OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4767

Statistics 2

Thursday **9 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 A student is collecting data on traffic arriving at a motorway service station during weekday lunchtimes. The random variable X denotes the number of cars arriving in a randomly chosen period of ten seconds.

(i) State two assumptions necessary if a Poisson distribution is to provide a suitable model for the distribution of X. Comment briefly on whether these assumptions are likely to be valid. [4]

The student counts the number of arrivals, x, in each of 100 ten-second periods. The data are shown in the table below.

x	0	1	2	3	4	5	>5
Frequency, f	18	39	20	12	8	3	0

(ii) Show that the sample mean is 1.62 and calculate the sample variance. [3]

(iii) Do your calculations in part **(ii)** support the suggestion that a Poisson distribution is a suitable model for the distribution of X? Explain your answer. [1]

For the remainder of this question you should assume that X may be modelled by a Poisson distribution with mean 1.62.

(iv) Find $\mathrm{P}(X = 2)$. Comment on your answer in relation to the data in the table. [4]

(v) Find the probability that at least ten cars arrive in a period of 50 seconds during weekday lunchtimes. [3]

(vi) Use a suitable approximating distribution to find the probability that no more than 550 cars arrive in a randomly chosen period of one hour during weekday lunchtimes. [4]

2 The fuel economy of a car varies from day to day according to weather and driving conditions. Fuel economy is measured in miles per gallon (mpg).

The fuel economy of a particular petrol-fuelled type of car is known to be Normally distributed with mean 38.5 mpg and standard deviation 4.0 mpg.

(i) Find the probability that on a randomly selected day the fuel economy of a car of this type will be above 45.0 mpg. [4]

(ii) The manufacturer wishes to quote a fuel economy figure which will be exceeded on 90% of days. What figure should be quoted? [3]

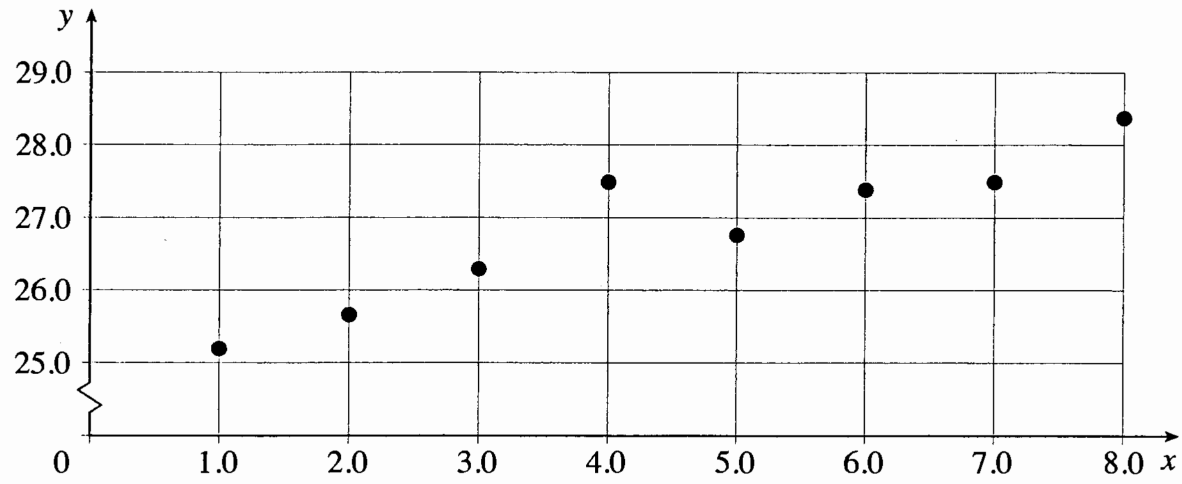
The daily fuel economy of a similar type of car which is diesel-fuelled is known to be Normally distributed with mean 51.2 mpg and unknown standard deviation σ mpg.

(iii) Given that on 75% of days the fuel economy of this type of car is below 55.0 mpg, show that $\sigma = 5.63$. [3]

(iv) Draw a sketch to illustrate both distributions on a single diagram. [4]

(v) Find the probability that the fuel economy of either the petrol or the diesel model (or both) will be above 45.0 mpg on a randomly selected day. You may assume that the fuel economies of the two models are independent. [4]

3 In a triathlon, competitors have to swim 600 metres, cycle 40 kilometres and run 10 kilometres. To improve her strength, a triathlete undertakes a training programme in which she carries weights in a rucksack whilst running. She runs a specific course and notes the total time taken for each run. Her coach is investigating the relationship between time taken and weight carried. The times taken with eight different weights are illustrated on the scatter diagram below, together with the summary statistics for these data. The variables x and y represent weight carried in kilograms and time taken in minutes respectively.



Summary statistics: $n = 8, \Sigma x = 36, \Sigma y = 214.8, \Sigma x^2 = 204, \Sigma y^2 = 5775.28, \Sigma xy = 983.6$.

(i) Calculate the equation of the regression line of y on x. [5]

On one of the eight runs, the triathlete was carrying 4 kilograms and took 27.5 minutes. On this run she was delayed when she tripped and fell over.

(ii) Calculate the value of the residual for this weight. [3]

(iii) The coach decides to recalculate the equation of the regression line without the data for this run. Would it be preferable to use this recalculated equation or the equation found in part **(i)** to estimate the delay when the triathlete tripped and fell over? Explain your answer. [2]

The triathlete's coach claims that there is positive correlation between cycling and swimming times in triathlons. The product moment correlation coefficient of the times of twenty randomly selected competitors in these two sections is 0.209.

(iv) Carry out a hypothesis test at the 5% level to examine the coach's claim, explaining your conclusions clearly. [5]

(v) What distributional assumption is necessary for this test to be valid? How can you use a scatter diagram to decide whether this assumption is likely to be true? [2]

4 **(a)** The selling prices of semi-detached houses in the suburbs of a particular city are known to be Normally distributed with mean £166 500 and standard deviation £14 200. A householder on one large estate claims that houses on her estate have a higher mean selling price. The selling prices of six randomly selected houses on her estate are

£180 000, £152 000, £156 500, £172 000, £189 000, £169 000.

(i) State suitable null and alternative hypotheses to test her claim. [2]

(ii) Carry out the test at the 5% level of significance, stating your conclusions clearly. You may assume that the standard deviation of the selling prices of houses on this estate is £14 200. [6]

(b) The manager of a restaurant undertakes a survey of the numbers and types of drinks ordered by a random sample of 400 customers. Customers are categorized as business, tourist or local. The drinks are categorized as alcoholic or soft drinks. A table of results of the survey is as follows.

		Type of drink		**Row totals**
		Alcoholic	Soft drinks	
Type of customer	Business	54	63	117
	Tourist	95	41	136
	Local	71	76	147
Column totals		220	180	400

Carry out a test at the 5% level of significance to examine whether there is any association between type of customer and type of drink. State carefully your null and alternative hypotheses.

[10]

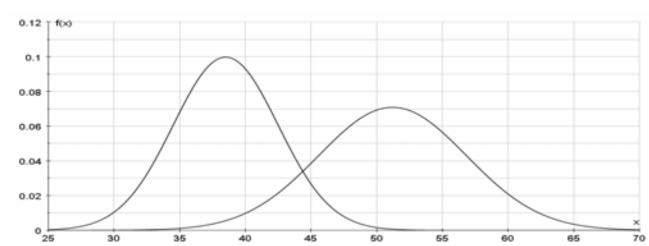
Mark Scheme

Question 1

(i)	Uniform average rate of occurrence; Successive arrivals are independent. Suitable arguments for/against each assumption: Eg Rate of occurrence could vary depending on the weather (any reasonable suggestion)	E1,E1 for suitable assumptions E1, E1 must be in context	**4**
(ii)	Mean $= \frac{\Sigma xf}{n} = \frac{39+40+36+32+15}{100} = \frac{162}{100} = 1.62$ Variance $= \frac{1}{n-1}\left(\Sigma fx^2 - n\bar{x}^2\right)$ $= \frac{1}{99}\left(430 - 100\times 1.62^2\right) = 1.69$ (to 2 d.p.)	B1 for mean ***NB answer given*** M1 for calculation A1	**3**
(iii)	Yes, since mean is close to variance	B1FT	**1**
(iv)	$\mathrm{P}(X=2) = \mathrm{e}^{-1.62}\frac{1.62^2}{2!}$ $= 0.260$ (3 s.f.) *Either:* Thus the expected number of 2's is 26 which is reasonably close to the observed value of 20. *Or*: This probability compares reasonably well with the relative frequency 0.2	M1 for probability calc. M0 for tables unless interpolated A1 B1 for expectation of 26 or r.f. of 0.2 E1	**4**
(v)	$\lambda = 5\times 1.62 = 8.1$ Using tables: $\mathrm{P}(X \geq 10) = 1 - \mathrm{P}(X \leq 9)$ $= 1 - 0.7041 = 0.2959$	B1FT for mean (SOI) M1 for probability from using tables to find $1 - \mathrm{P}(X \leq 9)$ A1 FT	**3**
(vi)	Mean no. of items in 1 hour $= 360 \times 1.62 = 583.2$ Using Normal approx. to the Poisson, $X \sim \mathrm{N}(583.2, 583.2)$: $\mathrm{P}(X \leq 550.5) = \mathrm{P}\left(Z \leq \frac{550.5 - 583.2}{\sqrt{583.2}}\right)$ $= \mathrm{P}(Z \leq -1.354) = 1 - \Phi(1.354) = 1 - 0.9121$	B1 for Normal approx. with correct parameters (SOI) B1 for continuity corr. M1 for probability	**4**

	$= 0.0879$ (3 s.f.)	using correct tail A1 CAO, (but FT wrong or omitted CC)	
			19

Question 2

(i)	$X \sim \mathrm{N}(38.5, 16)$ $\mathrm{P}(X > 45) = \mathrm{P}\left(Z > \dfrac{45 - 38.5}{4}\right)$ $= \mathrm{P}(Z > 1.625)$ $= 1 - \Phi(1.625) = 1 - 0.9479$ $= 0.0521$ (3 s.f.) *or* 0.052 (to 2 s.f.)	M1 for standardizing A1 for 1.625 M1 for prob. with tables and correct tail A1 CAO (min 2 s.f.)	**4**
(ii)	From tables $\Phi^{-1}(0.90) = 1.282$ $\dfrac{x - 38.5}{4} = -1.282$ $x = 38.5 - 1.282 \times 4 = 33.37$ So 33.4 should be quoted	B1 for 1.282 seen M1 for equation in x and negative z-value A1 CAO	**3**
(iii)	$Y \sim \mathrm{N}(51.2, \sigma^2)$ From tables $\Phi^{-1}(0.75) = 0.6745$ $\dfrac{55 - 51.2}{\sigma} = 0.6745$ $3.8 = 0.6745\,\sigma$ $\sigma = 5.63$	B1 for 0.6745 seen M1 for equation in σ with z-value A1 ***NB answer given***	**3**
(iv)		G1 for shape G1 for means, shown explicitly or by scale G1 for lower max height in diesel G1 for higher variance in diesel	**4**
(v)	$\mathrm{P(Diesel} > 45) = \mathrm{P}\left(Z > \dfrac{45 - 51.2}{5.63}\right)$	M1 for prob. calc. for diesel	

	$= \text{P}(Z > -1.101) = \Phi(1.101) = 0.8646$ P(At least one over 45) = 1 – P(Both less than 45) $= 1 - (1 - 0.0521) \times (1 - 0.8646)$ $= 1 - 0.9479 \times 0.1354 = 0.8717$ NB allow correct alternatives based on: P(D over, P under)+P(D under, P over)+ P(both over) *or* P(D over) + P(P over) – P(both over)	M1 for correct structure M1*dep* for correct probabilities A1 CAO (2 s.f. min)	**4**
			18

Question 3

(i)	$\bar{x} = 4.5, \quad \bar{y} = 26.85$ $b = \dfrac{Sxy}{Sxx} = \dfrac{983.6 - 36 \times 214.8/8}{204 - 36^2/8} = \dfrac{17}{42} = 0.405$ OR $b = \dfrac{983.6/8 - 4.5 \times 26.85}{204/8 - 4.5^2} = \dfrac{2.125}{5.25} = 0.405$ hence least squares regression line is: $y - \bar{y} = b(x - \bar{x})$ $\Rightarrow \quad y - 26.85 = 0.405(x - 4.5)$ $\Rightarrow \quad y = 0.405x + 25.03$	B1 for $\bar{x}$ and $\bar{y}$ used (SOI) M1 for attempt at gradient (b) A1 for 0.405 **cao** M1 *indep* for equation of line A1FT for complete equation	**5**
(ii)	$x = 4 \Rightarrow$ predicted $y = 0.405 \times 4 + 25.03 = 26.65$ Residual = 27.5 – 26.65 = 0.85	M1 for prediction A1FT for ± 0.85 B1FT for sign (+)	**3**
(iii)	The new equation would be preferable, since the equation in part (i) is influenced by the unrepresentative point (4,27.5)	B1 E1	**2**
(iv)	H_0: $\rho = 0$; H_1: $\rho > 0$ where ρ represents the population correlation coefficient Critical value at 5% level is 0.3783 Since 0.209 < 0.3783, there is not sufficient evidence to reject H_0, i.e. there is not sufficient evidence to conclude that there is any correlation between cycling and swimming times.	B1 for H_0 and H_1 B1 for defining ρ B1 for 0.3783 M1 for comparison leading to conclusion A1*dep on cv* for conclusion in words	**5**

		in context	
(v)	Underlying distribution must be bivariate normal. The distribution of points on the scatter diagram should be approximately elliptical.	B1 E1	**2**
			17

Question 4

(a) **(i)**	H_0: $\mu = 166500$; H_1: $\mu > 166500$ Where μ denotes the mean selling price in pounds of the population of houses on the large estate	B1 for both correct B1 for definition of μ	**2**
(ii)	n = 6, $\Sigma x = 1018500$, $\bar{x} = £169750$ Test statistic = $\dfrac{169750 - 166500}{14200/\sqrt{6}} = \dfrac{3250}{5797}$ $= 0.5606$ 5% level 1 tailed critical value of z = 1.645 0.5606 < 1.645 so not significant. There is insufficient evidence to reject H_0 It is reasonable to conclude that houses on this estate are not more expensive than in the rest of the suburbs.	B1CAO M1 must include √6 A1FT B1 for 1.645 M1 for comparison leading to a conclusion A1 for conclusion in words in context	**6**

(b) H_0: no association between customer and drink types;
H_1: some association between customer and drink types;

B1

Observed		Type of drink		Row totals
		Alcoholic	Soft drinks	
Type of customer	Business	54	63	117
	Tourist	95	41	136
	Local	71	76	147
Column totals		220	180	400

Expected		Type of drink		Row totals
		Alcoholic	Soft drinks	
Type of customer	Business	64.35	52.65	117
	Tourist	74.80	61.20	136
	Local	80.85	66.15	147
Column totals		220	180	400

M1 A1 for expected values (to 2dp)

Chi squared contribution		Type of drink		Row totals
		Alcoholic	Soft drinks	
Type of customer	Business	1.665	2.035	3.699
	Tourist	5.455	6.667	12.122
	Local	1.200	1.467	2.667

M1 for valid attempt at $(O-E)^2/E$

M1dep for summation **6**

$X^2 = 18.49$

A1CAO for X^2

Refer to $\mathcal{X}_2^2$
Critical value at 5% level = 5.991
Result is significant
There is some association between customer type and type of drink.
NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final B1 or final E1

B1 for 2 deg of f **4**
B1 CAO for cv
B1*dep on cv*
E1

18