OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

G241/01

MEI STATISTICS

Statistics 1 (Z1)

TUESDAY 5 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

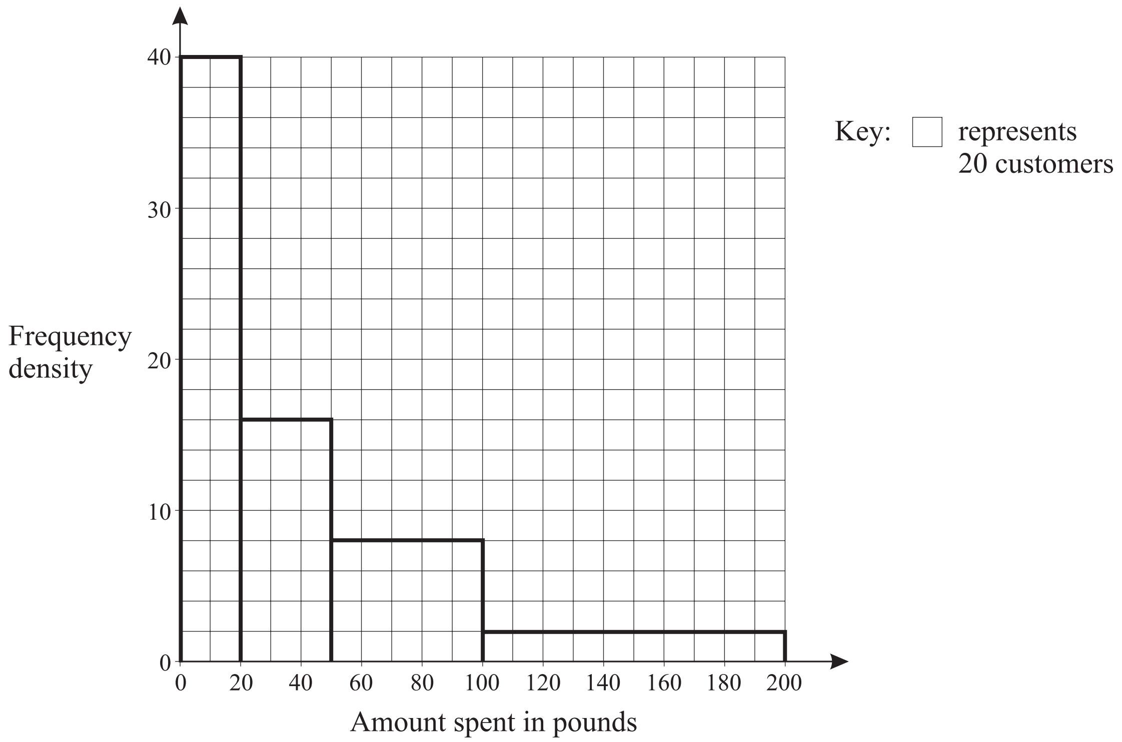
Section A (36 marks)

1 A girl is choosing tracks from an album to play at her birthday party. The album has 8 tracks and she selects 4 of them.

(i) In how many ways can she select the 4 tracks? [2]

(ii) In how many different orders can she arrange the 4 tracks once she has chosen them? [1]

2 The histogram shows the amount of money, in pounds, spent by the customers at a supermarket on a particular day.



(i) Express the data in the form of a grouped frequency table. [2]

(ii) Use your table to estimate the total amount of money spent by customers on that day. [2]

3 The marks x scored by a sample of 56 students in an examination are summarised by

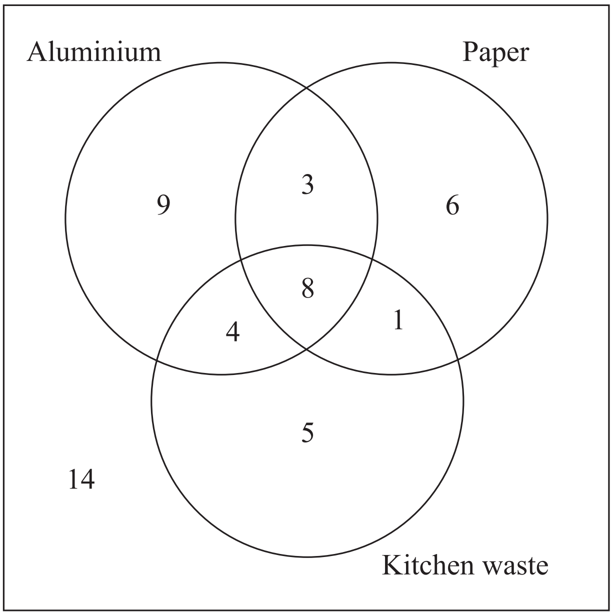
$$n = 56, \quad \Sigma x = 3026, \quad \Sigma x^2 = 178\,890.$$

(i) Calculate the mean and standard deviation of the marks. [3]

(ii) The highest mark scored by any of the 56 students in the examination was 93. Show that this result may be considered to be an outlier. [2]

(iii) The formula $y = 1.2x - 10$ is used to scale the marks. Find the mean and standard deviation of the scaled marks. [3]

4 A local council has introduced a recycling scheme for aluminium, paper and kitchen waste. 50 residents are asked which of these materials they recycle. The numbers of people who recycle each type of material are shown in the Venn diagram.



One of the residents is selected at random.

(i) Find the probability that this resident recycles

(A) at least one of the materials, [1]

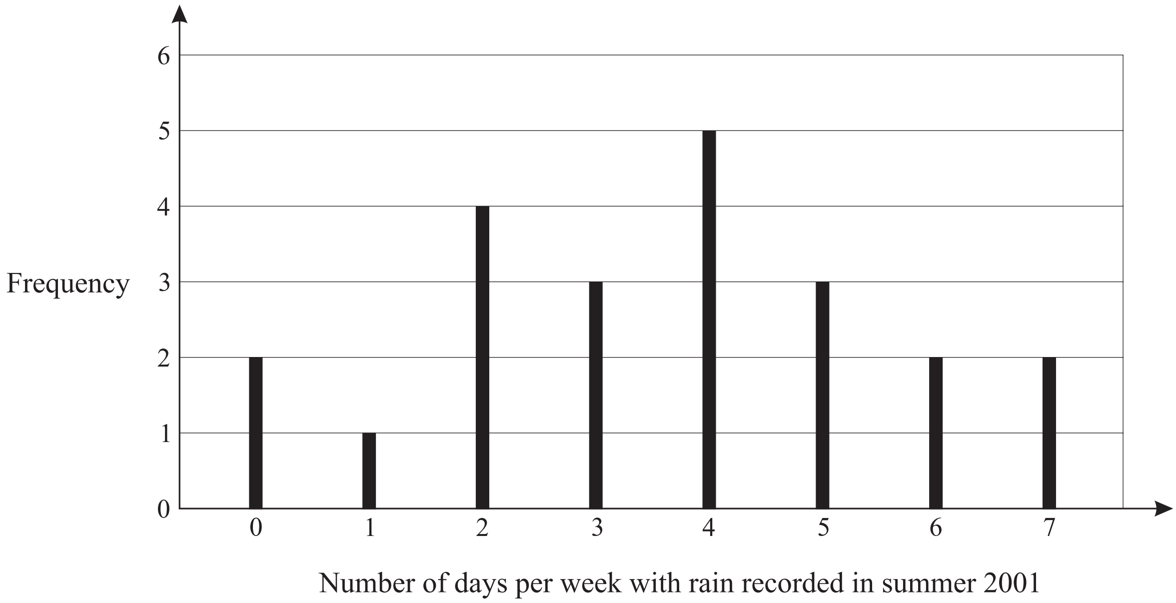
(B) exactly one of the materials. [2]

(ii) Given that the resident recycles aluminium, find the probability that this resident does not recycle paper. [2]

Two residents are selected at random.

(iii) Find the probability that exactly one of them recycles kitchen waste. [3]

5 A GCSE geography student is investigating a claim that global warming is causing summers in Britain to have more rainfall. He collects rainfall data from a local weather station for 2001 and 2006. The vertical line chart shows the number of days per week on which some rainfall was recorded during the 22 weeks of summer 2001.



(i) Show that the median of the data is 4, and find the interquartile range. [3]

(ii) For summer 2006 the median is 3 and the interquartile range is also 3. The student concludes that the data demonstrate that global warming is causing summer rainfall to decrease rather than increase. Is this a valid conclusion from the data? Give two brief reasons to justify your answer. [3]

6 In a phone-in competition run by a local radio station, listeners are given the names of 7 local personalities and are told that 4 of them are in the studio. Competitors phone in and guess which 4 are in the studio.

(i) Show that the probability that a randomly selected competitor guesses all 4 correctly is $\frac{1}{35}$. [2]

Let X represent the number of correct guesses made by a randomly selected competitor. The probability distribution of X is shown in the table.

r	0	1	2	3	4
$\mathrm{P}(X = r)$	0	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

(ii) Find the expectation and variance of X. [5]

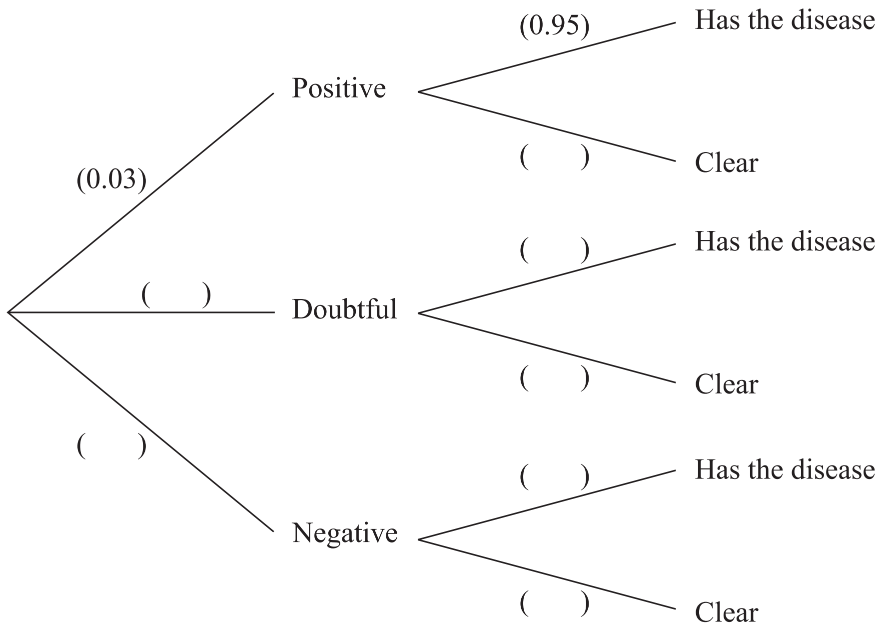
Section B (36 marks)

7 A screening test for a particular disease is applied to everyone in a large population. The test classifies people into three groups: 'positive', 'doubtful' and 'negative'. Of the population, 3% is classified as positive, 6% as doubtful and the rest negative.

In fact, of the people who test positive, only 95% have the disease. Of the people who test doubtful, 10% have the disease. Of the people who test negative, 1% actually have the disease.

People who do not have the disease are described as 'clear'.

(i) Copy and complete the tree diagram to show this information. [4]



(ii) Find the probability that a randomly selected person tests negative and is clear. [2]

(iii) Find the probability that a randomly selected person has the disease. [3]

(iv) Find the probability that a randomly selected person tests negative **given** that the person has the disease. [3]

(v) Comment briefly on what your answer to part **(iv)** indicates about the effectiveness of the screening test. [2]

Once the test has been carried out, those people who test doubtful are given a detailed medical examination. If a person has the disease the examination will correctly identify this in 98% of cases. If a person is clear, the examination will always correctly identify this.

(vi) A person is selected at random. Find the probability that this person either tests negative originally or tests doubtful and is then cleared in the detailed medical examination. [4]

8 A multinational accountancy firm receives a large number of job applications from graduates each year. On average 20% of applicants are successful.

A researcher in the human resources department of the firm selects a random sample of 17 graduate applicants.

(i) Find the probability that at least 4 of the 17 applicants are successful. [3]

(ii) Find the expected number of successful applicants in the sample. [2]

(iii) Find the most likely number of successful applicants in the sample, justifying your answer. [3]

It is suggested that mathematics graduates are more likely to be successful than those from other fields. In order to test this suggestion, the researcher decides to select a new random sample of 17 mathematics graduate applicants. The researcher then carries out a hypothesis test at the 5% significance level.

(iv) (A) Write down suitable null and alternative hypotheses for the test.

(B) Give a reason for your choice of the alternative hypothesis. [4]

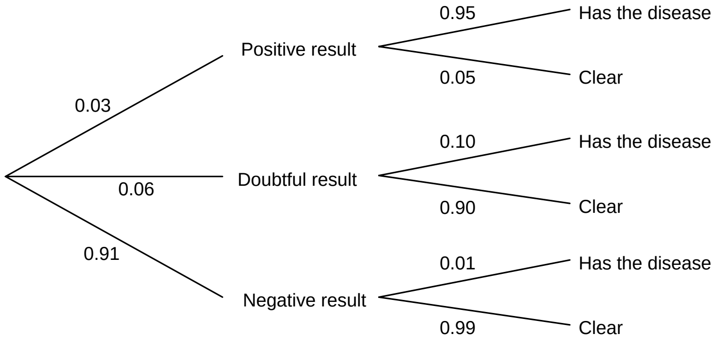
(v) Find the critical region for the test at the 5% level, showing all of your calculations. [4]

(vi) Explain why the critical region found in part **(v)** would be unaltered if a 10% significance level were used. [2]

Mark Scheme June 2007

Q1 (i)	$\binom{8}{4}$ ways to select $= 70$	M1 for $\binom{8}{4}$ A1 CAO	**2**
(ii)	$4! = 24$	B1 CAO	**1**
		TOTAL	**3**
Q2 (i)	<table><tr><td>Amount</td><td>0- <20</td><td>20- <50</td><td>50- <100</td><td>100- <200</td></tr><tr><td>Frequency</td><td>800</td><td>480</td><td>400</td><td>200</td></tr></table>	B1 for amounts B1 for frequencies	**2**
(ii)	Total ≈ $10\times800+35\times480+75\times400+150\times200=£84800$	M1 for their midpoints × their frequencies A1 CAO	**2**
		TOTAL	**4**
Q3 (i)	Mean $= \dfrac{3026}{56} = 54.0$ $S_{xx} = 178890 - \dfrac{3026^2}{56} = 15378$ $s = \sqrt{\dfrac{15378}{55}} = 16.7$	B1 for mean M1 for attempt at S_{xx} A1 CAO	**3**
(ii)	$\bar{x} + 2s = 54.0 + 2\times16.7 = 87.4$ So 93 is an outlier	M1 for their $\bar{x}$ +2×their s A1 FT for 87.4 and comment	**2**
(iii)	New mean = $1.2 \times 54.0 - 10 = 54.8$ New $s = 1.2 \times 16.7 = 20.1$	B1 FT M1A1 FT	**3**
		TOTAL	**8**
Q4 (i)	(A) P(at least one) $=\dfrac{36}{50}=\dfrac{18}{25}=0.72$ (B) P(exactly one) $=\dfrac{9+6+5}{50}=\dfrac{20}{50}=\dfrac{2}{5}=0.4$	B1 aef M1 for (9+6+5)/50 A1 aef	**3**
(ii)	P(not paper | aluminium) $=\dfrac{13}{24}$	M1 for denominator 24 or 24/50 or 0.48 A1 CAO	**2**
(iii)	P(one kitchen waste) $=2\times\dfrac{18}{50}\times\dfrac{32}{49}=\dfrac{576}{1225}=0.470$	M1 for both fractions M1 for 2 × product of both, or sum of 2 pairs A1	**3**
		TOTAL	**8**

Q5 (i)	11^{th} value is 4,12^{th} value is 4 so median is 4 Interquartile range = 5 – 2 = 3	B1 M1 for either quartile A1 CAO	**3**
(ii)	No, not valid any two valid reasons such as : • the sample is only for two years, which may not be representative • the data only refer to the local area, not the whole of Britain • even if decreasing it may have nothing to do with global warming • more days with rain does not imply more total rainfall • a five year timescale may not be enough to show a long term trend	B1 E1 E1	**3**
		TOTAL	**6**
Q6 (i)	Either P(all 4 correct) = $\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{35}$ or P(all 4 correct) = $\frac{1}{{}^7C_4}=\frac{1}{35}$	M1 for fractions, or ${}^7C_4$ seen A1 **NB answer given**	**2**
(ii)	E(X) = $1\times\frac{4}{35}+2\times\frac{18}{35}+3\times\frac{12}{35}+4\times\frac{1}{35}=\frac{80}{35}=2\frac{2}{7}=2.29$ E(X^2) = $1\times\frac{4}{35}+4\times\frac{18}{35}+9\times\frac{12}{35}+16\times\frac{1}{35}=\frac{200}{35}=5.714$ Var(X) = $\frac{200}{35}-\left(\frac{80}{35}\right)^2=\frac{24}{49}$ = 0.490 (to 3 s.f.)	M1 for Σrp (at least 3 terms correct) A1 CAO M1 for Σx^2p (at least 3 terms correct) M1*dep* for – their E(X)² A1 FT their E(X) provided Var(X) > 0	**5**
		TOTAL	**7**

	Section B		
Q7 (i)		G1 probabilities of result G1 probabilities of disease G1 probabilities of clear G1 labels	**4**
(ii)	P(negative and clear) $= 0.91 \times 0.99$ $= 0.9009$	M1 for their 0.91×0.99 A1 CAO	**2**
(iii)	P(has disease) $= 0.03 \times 0.95 + 0.06 \times 0.10 + 0.91 \times 0.01$ $= 0.0285 + 0.006 + 0.0091$ $= 0.0436$	M1 three products M1*dep* sum of three products A1 FT their tree	**3**
(iv)	P(negative | has disease) $= \dfrac{\text{P(negative } \textit{and} \text{ has disease)}}{\text{P(has disease)}} = \dfrac{0.0091}{0.0436} = 0.2087$	M1 for their 0.01×0.91 or 0.0091 on its own or as numerator M1 *indep* for their 0.0436 as denominator A1 FT their tree	**3**
(v)	Thus the test result is not very reliable. A relatively large proportion of people who have the disease will test negative.	E1 FT for idea of 'not reliable' or 'could be improved', etc E1 FT	**2**
(vi)	P(negative or doubtful and declared clear) $= 0.91 + 0.06 \times 0.10 \times 0.02 + 0.06 \times 0.90 \times 1$ $= 0.91 + 0.00012 + 0.054 = 0.96412$	M1 for their 0.91 + M1 for either triplet M1 for second triplet A1 CAO	**4**
		TOTAL	**18**

Q8 (i)	$X \sim B(17, 0.2)$ $P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.5489 = 0.4511$	B1 for 0.5489 M1 for 1 – their 0.5489 A1 CAO	**3**
(ii)	$E(X) = np = 17 \times 0.2 = 3.4$	M1 for product A1 CAO	**2**
(iii)	$P(X = 2) = 0.3096 - 0.1182 = 0.1914$ $P(X = 3) = 0.5489 - 0.3096 = 0.2393$ $P(X = 4) = 0.7582 - 0.5489 = 0.2093$ So 3 applicants is most likely	B1 for 0.2393 B1 for 0.2093 A1 CAO *dep* on both B1s	**3**
(iv)	(*A*) Let p = probability of a randomly selected maths graduate applicant being successful (for population) H_0: $p = 0.2$ H_1: $p > 0.2$ (*B*) H_1 has this form as the suggestion is that mathematics graduates are <u>more</u> likely to be successful.	B1 for definition of p in context B1 for H_0 B1 for H_1 E1	**4**
(v)	Let $X \sim B(17, 0.2)$ $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.8943 = 0.1057 > 5\%$ $P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9623 = 0.0377 < 5\%$ So critical region is {7,8,9,10,11,12,13,14,15,16,17}	B1 for 0.1057 B1 for 0.0377 M1 for at least one comparison with 5% A1 CAO for critical region *dep* on M1 and at least one B1	**4**
(vi)	Because $P(X \geq 6) = 0.1057 > 10\%$ Either: comment that 6 is still outside the critical region Or comparison $P(X \geq 7) = 0.0377 < 10\%$	E1 E1	**2**
		TOTAL	**18**