OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4766

Statistics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

4766

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1 A supermarket chain buys a batch of 10 000 scratchcard draw tickets for sale in its stores. 50 of these tickets have a £10 prize, 20 of them have a £100 prize, one of them has a £5000 prize and all of the rest have no prize. This information is summarised in the frequency table below.

Prize money	£0	£10	£100	£5000
Frequency	9929	50	20	1

(i) Find the mean and standard deviation of the prize money per ticket. **[4]**

(ii) I buy two of these tickets at random. Find the probability that I win either two £10 prizes or two £100 prizes. **[3]**

2 Thomas has six tiles, each with a different letter of his name on it.

(i) Thomas arranges these letters in a random order. Find the probability that he arranges them in the correct order to spell his name. **[2]**

(ii) On another occasion, Thomas picks three of the six letters at random. Find the probability that he picks the letters T, O and M (in any order). **[3]**

3 A zoologist is studying the feeding behaviour of a group of 4 gorillas. The random variable X represents the number of gorillas that are feeding at a randomly chosen moment. The probability distribution of X is shown in the table below.

r	0	1	2	3	4
$P(X = r)$	p	0.1	0.05	0.05	0.25

(i) Find the value of p. **[1]**

(ii) Find the expectation and variance of X. **[5]**

(iii) The zoologist observes the gorillas on two further occasions. Find the probability that there are at least two gorillas feeding on both occasions. **[2]**

4 A pottery manufacturer makes teapots in batches of 50. On average 3% of teapots are faulty.

(i) Find the probability that in a batch of 50 there is

(A) exactly one faulty teapot, **[3]**

(B) more than one faulty teapot. **[3]**

(ii) The manufacturer produces 240 batches of 50 teapots during one month. Find the expected number of batches which contain exactly one faulty teapot. **[2]**

5 Each day Anna drives to work.

- R is the event that it is raining.
- L is the event that Anna arrives at work late.

You are given that $P(R) = 0.36$, $P(L) = 0.25$ and $P(R \cap L) = 0.2$.

(i) Determine whether the events R and L are independent. **[2]**

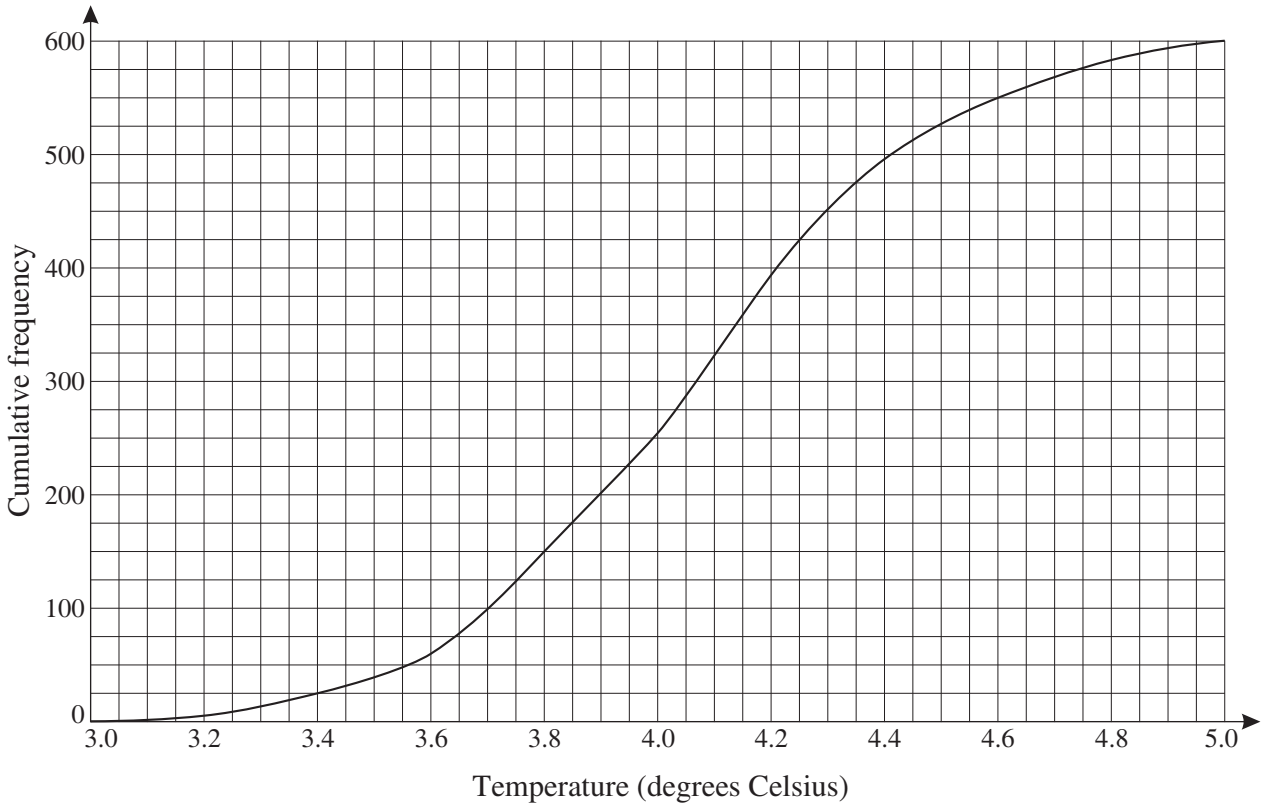
(ii) Draw a Venn diagram showing the events R and L. Fill in the probability corresponding to each of the four regions of your diagram. **[3]**

(iii) Find $P(L \mid R)$. State what this probability represents. **[3]**

[Question 6 is printed overleaf.]

Section B (36 marks)

6 The temperature of a supermarket fridge is regularly checked to ensure that it is working correctly. Over a period of three months the temperature (measured in degrees Celsius) is checked 600 times. These temperatures are displayed in the cumulative frequency diagram below.



(i) Use the diagram to estimate the median and interquartile range of the data. **[3]**

(ii) Use your answers to part **(i)** to show that there are very few, if any, outliers in the sample. **[4]**

(iii) Suppose that an outlier is identified in these data. Discuss whether it should be excluded from any further analysis. **[2]**

(iv) Copy and complete the frequency table below for these data. **[3]**

Temperature (t degrees Celsius)	$3.0 \leqslant t \leqslant 3.4$	$3.4 < t \leqslant 3.8$	$3.8 < t \leqslant 4.2$	$4.2 < t \leqslant 4.6$	$4.6 < t \leqslant 5.0$
Frequency			243	157	

(v) Use your table to calculate an estimate of the mean. **[2]**

(vi) The standard deviation of the temperatures in degrees Celsius is 0.379. The temperatures are converted from degrees Celsius into degrees Fahrenheit using the formula $F = 1.8C + 32$. Hence estimate the mean and find the standard deviation of the temperatures in degrees Fahrenheit. **[3]**

7 An online shopping company takes orders through its website. On average 80% of orders from the website are delivered within 24 hours. The quality controller selects 10 orders at random to check when they are delivered.

(i) Find the probability that

(A) exactly 8 of these orders are delivered within 24 hours, **[3]**

(B) at least 8 of these orders are delivered within 24 hours. **[2]**

The company changes its delivery method. The quality controller suspects that the changes will mean that fewer than 80% of orders will be delivered within 24 hours. A random sample of 18 orders is checked and it is found that 12 of them arrive within 24 hours.

(ii) Write down suitable hypotheses and carry out a test at the 5% significance level to determine whether there is any evidence to support the quality controller's suspicion. **[7]**

(iii) A statistician argues that it is possible that the new method could result in either better or worse delivery times. Therefore it would be better to carry out a 2-tail test at the 5% significance level. State the alternative hypothesis for this test. Assuming that the sample size is still 18, find the critical region for this test, showing all of your calculations. **[7]**

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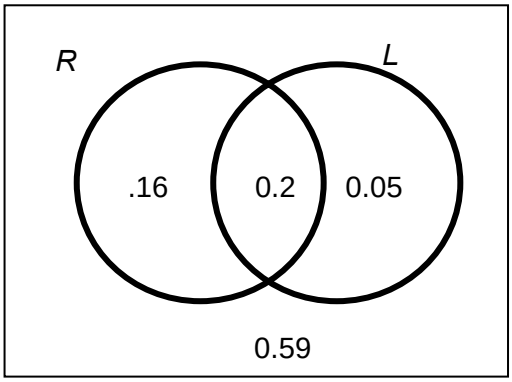
Section A

Q1 (i)	(With $\sum fx = 7500$ and $\sum f = 10000$ then arriving at the mean) (i) £0.75 scores (B1, B1) (ii) 75p scores (B1, B1) (iii) 0.75p scores (B1, B0) (incorrect units) (iv) £75 scores (B1, B0) (incorrect units) **After B0, B0** then sight of $\frac{7500}{10000}$ scores SC1. SC1or an answer in the range £0.74 - £0.76 or 74p – 76p (both inclusive) scores SC1 (units essential to gain this mark) Standard Deviation: (CARE NEEDED here with close proximity of answers) • 50.2(0) using divisor 9999 scores B2 (50.20148921) • 50.198 (= 50.2) using divisor 10000 scores B1(*rmsd*) • If divisor is not shown (or calc used) and only an answer of 50.2 (i.e. not coming from 50.198) is seen then award B2 on b.o.d. (default) **After B0 scored** then an attempt at S_{xx} as evident by either $S_{xx} = (5000 + 200000 + 25000000) - \frac{7500^2}{10000}$ (= 25199375) or $S_{xx} = (5000 + 200000 + 25000000) - 10000(0.75)^2$ **scores (M1) or M1ft 'their 7500^2' or 'their 0.75^2'** NB The structure must be correct in both above cases with a max of 1 slip only after applying the f.t.	B1 for numerical mean (0.75 or 75 seen) B1dep for correct units attached B2 correct s.d. (B1) correct rmsd (B2) default $\sum fx^2 = 25{,}205{,}000$ **Beware** $\sum x^2 = 25,010,100$ **After B0** scored then (M1) or M1f.t. for attempt at S_{xx} *NB full marks for correct results from recommended method which is use of calculator functions*	**4**

(ii)	P(Two £10 or two £100) $= \frac{50}{10000}\times\frac{49}{9999}+\frac{20}{10000}\times\frac{19}{9999}$ $= 0.0000245 + 0.0000038$ $= (0.00002450245 + 0.00000380038)$ $= 0.000028(3)$ o.e. $= (0.00002830283)$ After M0, M0 then $\frac{\mathbf{50}}{\mathbf{10000}}\times\frac{\mathbf{50}}{\mathbf{10000}}+\frac{\mathbf{20}}{\mathbf{10000}}\times\frac{\mathbf{20}}{\mathbf{10000}}$ o.e. Scores SC1 (ignore final answer but SC1 may be implied by sight of 2.9×10^{-5} o.e.) Similarly, $\frac{\mathbf{50}}{\mathbf{10000}}\times\frac{\mathbf{49}}{\mathbf{10000}}+\frac{\mathbf{20}}{\mathbf{10000}}\times\frac{\mathbf{19}}{\mathbf{10000}}$ scores SC1	M1 for either correct product seen (ignore any multipliers) M1 sum of both correct (ignore any multipliers) A1 CAO (as opposite with no rounding) (SC1 case #1) (SC1 case #2) **CARE** answer is also 2.83×10^{-5}	**3**
		TOTAL	**7**
Q2 (i)	Either P(all correct) $= \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times\frac{1}{1}=\frac{1}{720}$ or P(all correct) $= \frac{1}{6!}=\frac{1}{720}=0.00139$	M1 for 6! Or 720 (sioc) or product of fractions A1 CAO (accept 0.0014)	**2**
(ii)	Either P(picks T, O, M) $= \frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{20}$ or P(picks T, O, M) $= \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times3!=\frac{1}{20}$ or P(picks T, O, M) $= \frac{1}{\binom{6}{3}}=\frac{1}{20}$	M1 for denominators M1 for numerators or 3! A1 CAO Or M1 for $\binom{6}{3}$ or 20 sioc M1 for $1/\binom{6}{3}$ A1 CAO	**3**
		TOTAL	**5**
Q3 (i)	$p = 0.55$	B1 cao	**1**
(ii)	E(X) = $0\times0.55+1\times0.1+2\times0.05+3\times0.05+4\times0.25=1.35$ $E(X^2) = 0\times0.55+1\times0.1+4\times0.05+9\times0.05+16\times0.25$ $= 0 + 0.1 + 0.2 + 0.45 + 4$ $= (4.75)$ Var(X) = 'their' $4.75 - 1.35^2 = 2.9275$ awfw (2.9275 – 2.93)	M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) M1 for Σr^2p (at least 3 non zero terms correct) M1dep for – their $E(X)^2$ provided $Var(X) > 0$ A1 cao (no 'n' or 'n-1' divisors)	**5**
(iii)	P(At least 2 both times) $= (0.05+0.05+0.25)^2 = 0.1225$ o.e.	M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen A1cao: awfw (0.1225 - 0.123) or 49/400	**2**

Q4 (i)	$X \sim B(50, 0.03)$ **(A)** $P(X = 1) = \binom{50}{1} \times 0.03 \times 0.97^{49} = 0.3372$ **(B)** $P(X = 0) = 0.97^{50} = 0.2181$ $P(X > 1) = 1 - 0.2181 - 0.3372 = 0.4447$	M1 0.03×0.97^{49} or 0.0067(4)…. M1 $\binom{50}{1} \times pq^{49}$ (p+q =1) A1 CAO **(awfw 0. 337 to 0. 3372) or 0.34(2s.f.) or 0.34(2d.p.) but not just 0.34** B1 for 0.97^{50} or 0.2181 (awfw 0.218 to 0.2181) M1 for 1 – ('their' p (X = 0) + 'their' p(X = 1)) must have both probabilities A1 CAO (awfw 0.4447 to 0.445)	**3** **3**
(ii)	Expected number = np = 240 × 0.3372 = 80.88 – 80.93 = (81) *Condone 240 × 0.34 = 81.6 = (82) but for M1 A1f.t.*	M1 for 240 × prob (A) A1FT	**2**
		TOTAL	**8**
Q5 (i)	P(R) × P(L) = 0.36 × 0.25 = 0.09 ≠ P($R \cap L$) Not equal so not independent. (Allow 0.36 × 0.25 ≠ 0.2or 0.09 ≠ 0.2 or ≠ p(R ∩ L) so not independent)	M1 for 0.36 × 0.25 or 0.09 seen A1 (numerical justification needed)	**2**
(ii)		G1 for two overlapping circles labelled G1 for 0.2 and either 0.16 *or* 0.05 in the **correct places** G1 for all 4 **correct** probs in the **correct** places (including the 0.59) The last two G marks are independent of the labels	**3**
(iii)	$P(L \mid R) = \frac{P(L \cap R)}{P(R)} = \frac{0.2}{0.36} = \frac{5}{9} = 0.556$ (awrt 0.56) This is the probability that Anna is late given that it is raining. (must be in context) Condone 'if ' or 'when' or 'on a rainy day' for 'given that' but **not** the words 'and' or 'because' or 'due to'	M1 for 0.2/0.36 o.e. A1 cao E1 (indep of M1A1) **Order/structure must be correct i.e. no reverse statement**	**3**
		TOTAL	**8**

Section B

Q6 (i)	Median = 4.06 – 4.075 (inclusive) $Q_1 = 3.8$ $Q_3 = 4.3$ Inter-quartile range = 4.3 – 3.8 = 0.5	B1cao B1 for Q_1 (cao) B1 for Q_3 (cao) B1 ft for IQR must be using t-values not locations to earn this mark	**4**
(ii)	Lower limit ' their 3.8' – 1.5 × 'their 0.5' = (3.05) Upper limit ' their 4.3' + 1.5 × 'their 0.5' = (5.05) Very few if any temperatures <u>below 3.05 (but not zero)</u> None <u>above 5.05</u> 'So few, if any outliers' scores SC1	B1ft: must have -1.5 B1ft: must have +1.5 E1ft dep on -1.5 and Q_1 E1ft dep on+1.5 and Q_3 Again, must be using t-values NOT locations to earn these 4 marks	**4**
(iii)	Valid argument such as 'Probably not, because there is nothing to suggest that they are not genuine data items; (they do not appear to form a separate pool of data.') Accept: exclude outlier – 'measuring equipment was wrong' or 'there was a power cut' or ref to hot / cold day [Allow suitable valid alternative arguments]	E1	**1**
(iv)	Missing frequencies 25, 125, 50	B1, B1, B1 (all cao)	**3**
(v)	Mean = (3.2×25 + 3.6×125 + 4.0×243 + 4.4×157 + 4.8×50)/600 = 2432.8/600 = 4.05(47)	M1 for at least 4 midpoints correct and being used in attempt to find $\sum ft$ A1cao: awfw (4.05 – 4.055) ISW or rounding	**2**
(vi)	New mean = 1.8 × 'their 4.05(47)' + 32 = 39.29(84) to 39.3 New s = 1.8 × 0.379 = 0.682	B1 FT M1 for 1.8 × 0.379 A1 CAO awfw (0.68 – 0.6822)	**3**
		TOTAL	**17**

Q7 **(i)**	$X \sim B(10, 0.8)$ **(A)** *Either* $P(X = 8) = \binom{10}{8} \times 0.8^8 \times 0.2^2 = 0.3020$ (awrt) *or* $P(X = 8) = P(X \le 8) - P(X \le 7)$ $= 0.6242 - 0.3222 = 0.3020$ **(B)** *Either* $P(X \ge 8) = 1 - P(X \le 7)$ $= 1 - 0.3222 = 0.6778$ *or* $P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$ $= 0.3020 + 0.2684 + 0.1074 = 0.6778$	M1 $0.8^8 \times 0.2^2$ or 0.00671… M1 $\binom{10}{8} \times p^8 q^2$; (p +q =1) Or $45 \times p^8 q^2$; (p +q =1) A1 CAO **(0.302) not 0.3** OR: M2 for 0.6242 – 0.3222 A1 CAO M1 for 1 – 0.3222 (s.o.i.) A1 CAO awfw 0.677 – 0.678 or M1 for sum of 'their' p(X=8) plus correct expressions for p(x=9) and p(X=10) A1 CAO awfw 0.677 – 0.678	**3** **2**
(ii)	Let $X \sim B(18, p)$ Let p = probability of delivery (within 24 hours) (for population) H_0: $p = 0.8$ H_1: $p < 0.8$ $P(X \le 12) = 0.1329 > 5\%$ ref: [pp =0.0816] So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that less than 80% of orders will be delivered within 24 hours Note: use of critical region method scores M1 for region {0,1,2,…,9, 10} M1dep for 12 does not lie in critical region then A1dep E1dep as per scheme	B1 for definition of p B1 for H_0 B1 for H_1 M1 for probability 0.1329 M1dep strictly for comparison of 0.1329 with 5% (seen or clearly implied) A1dep on both M's E1dep on M1,M1,A1 for conclusion in context	**7**

(iii)	Let $X \sim B(18, 0.8)$ H_1: $p \neq 0.8$ LOWER TAIL $P(X \leq 10) = 0.0163 < 2.5\%$ $P(X \leq 11) = 0.0513 > 2.5\%$ UPPER TAIL $P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9009 = 0.0991 > 2.5\%$ $P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9820 = 0.0180 < 2.5\%$ So critical region is {0,1,2,3,4,5,6,7,8,9,10,18} o.e. Condone $X \leq 10$ and $X \geq 18$ or $X = 18$ but **not** $p(X \leq 10)$ and $p(X \geq 18)$ Correct CR without supportive working scores SC2 max after the 1st B1 (SC1 for each fully correct tail of CR)	B1 for H_1 B1 for 0.0163 or 0.0513 seen M1dep for either correct comparison with **2.5% (not 5%)** (seen or clearly implied) A1dep for correct lower tail CR (must have zero) B1 for 0.0991 or 0.0180 seen M1dep for either correct comparison with **2.5% (not 5%)** (seen or clearly implied) A1dep for correct upper tail CR	**7**
		TOTAL	**19**