OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4767
Statistics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

4767

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A pilot records the take-off distance for his light aircraft on runways at various altitudes. The data are shown in the table below, where a metres is the altitude and t metres is the take-off distance. Also shown are summary statistics for these data.

a	0	300	600	900	1200	1500	1800
t	635	704	776	836	923	1008	1105

$n = 7 \quad \Sigma a = 6300 \quad \Sigma t = 5987 \quad \Sigma a^2 = 8\,190\,000 \quad \Sigma t^2 = 5\,288\,931 \quad \Sigma at = 6\,037\,800$

(i) Draw a scatter diagram to illustrate these data. **[3]**

(ii) State which of the two variables a and t is the independent variable and which is the dependent variable. Briefly explain your answer. **[3]**

(iii) Calculate the equation of the regression line of t on a. **[5]**

(iv) Use the equation of the regression line to calculate estimates of the take-off distance for altitudes

(A) 800 metres,

(B) 2500 metres.

Comment on the reliability of each of these estimates. **[4]**

(v) Calculate the value of the residual for the data point where $a = 1200$ and $t = 923$, and comment on its sign. **[4]**

2 On average 2% of a particular model of laptop computer are faulty. Faults occur independently and randomly.

(i) Find the probability that exactly 1 of a batch of 10 laptops is faulty. **[3]**

(ii) State the conditions under which the use of a Poisson distribution is appropriate as an approximation to a binomial distribution. **[2]**

(iii) A school buys a batch of 150 of these laptops. Use a Poisson approximating distribution to find the probability that

(A) there are no faulty laptops in the batch, **[3]**

(B) there are more than the expected number of faulty laptops in the batch. **[3]**

(iv) A large company buys a batch of 2000 of these laptops for its staff.

(A) State the exact distribution of the number of faulty laptops in this batch. **[2]**

(B) Use a suitable approximating distribution to find the probability that there are at most 50 faulty laptops in this batch. **[5]**

3 In an English language test for 12-year-old children, the raw scores, X, are Normally distributed with mean 45.3 and standard deviation 11.5.

(i) Find

(A) $\mathrm{P}(X < 50)$, **[3]**

(B) $\mathrm{P}(45.3 < X < 50)$. **[2]**

(ii) Find the least raw score which would be obtained by the highest scoring 10% of children. **[3]**

(iii) The raw score is then scaled so that the scaled score is Normally distributed with mean 100 and standard deviation 15. This scaled score is then rounded to the nearest integer. Find the probability that a randomly selected child gets a rounded score of exactly 111. **[4]**

(iv) In a Mathematics test for 12-year-old children, the raw scores, Y, are Normally distributed with mean μ and standard deviation σ. Given that $\mathrm{P}(Y < 15) = 0.3$ and $\mathrm{P}(Y < 22) = 0.8$, find the values of μ and σ. **[5]**

[Question 4 is printed overleaf.]

4 A council provides waste paper recycling services for local businesses. Some businesses use the standard service for recycling paper, others use a special service for dealing with confidential documents, and others use both. Businesses are classified as small or large. A survey of a random sample of 285 businesses gives the following data for size of business and recycling service.

		Recycling Service		
		Standard	Special	Both
Size of business	Small	35	26	44
	Large	55	52	73

(i) Write down null and alternative hypotheses for a test to examine whether there is any association between size of business and recycling service used. **[1]**

The contributions to the test statistic for the usual χ^2 test are shown in the table below.

		Recycling Service		
		Standard	Special	Both
Size of business	Small	0.1023	0.2607	0.0186
	Large	0.0597	0.1520	0.0108

The sum of these contributions is 0.6041.

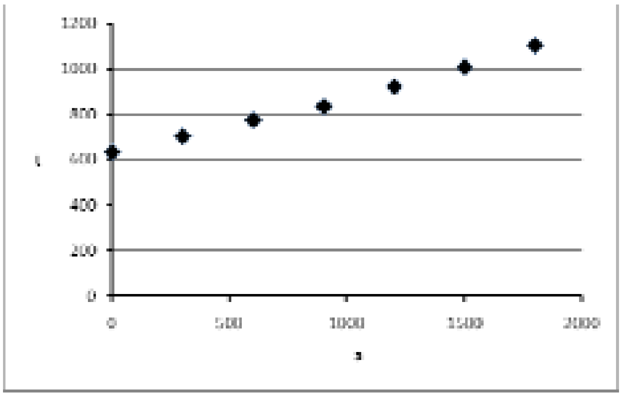
(ii) Calculate the expected frequency for large businesses using the special service. Verify the corresponding contribution 0.1520 to the test statistic. **[4]**

(iii) Carry out the test at the 5% level of significance, stating your conclusion clearly. **[5]**

The council is also investigating the weight of rubbish in domestic dustbins. In 2008 the average weight of rubbish in bins was 32.8 kg. The council has now started a recycling initiative and wishes to determine whether there has been a reduction in the weight of rubbish in bins. A random sample of 50 domestic dustbins is selected and it is found that the mean weight of rubbish per bin is now 30.9 kg, and the standard deviation is 3.4 kg.

(iv) Carry out a test at the 5% level to investigate whether the mean weight of rubbish has been reduced in comparison with 2008. State carefully your null and alternative hypotheses. **[8]**

4767 Statistics 2

1	**(i)**		G1 For values of a G1 for values of t G1 for axes	**[3]**
	(ii)	a is independent, t is dependent since the values of a are not subject to random variation, but are determined by the runways which the pilot chooses, whereas the values of t are subject to random variation.	B1 E1dep E1dep	**[3]**
	(iii)	$\bar{a} = 900$, $\bar{t} = 855.2$ $b = \frac{S_{at}}{S_{aa}} = \frac{6037800 - 5987 \times 6300/7}{8190000 - 6300^2/7} = \frac{649500}{2520000} = 0.258$ OR $b = \frac{6037800/7 - 855.29 \times 900}{8190000/7 - 900^2} = \frac{92785}{360000} = 0.258$ hence least squares regression line is: $t - \bar{t} = b(a - \bar{a})$ $\Rightarrow$ $t - 855.29 = 0.258\,(a - 900)$ $\Rightarrow$ $t = 0.258a + 623$	B1 for $\bar{a}$ and $\bar{t}$ used (SOI) M1 for attempt at gradient (b) A1 for 0.258 **cao** M1 for equation of line A1 FT for complete equation	**[5]**
	(iv)	(A) For $a = 800$, predicted take–off distance $= 0.258 \times 800 + 623 = 829$ (B) For $a = 2500$, predicted take–off distance $= 0.258 \times 2500 + 623 = 1268$ Valid relevant comments relating to the predictions such as: First prediction is interpolation so should be reasonable Second prediction is extrapolation and may not be reliable	M1 for at least one prediction attempted A1 for both answers (FT their equation if b>0) E1 (first comment) E1 (second comment)	**[4]**
	(v)	$a = 1200 \Rightarrow$ predicted $t = 0.258 \times 1200 + 623 = 933$ Residual = 923 – 933 = –10 The residual is negative because the observed value is less than the predicted value.	M1 for prediction M1 for subtraction A1 FT E1 **Total**	**[4]** **[19]**

2	**(i)**	P(1 of 10 is faulty) $= \binom{10}{1} \times 0.02^1 \times 0.98^9 = 0.1667$	M1 for coefficient M1 for probabilities A1	**[3]**
	(ii)	n is large and p is small	B1, B1 Allow appropriate numerical ranges	**[2]**
	(iii)	$\lambda = 150 \times 0.02 = 3$ (A) P(X = 0) = $\text{e}^{-3}\dfrac{3^0}{0!}$ = 0.0498 (3 s.f.) or from tables = 0.0498	B1 for mean (soi) M1 for calculation or use of tables A1	**[3]**
		(B) Expected number = 3 Using tables: P(X > 3) = 1 – P($X \leq$ 3) = 1 – 0.6472 = 0.3528	B1 expected no = 3 (soi) M1 A1	**[3]**
	(iv)	(A) Binomial(2000,0.02)	B1 for binomial B1 for parameters	**[2]**
		(B) Use Normal approx with $\mu = np = 2000 \times 0.02 = 40$ $\sigma^2 = npq = 2000 \times 0.02 \times 0.98 = 39.2$ P($X \leq$ 50) = $\text{P}\left(Z \leq \dfrac{50.5-40}{\sqrt{39.2}}\right)$ = P($Z \leq$ 1.677) = Φ(1.677) = 0.9532 NB Poisson approximation also acceptable for full marks	B1 B1 B1 for continuity corr. M1 for probability using correct tail A1 CAO	**[5]**
			Total	**[18]**

3	**(i)**	(A) P($X < 50$) $= \mathrm{P}\left(Z < \frac{50-45.3}{11.5}\right)$ $= \mathrm{P}(Z < 0.4087)$ $= \Phi(0.4087)$ $= 0.6585$	M1 for standardising M1 for correct structure of probability calc' A1 CAO inc use of diff tables NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only	**[3]**
		(B) P($45.3 < X < 50$) $= 0.6585 - 0.5$ $=0.1585$	M1 A1	**[2]**
	(ii)	From tables $\Phi^{-1}(0.9) = 1.282$ $\frac{k-45.3}{11.5} = 1.282$ $k = 45.3 + 1.282 \times 11.5 = 60.0$	B1 for 1.282 seen M1 for equation in k A1 CAO	**[3]**
	(iii)	P(score = 111) $=\mathrm{P}(110.5 < Y < 111.5)$ $= \mathrm{P}\left(\frac{110.5-100}{15} < Z < \frac{111.5-100}{15}\right)$ $= \mathrm{P}(0.7 < Z < 0.7667)$ $= \Phi(0.7667) - \Phi(0.7)$ $= 0.7784 - 0.7580$ $= 0.0204$	B1 for both continuity corrections M1 for standardising M1 for correct structure of probability calc' A1 CAO	**[4]**
	(iv)	From tables, $\Phi^{-1}(0.3) = -0.5244$, $\Phi^{-1}(0.8) = 0.8416$ $22 = \mu + 0.8416\,\sigma$ $15 = \mu - 0.5244\,\sigma$ $7 = 1.3660\,\sigma$ $\sigma = 5.124$, $\mu = 17.69$	B1 for 0.5244 or 0.8416 seen M1 for at least one equation in z, μ & σ A1 for both correct M1 for attempt to solve two appropriate equations A1 CAO for both	**[5]**
			TOTAL	**[17]**

4	(i)	H_0: no association between size of business and recycling service used. H_1: some association between size of business and recycling service used.	B1 for both	[1]
	(ii)	Expected frequency = 78/285 × 180 = 49.2632 Contribution = $(52 - 49.2632)^2$ / 49.2632 = 0.1520	M1 A1 M1 for valid attempt at $(O\text{-}E)^2/E$ A1 ***NB Answer given*** Allow 0.152	[4]
	(iii)	Test statistic X^2 = 0.6041 Refer to $\mathcal{X}_2^2$ Critical value at 5% level = 5.991 Result is not significant There is no evidence to suggest any association between size of business and recycling service used. NB if H_0 H_1 reversed, or 'correlation' mentioned in part (i), do not award B1in part (i) or E1 in part (iii).	B1 B1 for 2 deg of f(seen) B1 CAO for cv B1 for not significant E1	[5]
	(iv)	H_0: $\mu = 32.8$; H_1: $\mu < 32.8$ Where μ denotes the population mean weight of rubbish in the bins. Test statistic = $\dfrac{30.9-32.8}{3.4/\sqrt{50}} = -\dfrac{1.9}{0.4808} = -3.951$ 5% level 1 tailed critical value of z = –1.645 –3.951 < –1.645 so significant. There is sufficient evidence to reject H_0 There is evidence to suggest that the weight of rubbish in dustbins has been reduced.	B1 for use of 32.8 B1 for both correct B1 for definition of μ M1 must include √50 A1 B1 for ±1.645 M1 for sensible comparison leading to a conclusion A1 for conclusion in words in context	[8]
			TOTAL	**[18]**