www.	
2	(iii)		Because the number of mutating genes/X is binomially distributed n is large and p is small.	E1 E1 **[2]**	Allow B(500, 0.012) or B(n, p). Allow the sample is large & $np \approx np(1-p)$ or np not too large. Condone suitable numerical ranges – e.g. $n > 30$, $p < 0.1$ Do not allow "the number is large and probability is small". Allow "probability of success/a gene mutating is small" for p is small	
2	(iv)	(*A*)	$\lambda = 500 \times 0.012 = 6$ P(2 mutations) = $e^{-0.85}\frac{0.85^1}{1!}$ [0.0620 – 0.0174 from tables] = 0.0446	B1 M1 A1 **[3]**	For mean Correct structure for P(= 2) using Poisson pdf or tables. CAO Allow 0.04462 or 0.045www	

Question			Answer	Marks	Guidance
2	(iv)	(*B*)	From tables P(At least two) = 1 – P(≤1) = 1 – 0.0174 = 0.9826	 M1 A1 **[2]**	 For using 1 – P(≤ 1) using their mean. CAO Allow 0.983. Allow 0.98 www.
2	(v)		Mean 50000 × 0.012 = 600 , Var = 50000 × 0.012 × 0.988 = 592.8 Using Normal approx. to the binomial, $X \sim N(600, 592.8)$ $P(X \geq 650) = P\left(Z \leq \frac{30.5-25.5}{\sqrt{25.5}}\right)$ $= P(Z > 2.033) = 1 - \Phi(2.033) = 1 - 0.9789$ = 0.0211	 B1 B1 B1 M1 A1 **[5]**	 For Normal approximation (SOI). For correct parameters (SOI). For 649.5 For standardisation and probability calculation using correct tail. CAO (Allow answer from calculator 0.0210)
2	(v)		**Alternative solution using Normal approx. to Poisson** Mean 100 × 6 = 600 Using Normal approx. to the Poisson, $X \sim N(600, 600)$ $P(X \geq 650) = P\left(Z \leq \frac{30.5-25.5}{\sqrt{25.5}}\right)$ $= P(Z > 2.021) = 1 - \Phi(2.021) = 1 - 0.9783$ = 0.0217	 B1 B1 B1 M1 A1 **[5]**	 For Normal approximation (SOI). For correct parameters (SOI). For 649.5 For standardisation and probability calculation using correct tail. CAO (Allow answer from calculator 0.0216)
3	(i)		$P(50000 < X < 55000) =$ $P\left(Z \geq \frac{750-751.4}{2.5}\right)\left(\frac{50000-50600}{3400} < Z < \frac{55000-50600}{3400}\right)$ $= P(-0.176 < Z < 1.294) = \Phi(1.294) - (1 - \Phi(0.176)) = 0.9022 - 1 + 0.5699$ = 0.4721	M1 M1 A1 **[3]**	For standardising both. SOI. Penalise erroneous continuity corrections and wrong sd. Condone numerator(s) reversed. For correct structure Φ(positive z) - Φ(negative z) CAO including use of difference tables (Answer from calculator 0.4722 and from tables interpolated 0.4723)

Question			Answer	Marks	Guidance	
3	(ii)		P(X > 45000) = P$\left(Z \geq \frac{750-751.4}{2.5}\right)\left(Z > \frac{45000-50600}{3400}\right)$ = P(Z > -1.647) = Φ(1.647) = 0.9502 0.9502 > 95% so agree with claim	B1* B1* depE1* **[3]**	For -1.647 or -Φ^{-1}(0.95) = -1.645 or 1.647 seen with P(X < 56200) or numerator reversed For 0.9502 or 45007 or 0.0498, or B1 for -1.645 if B1 for -1.647 already awarded. **For comparison seen** e.g. -1.647 < -1.645 or 0.0498 < 0.05 or 1.647 > 1.645 or 95% last longer than 45007 hours, **and correct conclusion**. Dependent on B1, B1 awarded	
3	(iii)		From tables Φ^{-1} (0.999) = 3.09 $\frac{h-50600}{3400} = -3.09$ $k = 50600 - (3.09 \times 3400)$ = 40100 www	B1 M1 A1 **[3]**	±3.09 seen For equation as seen with their negative z-value CAO Allow 40094, 40090	

Question		Answer	Marks	Guidance
3	(iv)	$\text{P}(Y < 60000) = 0.6 \Rightarrow \text{P}\left(Z < \dfrac{60000-\mu}{\sigma}\right) = 0.6$ $\Rightarrow \dfrac{60000-\mu}{\sigma} = \Phi^{-1}(0.6) = 0.2533$ $\Rightarrow 60000 = \mu + 0.2533\sigma$ $\text{P}(Y > 50000) = 0.9 \Rightarrow \text{P}\left(Z > \dfrac{50000-\mu}{\sigma}\right) = 0.9$ $\Rightarrow \dfrac{50000-\mu}{\sigma} = \Phi^{-1}(0.1) = -1.282$ $\Rightarrow 50000 = \mu - 1.282\sigma$ $1.5353\sigma = 10000$ $\sigma = 6513$ $\Rightarrow \mu = 50000 + (1.282 \times 6513) = 58350$	B1 M1 A1 A1 A1 **[5]**	For ±0.2533 or ±1.282 seen For an equation ito μ, σ, *z* and *y* formed. NB using *z* = ±0.2533 with *y* = 60 000 or ±1.282 with *y* = 50 000 For two correct equations seen. CAO Allow 6510, 6515 CAO Allow 58400
3	(v)		G1 G1 G1 G1 **[4]**	For two Normal shapes including attempt at asymptotic behaviour with horizontal axis at each of the four ends. Penalise clear asymmetry. For means, shown explicitly or by scale **on a single diagram.** If shown explicitly, the positions must be consistent with horizontal scale if present. FT part (iv). For greater width (variance) for Different model. FT part (iv). For lower max height for Different model. FT part (iv) If not labelled assume the larger mean represents Different model. FT part(iv).

Question			Answer	Marks	Guidance
4	**(a)**	**(i)**	Expected frequency = 42/80 × 29 = 15.225 Contribution = $(12 – 15.225)^2$ / 15.225 (= 0.6831 AG)	B1 M1 A1 **[3]**	for 15.225 For valid attempt at $(O\text{-}E)^2/E$ leading to correct answer. ***NB Answer given***
4		**(ii)**	H_0: no association between sex and attitude to Mathematics. H_1: some association between sex and attitude to Mathematics. Test statistic X^2 = 5.3236 Refer to χ_2^2 Critical value at 5% level = 5.991	B1 B1 B1 B1	For correct hypotheses in context (with context seen in at least one hypothesis). NB if H_0 H_1 reversed do not award first B1or final A1. Allow hypotheses expressed in terms of independence, and in context. Allow 5.324 or 5.32 Allow "2 degrees of freedom" or ν = 2 seen. No further marks from here if wrong or omitted.
			(5.3236 < 5.991 so result is) not significant There is insufficient evidence **to suggest** that there is association <u>between sex and attitude to Mathematics</u>	M1 A1 **[6]**	For not significant oe. FT their test statistic. Allow 'Accept H_0' or 'Reject H_1' For **non-assertive** conclusion <u>in context</u> FT their test statistic. Do not allow "relationship" or "correlation" for "association".

Question			Answer	Marks	Guidance
4	(b)		$\bar{x} = 373/60 \quad = 6.217$	B1	Allow 6.22
			$s = \sqrt{\dfrac{2498-(373)^2/60}{59}} = \sqrt{\dfrac{179.183}{59}}$	M1	For correctly structured calculation (divisor = 59) for the sample standard deviation or variance.
			$= \sqrt{3.0370} = 1.743$	A1	Allow answers which round to 1.74
			H_0: $\mu = 5.64$; H_1 : $\mu > 5.64$	B1	For both hypotheses correct.
			Where μ denotes the **mean** radioactivity level in (the population of) limpets	B1	For definition of μ in context. Do not allow other symbols unless clearly defined as population mean.
			Test statistic $= \dfrac{6.217-5.64}{1.743/\sqrt{60}} = \dfrac{0.5767}{0.2250}$ $\dfrac{4.995-5.0}{0.0072/\sqrt{8}} = -\dfrac{0.005}{0.002546} = -1.964$	M1*	Structure of test statistic using their sd and mean. Must include correct use of √60. Do not condone numerator reversed.
			= 2.563	A1	Allow answers between 2.56 and 2.57 inclusive.
			Upper 5% level 1 tailed critical value of z = 1.645	B1	For 1.645 No further marks from here if wrong.
			2.563 > 1.645 The result is…	depM1*	For sensible comparison leading to a conclusion (even if incorrect). FT their test statistic.
			…significant. There is sufficient evidence to reject H_0	A1	Correct conclusion. FT their test statistic.
			There is sufficient evidence **to suggest** that the **mean** level of radioactivity has increased.	A1 **[11]**	For correct **non-assertive** conclusion in words in context. FT their test statistic.

Additional Notes on Sensible Comparisons

e.g. In Q4 (b) Neither 2.563 > 0.05 nor 0.0052 < 2.326 are considered sensible as each compares a z-value with a probability.
- 2.563 < 1.645 is not considered to be sensible.
For 2.563 < 1.645 leading to a conclusion, award M0 A0.

Additional Notes on Conclusions to Hypothesis Tests

The following are examples of conclusions which are considered too assertive.

There is sufficient evidence to reject H_0 and **conclude** that…
"there is a positive association between…" or
"there seems to be evidence that there is a positive association between…" or
"the mean level of radioactivity is greater …."
"there doesn't appear to be association between…"

Also note that final conclusions **must refer to H_1 in context** for the final mark to be given.
e.g. In Q4 (a) part (ii), a conclusion just stating that "there is insufficient evidence to suggest that there is an association" gets A0 as this does not refer to the context.

Additional Notes on Alternative Methods in Q4 (b)

Critical value method	cv = 5.64 + 1.645 × 1.743÷ √60	gets M1* (structure) FT their sd. B1 for 1.645 used (otherwise B0M0A0A0)
	= 6.01	gets A1 (replacing the A1 for 2.563)
	6.217 > 6.01	gets depM1* if a conclusion is made, FT their mean only if 1.645 used. Then A1, A1 available as before.
Probability Method	P(Z > 2.563) = 0.0052	gets B1 for value rounding to 0.005 which replaces the B1 for 1.645 (otherwise B0depM0*A0A0).
	0.005 < 0.05	gets depM1* if a conclusion is made only if B1for 0.005 has been awarded. Then A1, A1 available as before.
		NOTE Condone B1 for 0.995 obtained from P(Z < 2.563) only if compared with 0.95 at which point the final depM1*A1A1are available. B0depM0*A0A0 if 0.995 obtained from P(z > -2.563).

4767 Statistics 2

General Comments:

The vast majority of candidates appeared to be well prepared for this examination. The overall performance was very good and the average score is once again very high. In hypothesis tests, most candidates provided appropriately worded hypotheses and conclusions. Most candidates were able to complete required calculations correctly and with suitable working provided. Over-specified answers were present though many candidates managed to choose suitable degrees of accuracy for their final answers. Most of the candidates with access to more advanced calculators managed to provide sufficient detail in their solutions to be awarded full credit.

Comments on Individual Questions:

Question No. 1

(i) The scatter diagram was well drawn by many, with some choosing more manageable scales than others.

(ii) The key points concerning the absence of any discernible elliptical shape and the corresponding questioning of the underlying bivariate Normal population were handled well by many. Though most candidates managed to comment on the lack of an elliptical spread of points, often with poor spelling of ‘ellipse’, there were still many candidates who struggled to differentiate between data and population.

(iii) Many achieved full marks here. Only a few candidates reversed the ranking of one of the sets of values. Errors tended to involve mistakes in ranking, in adding the squares of the differences or in rounding the final answer. Very few candidates failed to rank the data.

(iv) This part proved to be challenging for many candidates, especially in the identification of the underlying population of countries involved. Inappropriately worded hypotheses, using ‘correlation’ in place of ‘association’, were seen in a few cases. Many realised the one-sided nature of the test which investigated the possible negative association and went on to use the appropriate critical value to obtain a suitable conclusion in context. Few candidates expressed the conclusion in terms of the null hypothesis.

(v) There were many correct answers to this part of the question, either in terms of incorrectly rejecting H_0 or one of the equivalents. Many answers referring to ‘accuracy’ or ‘reliability’ were seen.

(vi) Few candidates appeared to understand that for this test no modelling assumptions about the underlying distribution are required.

Question No. 2

This proved to be a very straightforward question with most candidates scoring high marks.

(i) Many candidates succeeded in defining ‘random’ and ‘independent’ but many failed to define independence in terms of probability.

(ii) Well answered. A minority of candidates used a Poisson calculation here.

(iii) Though most candidates identified the usual ‘*n* is large and *p* is small’, very few explicitly related these values to the binomial distribution.

(iv) (*A*) Well answered.

(*B*) Well answered.

(v) Well answered. Common errors tended to involve either use of an incorrect standard deviation or omission of the required continuity correction.

Question No. 3

There were many good responses to this question. Spurious continuity corrections were rarely seen. It helps when candidates provide sketches for questions involving the Normal distribution.

(i) Well answered. Errors caused by lack of accuracy reading Normal tables were seen fairly regularly. Most candidates used the correct probability structure with their z values.

(ii) This was well done on the whole though many candidates did not provide the required comparison to justify their conclusion. In most cases the working provided was clear – diagrams were helpful to examiners in conveying the candidates' intentions – often more successfully than their wording.

(iii) Many correctly identified the z value of -3.09 and went on to find the appropriate value for h, rounded to a suitable level of accuracy.

(iv) There were some pleasing attempts at this question, marred only by an inappropriate degree of accuracy for the final answers. It was good to see that once the initial equations had been established with the correct z values, many could still solve the simultaneous equations. A few candidates failed to identify, for the given probabilities, the z values needed for the simultaneous equations.

(v) This was answered well, though many candidates could have made a greater effort to include symmetry in their sketches and to pay more attention to the asymptotic nature. Spurious labelling of axes was seen but only rarely.

Question No. 4

Most candidates scored well on this question. Pleasingly, overly-assertive conclusions to the hypotheses tests were rarely seen.

(a) (i) Most candidates found this to be very easy.

(ii) Well answered. Some candidates failed to word their hypotheses and conclusion in terms of 'association'. Most stated the correct number of degrees of freedom and critical value. Most candidates were able to finish off with appropriately worded conclusions.

(b) With a little more background work to be done, finding sample mean and standard deviation, candidates found this part of the question more difficult than part (a). The calculation of sample standard deviation caused problems for many. Issues with premature rounding of sample mean and standard deviation, leading to inaccuracy in the calculation of the test statistic, were quite common. A small number of candidates did not express their hypotheses in terms of μ. Definitions of μ as 'sample mean' were, thankfully, rare. Given that most candidates provided the correct alternative hypothesis it was disappointing to see many working with a negative test statistic and critical value – these were deemed inappropriate and thus penalised. It was again pleasing to see candidates taking care to word conclusions in an appropriately non-assertive manner.