RECOGNISING ACHIEVEMENT

## ADVANCED SUBSIDIARY GCE

Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

## INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.


## Section A (36 marks)

1 In a survey, a sample of 44 fields is selected. Their areas ( $x$ hectares) are summarised in the grouped frequency table.

| Area $(x)$ | $0<x \leqslant 3$ | $3<x \leqslant 5$ | $5<x \leqslant 7$ | $7<x \leqslant 10$ | $10<x \leqslant 20$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3 | 8 | 13 | 14 | 6 |

(i) Calculate an estimate of the sample mean and the sample standard deviation.
(ii) Determine whether there could be any outliers at the upper end of the distribution.

2 In the 2001 census, people living in Wales were asked whether or not they could speak Welsh. A resident of Wales is selected at random.

- $W$ is the event that this person speaks Welsh.
- $\quad C$ is the event that this person is a child.

You are given that $\mathrm{P}(W)=0.20, \mathrm{P}(C)=0.17$ and $\mathrm{P}(W \cap C)=0.06$.
(i) Determine whether the events $W$ and $C$ are independent.
(ii) Draw a Venn diagram, showing the events $W$ and $C$, and fill in the probability corresponding to each region of your diagram.
(iii) Find $\mathrm{P}(W \mid C)$.
(iv) Given that $\mathrm{P}\left(W \mid C^{\prime}\right)=0.169$, use this information and your answer to part (iii) to comment very briefly on how the ability to speak Welsh differs between children and adults.

3 In a game of darts, a player throws three darts. Let $X$ represent the number of darts which hit the bull's-eye. The probability distribution of $X$ is shown in the table.

| $r$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=r)$ | 0.5 | 0.35 | $p$ | $q$ |

(i) (A) Show that $p+q=0.15$.
(B) Given that the expectation of $X$ is 0.67 , show that $2 p+3 q=0.32$.
(C) Find the values of $p$ and $q$.
(ii) Find the variance of $X$.

4 A small business has 8 workers. On a given day, the probability that any particular worker is off sick is 0.05 , independently of the other workers.
(i) A day is selected at random. Find the probability that
$(A)$ no workers are off sick,
(B) more than one worker is off sick.
(ii) There are 250 working days in a year. Find the expected number of days in the year on which more than one worker is off sick.

5 A psychology student is investigating memory. In an experiment, volunteers are given 30 seconds to try to memorise a number of items. The items are then removed and the volunteers have to try to name all of them. It has been found that the probability that a volunteer names all of the items is 0.35 . The student believes that this probability may be increased if the volunteers listen to the same piece of music while memorising the items and while trying to name them.

The student selects 15 volunteers at random to do the experiment while listening to music. Of these volunteers, 8 name all of the items.
(i) Write down suitable hypotheses for a test to determine whether there is any evidence to support the student's belief, giving a reason for your choice of alternative hypothesis.
(ii) Carry out the test at the $5 \%$ significance level.

## Section B (36 marks)

6 In a large town, $79 \%$ of the population were born in England, $20 \%$ in the rest of the UK and the remaining $1 \%$ overseas. Two people are selected at random.

You may use the tree diagram below in answering this question.

(i) Find the probability that
(A) both of these people were born in the rest of the UK,
(B) at least one of these people was born in England,
(C) neither of these people was born overseas.
(ii) Find the probability that both of these people were born in the rest of the UK given that neither was born overseas.
(iii) (A) Five people are selected at random. Find the probability that at least one of them was not born in England.
(B) An interviewer selects $n$ people at random. The interviewer wishes to ensure that the probability that at least one of them was not born in England is more than $90 \%$. Find the least possible value of $n$. You must show working to justify your answer.

7 The histogram shows the age distribution of people living in Inner London in 2001.

(i) State the type of skewness shown by the distribution.
(ii) Use the histogram to estimate the number of people aged under 25.
(iii) The table below shows the cumulative frequency distribution.

| Age | 20 | 30 | 40 | 50 | 65 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative frequency (thousands) | 660 | 1240 | 1810 | $a$ | 2490 | 2770 |

(A) Use the histogram to find the value of $a$.
(B) Use the table to calculate an estimate of the median age of these people.

The ages of people living in Outer London in 2001 are summarised below.

| Age $(x$ years $)$ | $0 \leqslant x<20$ | $20 \leqslant x<30$ | $30 \leqslant x<40$ | $40 \leqslant x<50$ | $50 \leqslant x<65$ | $65 \leqslant x<100$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency (thousands) | 1120 | 650 | 770 | 590 | 680 | 610 |

(iv) Illustrate these data by means of a histogram.
(v) Make two brief comments on the differences between the age distributions of the populations of Inner London and Outer London.
(vi) The data given in the table for Outer London are used to calculate the following estimates.

Mean 38.5, median 35.7, midrange 50, standard deviation 23.7, interquartile range 34.4.
The final group in the table assumes that the maximum age of any resident is 100 years. These estimates are to be recalculated, based on a maximum age of 105 , rather than 100 . For each of the five estimates, state whether it would increase, decrease or be unchanged.

## 4766 Statistics 1

| $\begin{aligned} & \hline \text { Q1 } \\ & \text { (i) } \end{aligned}$ | Mean $=7.35$ (or better) <br> Standard deviation: 3.69-3.70 (awfw) <br> Allow s ${ }^{2}=13.62$ to 13.68 <br> Allow rmsd $=3.64-3.66$ (awfw) <br> After B0, B0 scored then if at least 4 correct mid-points seen or used. $\{1.5,4,6,8.5,15\}$ <br> Attempt of their mean $=\frac{\sum f x}{44}$, with $301 \leq \mathrm{fx} \leq 346$ and fx strictly from mid-points not class widths or top/lower boundaries. | B2cao $\sum f x=323.5$ <br> B2cao $\sum f x^{2}=2964.25$ <br> (B1) for variance s.o.i.o <br> (B1) for rmsd <br> (B1) mid-points <br> (B1) $6.84 \leq$ mean $\leq 7.86$ | 4 |
| :---: | :---: | :---: | :---: |
| (ii) | Upper limit $=7.35+2 \times 3.69=14.73$ or 'their sensible mean' $+2 \times$ 'their sensible s.d.' <br> So there could be one or more outliers | M1 ( with s.d. < mean) <br> E1dep on B2, B2 earned and comment | 2 |
|  |  | TOTAL | 6 |
| $\begin{aligned} & \text { Q2 } \\ & \text { (i) } \end{aligned}$ | $\mathrm{P}(W) \times \mathrm{P}(C)=0.20 \times 0.17=0.034$ <br> $P(W \cap C)=0.06$ (given in the question) <br> Not equal so not independent (Allow $0.20 \times 0.17 \neq 0.06$ or $\neq p(W \cap C)$ so not independent). | M1 for multiplying or 0.034 seen <br> A1 (numerical justification needed) | 2 |
| (ii) | The last two G marks are independent of the labels | G1 for two overlapping circles labelled <br> G1 for 0.06 and either 0.14 or 0.11 in the correct places <br> G1 for all 4 correct probs in the correct <br> places (including the 0.69) <br> NB No credit for <br> Karnaugh maps here | 3 |
| (iii) | $\mathrm{P}(W \mid C)=\frac{\mathrm{P}(W \cap C)}{\mathrm{P}(\mathrm{C})}=\frac{0.06}{0.17}=\frac{6}{17}=0.353(\text { awrt } 0.35)$ | M1 for 0.06 / 0.17 <br> A1 cao | 2 |


| (iv) | Children are more likely than adults to be able to speak <br> Welsh or 'proportionally more children speak Welsh than <br> adults' <br> Do not accept: 'more Welsh children speak Welsh than <br> adults' | E1FT Once the correct <br> idea is seen, apply ISW | 1 |
| :--- | :--- | :--- | :--- |



|  | Section B |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline \text { Q6 } \\ \text { (i) } \end{array}$ | (B) Either: All 5 case <br> $\mathrm{P}($ at least one England $)=$ $\begin{aligned} & (0.79 \times 0.20)+(0.79 \times 0.01)+(0.20 \times 0.79)+(0.01 \times 0.79)+ \\ & (0.79 \times 0.79) \\ & =0.158+0.0079+0.158+0.0079+0.6241=0.9559 \end{aligned}$ <br> Or $\begin{aligned} & P(\text { at least one England })=1-P(\text { neither England }) \\ & =1-(0.21 \times 0.21)=1-0.0441=0.9559 \\ & \text { or listing all } \\ & =1-\{(0.2 \times 0.2)+(0.2 \times 0.01)+(0.01 \times 0.20)+(0.01 \times \\ & 0.01)\} \\ & =1-\left({ }^{* *}\right) \\ & =1-\{0.04+0.002+0.002+0.0001) \\ & =1-0.0441 \\ & =0.9559 \end{aligned}$ <br> Or: All 3 case <br> P(at least one England) $=$ $=0.79 \times 0.21+0.21 \times 0.79+0.79^{2}$ $=0.1659+0.1659+0.6241$ $=0.9559$ <br> (C)Either $0.79 \times 0.79+0.79 \times 0.2+0.2 \times 0.79+0.2 \times 0.2=0.9801$ <br> Or $0.99 \times 0.99=0.9801$ <br> Or $\begin{aligned} & 1-\{0.79 \times 0.01+0.2 \times 0.01+0.01 \times 0.79+0.01 \times 0.02+ \\ & \left.0.01^{2}\right\}=1-0.0199 \\ & \quad=0.9801 \end{aligned}$ | M1 for multiplying <br> A1cao <br> M1 for any correct term (3case or 5case) <br> M1 for correct sum of all 3 (or of all 5) with no extras <br> A1cao (condone 0.96 www) <br> Or M1 for $0.21 \times 0.21$ <br> or for (**) fully enumerated or 0.0441 seen <br> M1dep for 1 - ( $1^{\text {st }}$ part) <br> A1cao <br> See above for 3 case <br> M1 for sight of all 4 correct terms summed <br> A1 cao (condone 0.98 www) <br> or <br> M1 for $0.99 \times 0.99$ <br> A1cao <br> Or <br> M1 for everything <br> 1 - \{.....\} <br> A1cao | 2 |
| (ii) | $\left.\begin{array}{l} \mathrm{P} \text { (both the rest of the UK \| neither overseas) } \\ \qquad=\frac{\mathrm{P}(\text { the rest of the UK and } \text { neither overseas })}{\mathrm{P}(\text { neither overseas })} \\ \quad=\frac{0.04}{0.9801}=0.0408 \end{array}\right\}$ | M1 for numerator of 0.04 or 'their answer to (i)(A)' <br> M1 for denominator of 0.9801 or 'their answer to (i) (C)' <br> A1 FT $(0<p<1) 0.041$ at least | 3 |

\begin{tabular}{|c|c|c|c|}
\hline (iii) \& \begin{tabular}{l}
(A)
\[
\begin{aligned}
\text { Probability } \& =1-0.79^{5} \\
\& =1-0.3077 \\
\& =0.6923 \text { (accept awrt } 0.69)
\end{aligned}
\] \\
see additional notes for alternative solution \\
(B) \(1-0.79^{n}>0.9\) \\
EITHER: \\
\(1-0.79^{n}>0.9\) or \(0.79^{n}<0.1\) \\
(condone \(=\) and \(\geq\) throughout) but not reverse inequality \\
\(n>\frac{\log 0.1}{\log 0.79}\), so \(n>9.768 \ldots\) \\
Minimum \(n=10\) Accept \(n \geq 10\) \\
OR (using trial and improvement): \\
Trial with \(0.79^{9}\) or \(0.79^{10}\)
\[
\begin{aligned}
\& 1-0.79^{9}=0.8801(<0.9) \text { or } 0.79^{9}=0.1198(>0.1) \\
\& 1-0.79^{10}=0.9053(>0.9) \text { or } 0.79^{10}=0.09468(<0.1)
\end{aligned}
\] \\
Minimum \(n=10\) Accept \(n \geq 10\) \\
NOTE: \(n=10\) unsupported scores SC1 only
\end{tabular} \& \begin{tabular}{l}
M1 for \(0.79^{5}\) or 0.3077... \\
M1 for \(1-0.79^{5}\) dep \\
A1 CAO \\
M1 for equation/inequality in \(n\) (accept either statement opposite) \\
M1 (indep) for process of using logs i.e. \(\frac{\log a}{\log b}\) \\
A1 CAO \\
M1 (indep) for sight of 0.8801 or 0.1198 \\
M1 (indep) for sight of 0.9053 or 0.09468 \\
A1 dep on both M's cao
\end{tabular} \& 3

3 <br>
\hline \& \& TOTAL \& 16 <br>
\hline
\end{tabular}

| Q7 (i) | Positive | B1 | 1 |
| :---: | :---: | :---: | :---: |
| (ii) | $\begin{aligned} & \text { Number of people }=20 \times 33(000)+5 \times 58(000) \\ & \quad=660(000)+290(000)=950000 \end{aligned}$ | M1 first term <br> M1 (indep) second term <br> A1 cao <br> NB answer of 950 scores M2A0 | 3 |
| (iii) | (A) $a=1810+340=2150$ <br> (B) Median = age of $1385\left(000^{\text {th }}\right)$ person or $1385.5(000)$ <br> Age 30, cf = 1240 (000); age 40, cf = 1810 (000) <br> Estimate median $=(30)+\frac{\mathbf{1 4 5}}{\mathbf{5 7 0}} \times 10$ <br> Median $=32.5$ years ( $32.54 \ldots$...) If no working shown then 32.54 or better is needed to gain the M1A1. If 32.5 seen with no previous working allow SC1 | M1 for sum <br> A1 cao 2150 or 2150 thousand but not 215000 <br> B1 for 1385 (000) or 1385.5 <br> M1 for attempt to interpolate $\frac{\mathbf{1 4 5} k}{\mathbf{5 7 0} k} \times 10$ <br> (2.54 or better suggests this) <br> A1 cao min 1dp | 2 3 |
| (iv) | Frequency densities: 56, 65, 77, 59, 45, 17 <br> (accept 45.33 and 17.43 for 45 and 17) | B1 for any one correct B1 for all correct (soi by listing or from histogram) <br> Note: all G marks below dep on attempt at frequency density, NOT frequency <br> G1 Linear scales on both axes (no inequalities) <br> G1 Heights FT their listed fds or all must be correct. Also widths. All blocks joined <br> G1 Appropriate label for vertical scale eg 'Frequency density (thousands)', 'frequency (thousands) per 10 years', 'thousands of people per 10 years'. (allow key). <br> OR f.d. | 5 |


| (v) | Any two suitable comments such as: <br> Outer London has a greater proportion (or \%) of people <br> under 20 (or almost equal proportion) | E1 |
| :--- | :--- | :--- | :--- |
| The modal group in Inner London is 20-30 but in Outer  <br> London it is 30-40  <br> Outer London has a greater proportion (14\%) of aged 65+  <br> All populations in each age group are higher in Outer  <br> London  <br> Outer London has a more evenly spread distribution or <br> balanced distribution (ages) o.e. E1 <br> (vi) Mean increase $\uparrow$ <br> median unchanged (-) <br> midrange increase $\uparrow$ <br> standard deviation increase $\uparrow$ <br> interquartile range unchanged. ( - ) <br>  Any one correct B1 <br> Any two correct B2 <br> Any three correct B3 <br> All five correct B4 | TOTAL |  |

