

Friday 25 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4767/01 Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4767/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 A manufacturer of playground safety tiles is testing a new type of tile. Tiles of various thicknesses are tested to estimate the maximum height at which people would be unlikely to sustain injury if they fell onto a tile. The results of the test are as follows.

Thickness (t mm)	20	40	60	80	100
Maximum height (h m)	0.72	1.09	1.62	1.97	2.34

- (i) Draw a scatter diagram to illustrate these data. [3]
- (ii) State which of the two variables is the independent variable, giving a reason for your answer. [1]
- (iii) Calculate the equation of the regression line of maximum height on thickness. [5]
- (iv) Use the equation of the regression line to calculate estimates of the maximum height for thicknesses of
- (A) 70 mm,
- (B) 120 mm.
- Comment on the reliability of each of these estimates. [4]
- (v) Calculate the value of the residual for the data point at which $t = 40$. [3]
- (vi) In a further experiment, the manufacturer tests a tile with a thickness of 200 mm and finds that the corresponding maximum height is 2.96 m. What can be said about the relationship between tile thickness and maximum height? [5]
- 2 John is observing butterflies being blown across a fence in a strong wind. He uses the Poisson distribution with mean 2.1 to model the number of butterflies he observes in one minute.
- (i) Find the probability that John observes
- (A) no butterflies in a minute, [2]
- (B) at least 2 butterflies in a minute, [2]
- (C) between 5 and 10 butterflies inclusive in a period of 5 minutes. [3]
- (ii) Use a suitable approximating distribution to find the probability that John observes at least 130 butterflies in a period of 1 hour. [5]
- In fact some of the butterflies John observes being blown across the fence are being blown in pairs.
- (iii) Explain why this invalidates one of the assumptions required for a Poisson distribution to be a suitable model. [1]
- John decides to revise his model for the number of butterflies he observes in one minute. In this new model, the number of pairs of butterflies is modelled by the Poisson distribution with mean 0.2, and the number of single butterflies is modelled by an independent Poisson distribution with mean 1.7.
- (iv) Find the probability that John observes no more than 3 butterflies altogether in a period of one minute. [5]

- 3 The amount of data, X megabytes, arriving at an internet server per second during the afternoon is modelled by the Normal distribution with mean 435 and standard deviation 30.

(i) Find

(A) $P(X < 450)$, [3]

(B) $P(400 < X < 450)$. [3]

- (ii) Find the probability that, during 5 randomly selected seconds, the amounts of data arriving are all between 400 and 450 megabytes. [2]

The amount of data, Y megabytes, arriving at the server during the evening is modelled by the Normal distribution with mean μ and standard deviation σ .

- (iii) Given that $P(Y < 350) = 0.2$ and $P(Y > 390) = 0.1$, find the values of μ and σ . [5]

- (iv) Find values of a and b for which $P(a < Y < b) = 0.95$. [4]

- 4 (a) A random sample of 60 students studying mathematics was selected. Their grades in the Core 1 module are summarised in the table below, classified according to whether they worked less than 5 hours per week or at least 5 hours per week. Test, at the 5% significance level, whether there is any association between grade and hours worked.

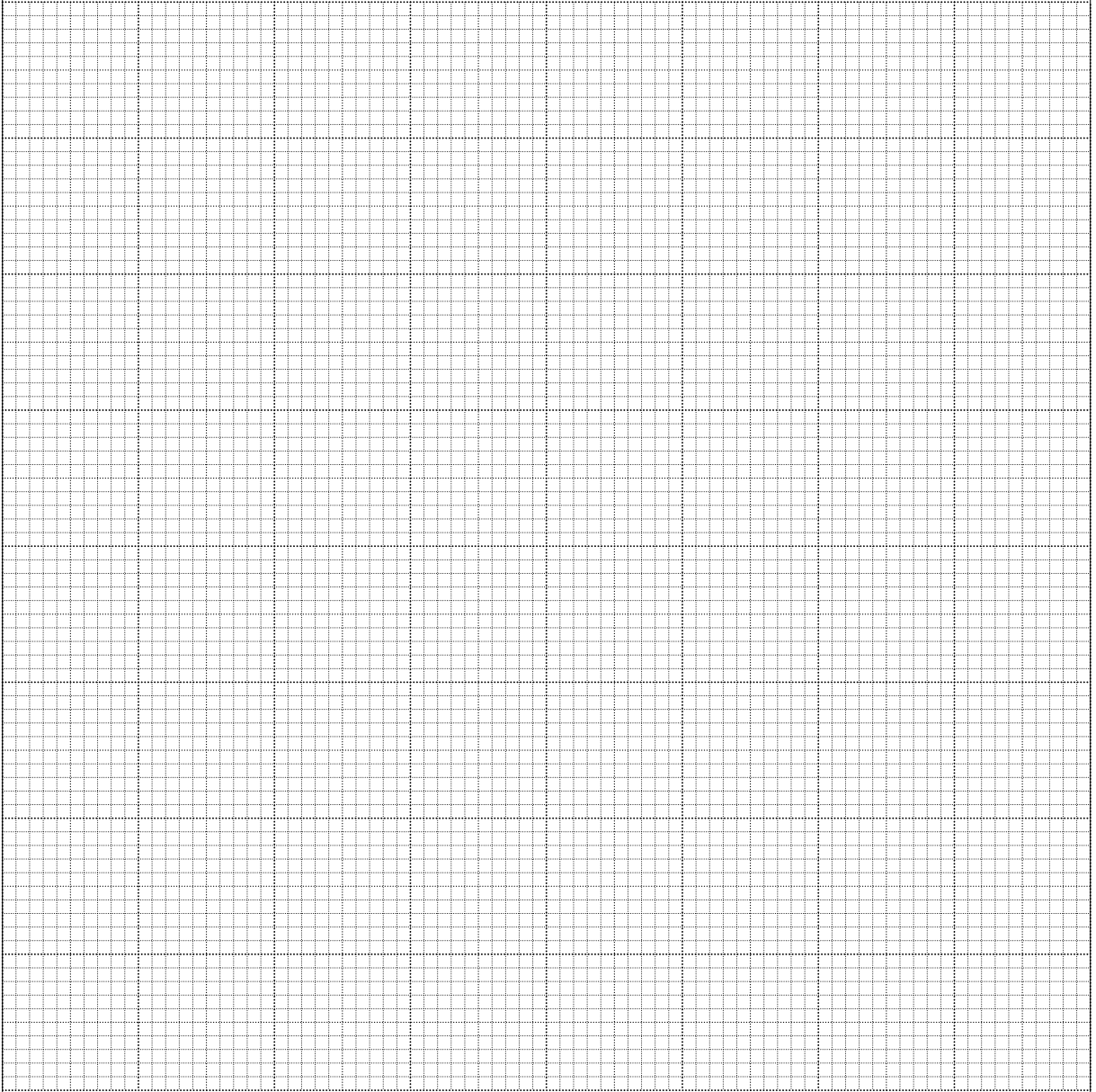
		Hours worked	
		Less than 5	At least 5
Grade	A or B	20	11
	C or lower	13	16

[9]

- (b) At a canning factory, cans are filled with tomato purée. The machine which fills the cans is set so that the volume of tomato purée in a can, measured in millilitres, is Normally distributed with mean 420 and standard deviation 3.5. After the machine is recalibrated, a quality control officer wishes to check whether the mean is still 420 millilitres. A random sample of 10 cans of tomato purée is selected and the volumes, measured in millilitres, are as follows.

417.2 422.6 414.3 419.6 420.4 410.0 418.3 416.9 418.9 419.7

Carry out a test at the 1% significance level to investigate whether the mean is still 420 millilitres. You should assume that the volumes are Normally distributed with unchanged standard deviation. [9]

1 (i)**1 (ii)**

Question		Answer	Marks	Guidance	
1	(i)		<p>G1</p> <p>G2,1,0</p> <p>[3]</p>	<p>G1 For axes suitably labelled with some indication of linear scale provided.</p> <p>G2 for points plotted correctly. G1 if 4 points plotted correctly. G0 if two or more incorrectly plotted/omitted points.</p> <p>Special Case SC1 for points visibly correct on axes where no indication of scale has been provided.</p>	<p>Allow x & y</p> <p>Allow axes reversed.</p>
1	(ii)	Thickness is the independent variable since the values of 'Thickness' are not subject to random variation, but are determined by the manufacturer.	<p>E1</p> <p>[1]</p>	<p>Allow explanations referring to thickness being "controlled" by the manufacturer. Allow equivalent interpretations.</p>	
1	(iii)	$\bar{t} = 60, \bar{h} = 1.548$ $b = \frac{S_{th}}{S_{tt}} = \frac{546.8 - (300 \times 7.74 / 5)}{22000 - 300^2 / 5} = \frac{82.4}{4000} = 0.0206$	<p>B1</p> <p>M1*</p> <p>A1</p>	<p>For \bar{t} and \bar{h} used. SOI (e.g. can be implied by $b = 0.0206$)</p> <p>For attempt at calculating gradient (b) for h on t.</p> <p>For 0.0206 cao</p>	
		<p>OR $b = \frac{546.8 / 5 - (60 \times 1.548)}{22000 / 5 - 60^2} = \frac{16.48}{800} = 0.0206$</p> <p>hence least squares regression line is:</p> $h - \bar{h} = b(t - \bar{t})$			

Question			Answer	Marks	Guidance	
2	(i)	(A)	$P(X=0) = \frac{e^{-2.1} 2.1^0}{0!}$ $= 0.1225$	M1 A1	For calculation CAO Allow 0.122	
			Or from tables $P(X=0) = 0.1225$			
				[2]		
2	(i)	(B)	$P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.3796$ $= 0.6204$	M1 A1 [2]	M1 for use of correct structure. i.e. M0 for use of $1 - P(X \leq 2)$ or $1 - 0.6796$ Using $\lambda = 2.0$ leading to $1 - 0.4060$ gets M1 CAO Allow 0.6203, 0.620	
2	(i)	(C)	New $\lambda = 5 \times 2.1 = 10.5$ P(Between 5 and 10 in 5 mins) $= 0.5207 - 0.0211$ $= 0.4996$	B1 M1 A1 [3]	For mean (SOI) For $P(X \leq 10) - P(X \leq 4)$ used. CAO Allow 0.500, 0.50. Condone 0.5 www.	e.g. $1 - 0.9379$ leads to B0M1A0
2	(ii)		Mean number in 60 minutes $= 60 \times 2.1 = 126$ Using Normal approx. to the Poisson, $X \sim N(126, 126)$ $P(X \geq 130) = P\left(Z \geq \frac{129.5 - 126}{\sqrt{126}}\right)$ $= P(Z > 0.3118) = 1 - \Phi(0.3118)$ $= 1 - 0.6224$ $= 0.3776$	B1 B1 B1 M1 A1 [5]	For Normal approx. For correct parameters (SOI) For correct continuity correction For correct probability structure CAO, (Do not FT wrong or omitted CC). Allow 0.378www & 0.3775	

Question		Answer	Marks	Guidance	
2	(iii)	(Because if butterflies are blown in pairs,) the events will no longer be occurring singly.	E1 [1]		Accept 'not independent'
2	(iv)	$P(3 \text{ or fewer}) = P(3 \text{ or fewer individuals and no pairs}) + P(0 \text{ or } 1 \text{ individual and } 1 \text{ pair})$ $= (0.9068 \times 0.8187) + (0.4932 \times (0.9825 - 0.8187))$ $= (0.9068 \times 0.8187) + (0.4932 \times 0.1638)$ $= 0.7424 + 0.0808$ $= 0.8232$ <p>Or</p> <p>using D for the number of pairs and S for the number of singles</p> $P(D=0) \times P(S=0) = e^{-0.2} \times e^{-1.7} = 0.1495\dots$ $P(D=0) \times P(S=1) = e^{-0.2} \times 1.7e^{-1.7} = 0.2542\dots$ $P(D=0) \times P(S=2) = e^{-0.2} \times 1.7^2 e^{-1.7} \div 2 = 0.2161\dots$ $P(D=0) \times P(S=3) = e^{-0.2} \times 1.7^3 e^{-1.7} \div 3! = 0.1224\dots$ $P(D=1) \times P(S=0) = 0.2e^{-0.2} \times e^{-1.7} = 0.0299\dots$ $P(D=1) \times P(S=1) = 0.2e^{-0.2} \times 1.7e^{-1.7} = 0.0508\dots$ <p>Or</p> $P(D=0) \times P(S=3) + P(D=1) \times P(S=1)$ $P(D=0) \times P(S=2) + P(D=1) \times P(S=0)$ $P(D=0) \times P(S=1)$ $P(D=0) \times P(S=0)$	M1 M1 M2 A1	For P(0 pairs) (= 0.8187) For P(1 pair) (= 0.1638 or 0.1637) For structure M2 for correct 6 combinations identified and their probabilities added, M1 for 5 combinations identified and their probabilities added. CAO Allow awrt 0.823	First two M1s can be awarded for 0.9825

Question			Answer	Marks	Guidance	
			Or $P(D \leq 1) \times P(S \leq 1) = 0.98247 \times 0.49324$ $D(D = 0) \times P(S = 2) = 0.21613$ $D(D = 0) \times P(S = 3) = 0.12247$	[5]		
3	(i)	(A)	$P(X < 450) = P\left(Z < \frac{450 - 435}{30}\right)$ $= P(Z < 0.5) = \Phi(0.5)$ $= 0.6915$	M1 M1 A1 [3]	For standardising. M0 if ‘continuity correction’ applied For correct structure CAO Allow 0.692	
3	(i)	(B)	$P(400 < X < 450)$ $= P\left(\frac{400 - 435}{30} < Z < \frac{450 - 435}{30}\right)$ $= P(-1.1667 < Z < 0.5)$ $= \Phi(0.5) - \Phi(-1.1667)$ $= 0.6915 - 0.1216$ $= 0.5699$	M1 B1 A1 [3]	For correct structure For use of difference column to obtain 0.8784, 0.8783, 0.1216 or 0.1217. Condone 0.8782 or 0.1218 FT “their 0.6915” – 0.1216 (or 0.1217)	
3	(ii)		$P(\text{all 5 between 400 and 450})$ $= 0.5699^5$ $= 0.0601$	M1 A1 [2]	FT Allow 0.060	

Question		Answer	Marks	Guidance										
4	(a)	H ₀ : no association between grade and hours worked H ₁ : some association between grade and hours worked;	B1	Hypotheses in context	NB These M1A1 marks cannot be implied by a correct final value of X^2									
		<table><tr><td></td><td>Less than 5hrs</td><td>At least 5hrs</td></tr><tr><td>A or B</td><td>17.05</td><td>13.95</td></tr><tr><td>C or lower</td><td>15.95</td><td>13.05</td></tr></table>		Less than 5hrs		At least 5hrs	A or B	17.05	13.95	C or lower	15.95	13.05	M1 A1	Any row/column correct For expected values (to 2 dp)
			Less than 5hrs	At least 5hrs										
		A or B	17.05	13.95										
		C or lower	15.95	13.05										
		<table><tr><td></td><td>Less than 5hrs</td><td>At least 5hrs</td></tr><tr><td>A or B</td><td>0.5104</td><td>0.6238</td></tr><tr><td>C or lower</td><td>0.5456</td><td>0.6669</td></tr></table>		Less than 5hrs		At least 5hrs	A or B	0.5104	0.6238	C or lower	0.5456	0.6669	M1 A1	For valid attempt at (O-E) ² /E. Any row column correct. For all correct
			Less than 5hrs	At least 5hrs										
		A or B	0.5104	0.6238										
		C or lower	0.5456	0.6669										
		$X^2 = 2.347$ Refer to χ^2_1 Critical value at 5% level = 3.841	B1 M1 A1	For 1 deg of freedom. No FT from here if wrong. CAO for cv or p -value = 0.1255. SC1 for cv or p -value if 1 dof not seen.										
Result is not significant. There is insufficient evidence to suggest that there is any association between hours worked and grade.	E1	For conclusion in context. NB if H ₀ H ₁ reversed, or ‘correlation’ mentioned, do not award first B1 or final E1												
	[9]													

Question		Answer	Marks	Guidance	
4	(b)	$\bar{x} = 417.79$ $H_0: \mu = 420;$ $H_1: \mu \neq 420$ Where μ denotes the mean volume of the cans of tomato purée (in the population) $\text{Test statistic} = \frac{417.79 - 420}{3.5 / \sqrt{10}} = \frac{-2.21}{1.107} = -1.997$ Lower 1% level 2 tailed critical value of $z = -2.576$ $-1.997 > -2.576$ So not significant. There is insufficient evidence to reject H_0 There is insufficient evidence to suggest that the average volumes of the cans of tomato purée is not 420ml	B1 B1 B1 B1 M1* A1 B1* M1 dep* A1 [9]	For \bar{x} For use of 420 in hypotheses. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population mean. For both correct For definition of μ . Condone omission of “population” if correct notation μ is used, but if μ is defined as the sample mean then award B0 . must include $\sqrt{10}$ FT their \bar{x} For -2.576 Must be -2.576 unless it is clear that absolute values are being used. For sensible comparison leading to a conclusion. For conclusion in words in context provided that correct cv used. FT only candidate’s test statistic.	

ADDITIONAL NOTES REGARDING QUESTION 4 (b)Critical Value Method

$420 - 2.576 \times 3.5 \div \sqrt{10}$ gets M1*B1*

= 417.148... gets A1

$417.79 > 417.148..$ gets M1dep* for sensible comparison

A1 still available for correct conclusion in words & context

Confidence Interval Method

CI centred on 417.79 + or $- 2.5756 \times 3.5 \div \sqrt{10}$ gets M1* B1*

= (414.93..., 420.64..) gets A1

NOTE that the final M1dep* A1 available only if 2.576 used.

“Contains 420” gets M1dep*

A1 still available for correct conclusion in words & context

Probability Method

Finding $P(\text{sample mean} < 417.79) = 0.0229$ gets M1* A1 B1*

$0.0229 > 0.005^*$ gets M1dep* for a sensible comparison if a conclusion is made.

A1 available for a correct conclusion in words & context.

Condone $P(\text{sample mean} > 417.79) = 0.9771$ for M1* but only allow A1 B1* if sensible comparison made, at which point the final M1dep* and A1 are still available

ADDITIONAL NOTE REGARDING OVER-SPECIFICATION OF ANSWERS

Over-specification by providing final answers correct to 5 or more significant figures will be penalised. When this applies, candidates may lose no more than 2 marks per question and no more than 4 marks in total. The only exception to this rule is in Question 3 parts (iii) & (iv) – see guidance notes.

4767 Statistics 2

General Comments

Once again, the overall level of ability shown by candidates taking this paper was high. Responses to questions requiring statistical interpretation were, on the whole, good. The majority of candidates coped well with the questions involving probability calculations. Question 4, involving hypothesis tests, was particularly well done; it was pleasing to see many suitably non-assertive conclusions. Over-specification of answers was seen, though most worked to appropriate levels of accuracy. Few candidates were penalised for under-specification.

Comments on Individual Questions

- 1(i) Most candidates produced an accurately drawn scatter diagram with suitably labelled axes. Few candidates neglected to label their axes. Those using unusual scales on the vertical axis often incorrectly plotted the second or third point.
- (ii) This was well answered. The majority of candidates identified “thickness” as the independent variable and provided a suitable reason for their choice; this often involved describing an element of control over the thickness of tile used.
- (iii) Most candidates obtained the correct equation for h on t (though many used variables y & x instead of h & t). A small number of candidates estimated the gradient of the line using points on the graph rather than the least squares regression formula provided in the Examination Formulae booklet. Few arithmetic slips were seen. Most candidates opted to give their gradient and intercept values correct to 3 significant figures. Candidates calculating the least squares regression line for t on h were few in number.
- (ivA&) Well answered. Most candidates showed awareness of interpolation and extrapolation, and provided suitable comments
- (v) This question was generally well answered. Many candidates scored full marks. A few candidates calculated the residual as “predicted value – observed value”.
- (vi) After calculating the predicted height for a tile of thickness 200 mm, most candidates realised that the linear relationship was reliable for thicknesses within the range of values of the data provided, and that the relationship appeared to break down for larger thicknesses. Most of these candidates communicated this idea well, but those candidates simply stating that the (overall) relationship was non-linear did not earn the final mark. A small proportion of candidates managed to make a suitable comment without showing that they had calculated the prediction for the 200 mm tile. Many candidates seemed unaware of the differences between linear relationships and relationships where one variable is proportional to another.
- 2(iA) This was well answered with most candidates obtaining both marks.
- (iB) Also well answered though occasional mistakes using tables, such as looking up the value of $P(X \leq 1)$ using $\lambda = 2.0$, were seen.

- (iC) Most candidates realised that the new mean, $\lambda = 10.5$, was to be used. Many correct answers were seen through a variety of incorrect methods for finding $P(5 \leq X \leq 10)$ followed; of which " $P(X \leq 10) - P(X \leq 5)$ " and " $P(X \geq 5) - P(X \leq 10)$ " were typical.
- (ii) Many candidates obtained full marks here though some failed to apply the required continuity correction. A few candidates lost the final accuracy mark through premature rounding of their z-value prior to using Normal tables.
- (iii) Most candidates provided a suitable comment here, with remarks about "independence" being the most popular.
- (iv) This proved to be one of the most challenging parts of the paper. Despite answering part (iii) correctly many candidates reverted back to the inappropriate model by combining the means rather than considering the different combinations of "pairs" and "singles". Of those attempting to consider combinations of pairs and singles only a small proportion obtained a fully correct solution; a variety of approaches was seen and those working systematically were the most successful.
- 3(iA) Well answered, though inappropriate "continuity corrections" were seen on occasion.
- (iB) Well answered, though arithmetic errors were quite common. In several cases, -1.667 was used rather than -1.1667 often as a result of candidates misreading their own figures. A few candidates lost accuracy by prematurely rounding their z-value before using the Normal tables.
- (ii) Very well answered. Most candidates scored both marks.
- (iii) On the whole, this was well answered. Many candidates provided clear, accurate methods leading to correct final answers. Some candidates started out with one of the required equations containing a sign error which was not picked up, even when the error led to a negative value for σ . Most candidates identified the correct z-values. In the poorest answers, continuity corrections were attempted and z-values were changed to absurd values, such as " $1 - 0.8416$ ", before substitution into equations. Over-specification of final answers was seen, on occasion, here.
- (iv) Though one of the more challenging parts, many candidates scored full marks here. A variety of correct, "non-symmetrical" solutions were seen though most opted to use z-values of ± 1.96 .
- 4(a) This question was well answered. Many candidates scored full marks. Marks lost typically for over-assertive conclusions, typically containing words such as "not enough evidence to prove that...". The small number of candidates referring to correlation in their hypotheses often lost the first and last marks. Most candidates managed to accurately calculate the test statistic though some did not show all working as required. Most candidates stated the correct number of degrees of freedom and identified the correct critical value, though some thought that this was a 2-tailed test.
- (b) Well answered. Most candidates accurately calculated the sample mean and provided hypotheses in terms of μ . Note that candidates should be discouraged from referring to the "sample population mean" when defining μ . In carrying out the test, the test statistic method proved the most popular; those who "reversed their numerator" needed to be very careful how they used their test statistic. Many appropriate, non-assertive conclusions were seen though some failed to include context in the final comments.