OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4767
Statistics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Friday 18 June 2010
Afternoon

Duration: 1 hour 30 minutes

4767

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Two celebrities judge a talent contest. Each celebrity gives a score out of 20 to each of a random sample of 8 contestants. The scores, x and y, given by the celebrities to each contestant are shown below.

Contestant	A	B	C	D	E	F	G	H
x	6	17	9	20	13	15	11	14
y	6	13	10	11	9	7	12	15

(i) Calculate the value of Spearman's rank correlation coefficient. **[5]**

(ii) Carry out a hypothesis test at the 5% significance level to determine whether there is positive association between the scores allocated by the two celebrities. **[6]**

(iii) State the distributional assumption required for a test based on the product moment correlation coefficient. Sketch a scatter diagram of the scores above, and discuss whether it appears that the assumption is likely to be valid. **[5]**

2 A radioactive source is decaying at a mean rate of 3.4 counts per 5 seconds.

(i) State conditions for a Poisson distribution to be a suitable model for the rate of decay of the source. **[2]**

You may assume that a Poisson distribution with a mean rate of 3.4 counts per 5 seconds is a suitable model.

(ii) State the variance of this Poisson distribution. **[1]**

(iii) Find the probability of

(A) exactly 3 counts in a 5-second period,

(B) at least 3 counts in a 5-second period. **[5]**

(iv) Find the probability of exactly 40 counts in a period of 60 seconds. **[3]**

(v) Use a suitable approximating distribution to find the probability of at least 40 counts in a period of 60 seconds. **[5]**

(vi) The background radiation rate also, independently, follows a Poisson distribution and produces a mean count of 1.4 per 5 seconds. Find the probability that the radiation source together with the background radiation give a total count of at least 8 in a 5-second period. **[3]**

3 In a men's cycling time trial, the times are modelled by the random variable X minutes which is Normally distributed with mean 63 and standard deviation 5.2.

(i) Find

(A) $P(X < 65)$,

(B) $P(60 < X < 65)$. **[6]**

(ii) Find the probability that 5 riders selected at random all record times between 60 and 65 minutes. **[2]**

(iii) A competitor aims to be in the fastest 5% of entrants (i.e. those with the lowest times). Find the maximum time that he can take. **[3]**

It is suggested that holding the time trial on a new course may result in lower times. To investigate this, a random sample of 15 competitors is selected. These 15 competitors do the time trial on the new course. The mean time taken by these riders is 61.7 minutes. You may assume that times are Normally distributed and the standard deviation is still 5.2 minutes. A hypothesis test is carried out to investigate whether times on the new course are lower.

(iv) Write down suitable null and alternative hypotheses for the test. Carry out the test at the 5% significance level. **[8]**

4 In a survey a random sample of 63 runners is selected. The category of runner and the type of running are classified as follows.

		Category of runner			Row totals
		Junior	Senior	Veteran	
Type of running	Track	9	8	2	19
	Road	4	8	12	24
	Both	4	10	6	20
Column totals		17	26	20	63

(i) Carry out a test at the 5% significance level to examine whether there is any association between category of runner and the type of running. State carefully your null and alternative hypotheses. Your working should include a table showing the contributions of each cell to the test statistic. **[12]**

(ii) For each category of runner, comment briefly on how the type of running compares with what would be expected if there were no association. **[6]**

GCE

Mathematics (MEI)

Advanced GCE **4767**

Statistics 2

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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Question 1

<table>
<tr><td>(i)</td><td>

x	6	17	9	20	13	15	11	14
y	6	13	10	11	9	7	12	15
Rank x	8	2	7	1	5	3	6	4
Rank y	8	2	5	4	6	7	3	1
d	0	0	2	-3	-1	-4	3	3
d^2	0	0	4	9	1	16	9	9

$\Sigma d^2 = 48$

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 48}{8\times 63}$$

= 0.429 (to 3 s.f.) [*allow* 0.43 to 2 s.f.]</td><td>M1 for attempt at ranking (allow all ranks reversed)

M1 for d^2

A1 CAO for Σd^2

M1 for method for r_s

A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td><td>5</td></tr>
<tr><td>(ii)</td><td>H_0: no association between X and Y in the population
H_1: some positive association between X and Y in the population

One tail test critical value at 5% level is 0.6429
Since 0.429 < 0.6429, there is insufficient evidence to reject H_0,

i.e. conclude that there is not enough evidence to show positive association between the two judges’ scores.</td><td>B1 for H_0
B1 for H_1
B1 for population SOI
NB H_0 H_1 <u>not</u> ito ρ
B1 for ± 0.6429
M1 for sensible comparison with c.v., provided that $|r_s| < 1$
A1 for conclusion in context f.t. their r_s and sensible cv</td><td>3

3</td></tr>
<tr><td>(iii)</td><td>A bivariate Normal distribution is required.

Scatter diagram.

Suitable discussion</td><td>B1

G1 labelled axes
G1 correct points
E1
E1</td><td>5</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>16</td></tr>
</table>

Question 2

(i)	Counts have a uniform average rate of occurrence All counts are independent	E1 E1	**2**
(ii)	Variance = 3.4	B1	**1**
(iii)	(A) *Either* $P(X = 3) = 0.5584 - 0.3397 = 0.2187$ *Or* $P(X = 3) = e^{-3.4}\frac{3.4^3}{3!} = 0.2186$ (B) Using tables: $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.3397$ $= 0.6603$	M1 for use of tables or calculation A1 M1 for 1 - $P(X \leq 2)$ M1 correct use of Poisson tables A1	**2** **3**
(iv)	$\lambda = 12 \times 3.4 = 40.8$ $P(X = 40) = e^{-40.8}\frac{40.8^{40}}{40!} = 0.0625$	B1 for mean M1 for calculation A1	**3**
(v)	Mean no. per hour = 12 × 3.4 = 40.8 Using Normal approx. to the Poisson, $X \sim N(40.8, 40.8)$ $P(X \geq 40) = P\left(Z > \frac{39.5 - 40.8}{\sqrt{40.8}}\right)$ $= P(Z > -0.2035) = \Phi(0.2035)$ $= 0.5806$	B1 for Normal approx. B1 for correct parameters (SOI) B1 for correct continuity corr. M1 for probability using correct tail A1 CAO (3 s.f.)	**5**
(vi)	Overall mean = 4.8 $P(X \geq 8) = 1 - P(X \leq 7)$ $= 1 - 0.8867 = 0.1133$	B1 for 4.8 M1 A1	**3**
		TOTAL	**19**

Question 3

(i)	(*A*) $P(X < 65) = P\left(Z < \frac{65-63}{5.2}\right)$ $= P(Z < 0.3846)$ $= \Phi(0.3846) = 0.6497$ (*B*) $P(60 < X < 65) = P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ $= P(-0.5769 < Z < 0.3846)$ $= \Phi(0.3846) - (1 - \Phi(0.5769))$ $= 0.6497 - (1 - 0.7181)$ $= 0.3678$	M1 for standardizing M1 for structure A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only M1 for standardizing both M1 for correct structure A1 CAO 3s.f.	**3** **3**
(ii)	P(All 5 between 60 and 65) $= 0.3678^5 = 0.00673$	M1 A1 FT (min 2sf)	**2**
(iii)	From tables $\Phi^{-1}(0.95) = 1.645$ $\frac{k-63}{5.2} = -1.645$ $x = 63 - 5.2 \times 1.645 = 54.45$ mins	B1 for ±1.645 seen M1 for correct equation in *k* A1 CAO	**3**
(iv)	H_0: $\mu = 63$ minutes; H_1: $\mu < 63$ minutes. Where μ denotes the population mean time on the new course. Test statistic $= \frac{61.7-63}{5.2/\sqrt{15}} = \frac{-1.3}{1.3426}$ $= -0.968$ 5% level 1 tailed critical value of $z = 1.645$ $-0.968 > -1.645$ so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the new course results in lower times.	B1 for use of 63 B1 for both correct B1 for definition of μ M1 must include √15 A1 B1 for ±1.645 M1 for sensible comparison leading to a conclusion A1 FT for correct conclusion in words in context	**3** **5**
			19

Question 4

(i)

H_0: no association between category of runner and type of running; H_1: some association between category of runner and type of running;	B1	1

EXPECTED	Junior	Senior	Veteran
Track	5.13	7.84	6.03
Road	6.48	9.90	7.62
Both	5.40	8.25	6.35

M1 A2 for expected values (to 2 dp)

(allow A1 for at least one row or column correct)

CONTRIBUTN	Junior	Senior	Veteran
Track	2.9257	0.0032	2.6949
Road	0.9468	0.3663	2.5190
Both	0.3615	0.3694	0.0192

M1 for valid attempt at $(O-E)^2/E$

A1 for all correct

NB These M1A1 marks cannot be implied by a correct final value of X^2

$X^2 = 10.21$	M1 for summation A1 for X^2	7
Refer to X_4^2	B1 for 4 deg of f	
Critical value at 5% level = 9.488	B1 CAO for cv	
Result is significant	B1 FT their 'sensible' X^2	
There is evidence to suggest that there is some association between category of runner and type of running. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final E1	E1 must be consistent with their X^2	4

(ii)

• Juniors appear be track runners more often than expected and road less often than expected.	E1 E1	
• Seniors tend to be as expected ín all three categories of running.	E1 E1	
• Veterans tend to be road runners more than expected and track runners less than expected.	E1 E1	6
	TOTAL	**18**

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Chief Examiners' Report

In this series, as always, the Principal Examiners' reports have tried to give teachers information to help them to evaluate the work of their students in the context of the strengths and weaknesses of the overall entry.

Some weaknesses are commonly mentioned: poor recognition and use of 'technical' language and notation, failure to present methods or reasons clearly and failure to set out work clearly.

Any candidate who does not know the meaning of technical words or notation given in the specification is at a great disadvantage. This is obviously the case when this lack of knowledge prevents the candidate from completely understanding what is required but also, poor or inaccurate use of technical terms or notation can impair a candidate's attempt to comment on an answer or explain a method.

Almost all solutions should include a clear indication of the method used. The rubric for each paper advises candidates that 'an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used'. Of course, when candidates are asked to establish a *given* answer, the detail required may be much greater that when the answer is not known.

Good, clear (and compact) display of working helps a candidate produce a coherent argument and reduces the chance of 'slips'. Candidates of all levels of ability can benefit from presenting their work and ideas well and there is often an association between good layout and high quality of work. It is to be hoped that the introduction of Printed Answer Books will encourage candidates to consider more carefully their setting out of solutions.

There are three matters that have been raised about how candidates should use the Printed Answer Books (that will be scanned). The first is that they should put their answers in the correct sections; the second is that they should not try to erase writing or drawing but should cross it out – the scanning process is sensitive and copies the faint images and marks that often are left after attempts at erasure. Finally it should be noted that the use of additional answer sheets should be unusual, and that sheets of rough working should not be handed in.

Note on accuracy in Statistics modules

The Principal Examiners' reports that follow discuss the candidates' performances on the individual modules. There is one matter that should be discussed in a general way as it applies to all the statistics modules. This is in respect of arithmetical accuracy in intermediate working and in quotation of final answers. Please note that these remarks are specific to the *statistics* modules; they do not necessarily apply to other modules, where it may be natural for somewhat different criteria to be appropriate.

Most candidates are sensible in their arithmetical work, but there is some unease as to exactly what level of accuracy the examiners are expecting. There is no general answer to this! The standard rubric for all the papers sums the situation up by including "final answers should be given to a degree of accuracy appropriate to the context". Three significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting from Normal tables, *some* evidence of interpolation is generally expected and so quotation to four decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice.

Talking now in general terms, the examiners always exercise sensible discretion in cases of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, a candidate is likely to lose an Accuracy mark; but if 4 significant figures are given, there would normally be no penalty. Likewise, answers which are slightly deviant from what is expected in a very minor manner are not penalised (for example, a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected). However, there are increasing numbers of cases where candidates give answers which are *grossly* over- or under-specified, such as insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happens to come off the candidate's calculator. **Such gross over-specification indicates a lack of appreciation of the nature of statistical work and, with effect from the January 2011 examinations, will be penalised by withholding of associated Accuracy marks.**

Candidates must however always be aware of the dangers of premature rounding if there are several steps in a calculation. If, say, a final answer is desired that is correct to 3 decimal places, this can in no way be guaranteed if only 3 decimal places are used in intermediate steps; indeed, it may not be safe to carry out the intermediate work even to 4 decimal places. The issue of over-specification may arise for the final answer but not for intermediate stages of the working.

It is worth repeating that most candidates act sensibly in all these respects, but it is hoped that this note may help those who are perhaps a little less confident in how to proceed.

4767 Statistics 2

General comments
A strong overall performance from candidates yet again. The vast majority demonstrated a good working knowledge and could convey their understanding to a reasonable degree. Many candidates were less than convincing when clear explanations were required.

Comments on individual questions

1) (i) Well answered, with most earning full marks. Odd slips with ranking meant some candidates lost a mark. Some lost the final M1A1 through errors in applying the Spearman's rank correlation formula; typically, neglecting to include "1 – ". A small number failed to rank the data and were awarded 0/5.

1) (ii) Well answered; the majority of candidates picked up 5 out of the 6 marks available. The hypotheses needed to be stated in words and refer to the "association between the judges' scores", or equivalent, to earn full credit. Many candidates lost a mark for failing to include the word "positive" in their alternative hypothesis, despite subsequently carrying out one-tailed tests. Very few candidates were awarded the mark for indicating that the test was for association between "*X* and *Y* in the population". The majority of candidates identified 0.6429 as the correct critical value and went on to compare their test statistic and make a suitable conclusion to earn the final 3 marks. Alternative critical values included pmcc critical values, incorrect Spearman's critical values (2-tail or incorrect *n* used) and, in a few cases, values from the *t* distribution.

1) (iii) Well answered. Awareness that the underlying distribution required to carry out a test based on the pmcc is "bivariate Normal" is still not wide amongst candidates. However, most candidates scored 4 out of the 5 available marks for sketching a scatter diagram and using it to comment on the validity of the distributional assumption. Candidates should be aware that their explanations should leave no doubt as to their meaning; answers which require examiners to refer to the question then deduce the meaning are not deemed worthy of credit. For example, stating "ellipse, so yes" is not considered to be a discussion.

2) (i) Again, candidates found it difficult to express what they wanted to say here. Examiners were looking for comments relating to the counts occurring independently and with a uniform average rate. Comments referring to the "data" or "results" or "values" or "variables" occurring independently, were seen frequently. Many comments suggested that the rate of decay was constant.

2) (ii) Well answered on the whole, but some doubt was introduced by those writing their answers as $\lambda = 3.4$, or Po(3.4). Examiners needed to be left in no doubt that the candidates answers showed that the variance was 3.4 to award the mark.

2) (iii)(A) Well answered.

2) (iii)(B) Well answered. A few made mistakes in using tables; typically, looking up $P(X \le 3)$ then finding $1 - P(X \le 3)$.

2) (iv) Well answered. Including some attempts to use the Normal distribution.

2) (v) Most candidates managed to score at least 3 out of the 5 available marks, and many earned 5/5. The most common mistake was in using a continuity correction; some used 40.5 and others neglected to use one at all. A small proportion of candidates were unsure what value to use for the standard deviation, with 3.4 seen frequently. It was not uncommon to see answers where candidates had found $P(X \leq 40)$; those making a sketch to show which tail they were using tended not to make this mistake.

2) (vi) Well answered, with most earning full marks. A few candidates found the overall mean of 4.8 but then did not know what to do with it.

3) (i)(A) Well answered. A few lost a mark through inaccurate use of the Normal probability tables; rounding of z values to 2d.p. and/or neglecting to use the difference column of the Normal distribution tables. A small number ended up finding $P(X > 65)$. Many candidates erroneously applied continuity corrections and were consequently penalised.

3) (i)(B) Well answered.

3) (ii) Well answered. A small number of candidates multiplied their previous answer by 5. Some found the probability that only one of the 5 riders recorded times between 60 and 65 minutes.

3) (iii) Well answered. Most candidates scored all three marks. Using +1.645, leading to an answer of 71.544 minutes, was seen frequently.

3) (iv) Well answered. Very few candidates were awarded the mark for defining μ as the population mean time on the new course. Otherwise, most of the remaining marks were usually given. In this hypothesis test, candidates were expected to write their hypotheses in terms of μ; other symbols were accepted only if defined as the population mean. Most candidates managed to correctly obtain the test statistic of –0.968, compare it with –1.645 and make a suitable conclusion. Alternative methods were allowed; for example, finding a p-value of 0.1665 and comparing it with 0.05. Some candidates revealed their lack of understanding by making inappropriate comparisons such as "–0.968 < +1.645" or "0.968 > 0.05" or "0.1665 < 0.968" and so on. For the final mark, candidates were required to answer in context.

4)(i) Most candidates scored full marks in this part of the question. The hypotheses needed to be written in words and to include reference to "category and type of runner". In calculating X^2, a small number of candidates lost marks for inaccurate working either through premature rounding or through odd slips. Some neglected to provide "a table showing the contributions of each cell to the test statistic" despite being requested to do so. Most candidates were awarded the final four marks in this part of the question, but some did not specify the number of degrees of freedom and gained three marks. Those using anything other than 4 degrees of freedom were awarded no marks here.

4) (ii) In this part of the question there were two marks available for comments relating to each of the three categories of runner. Candidates were required to refer to their table of contributions then make a judgement regarding the level of association between category of runner and type of running. To score full marks, candidates had to make clear, accurate comments; however, most found this difficult. Poorly-worded comments could achieve no more that one mark out of the two available for each category of runner. From the comments provided it was often difficult to tell whether the table of contributions had been considered; many made simple statements about whether there were more (or fewer) runners observed than expected, regardless of the level of association. From this it was difficult to conclude whether or not the candidates were aware that a very small contribution indicated that the observed results were “as expected”. Ambiguous answers such as “the Junior track is higher than expected” were not uncommon. Some candidates thought that if there were no association then there would be an even spread of results. Those candidates who made general statements giving reasons why “juniors prefer running on tracks” rather than commenting on the statistics were given no credit.