

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MEI STRUCTURED MATHEMATICS**

**4766**

Statistics 1

**Advanced Subsidiary General Certificate of Education**

**MEI STATISTICS**

**G241**

Statistics 1 (Z1)

Thursday

**12 JANUARY 2006**

Afternoon

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

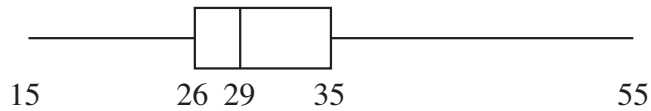
- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

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**This question paper consists of 6 printed pages and 2 blank pages.**

## Section A (36 marks)

- 1 The times taken, in minutes, by 80 people to complete a crossword puzzle are summarised by the box and whisker plot below.



- (i) Write down the range and the interquartile range of the times. [2]
- (ii) Determine whether any of the times can be regarded as outliers. [3]
- (iii) Describe the shape of the distribution of the times. [1]

- 2 Four letters are taken out of their envelopes for signing. Unfortunately they are replaced randomly, one in each envelope.

The probability distribution for the number of letters,  $X$ , which are now in the correct envelope is given in the following table.

$r$	0	1	2	3	4
$P(X = r)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

- (i) Explain why the case  $X = 3$  is impossible. [1]
- (ii) Explain why  $P(X = 4) = \frac{1}{24}$ . [2]
- (iii) Calculate  $E(X)$  and  $\text{Var}(X)$ . [5]

- 3 Over a long period of time, 20% of all bowls made by a particular manufacturer are imperfect and cannot be sold.

- (i) Find the probability that fewer than 4 bowls from a random sample of 10 made by the manufacturer are imperfect. [2]

The manufacturer introduces a new process for producing bowls. To test whether there has been an improvement, each of a random sample of 20 bowls made by the new process is examined. From this sample, 2 bowls are found to be imperfect.

- (ii) Show that this does not provide evidence, at the 5% level of significance, of a reduction in the proportion of imperfect bowls. You should show your hypotheses and calculations clearly. [6]

- 4 A company sells sugar in bags which are labelled as containing 450 grams.

Although the mean weight of sugar in a bag is more than 450 grams, there is concern that too many bags are underweight. The company can adjust the mean or the standard deviation of the weight of sugar in a bag.

- (i) State two adjustments the company could make. [2]

The weights,  $x$  grams, of a random sample of 25 bags are now recorded.

- (ii) Given that  $\sum x = 11\,409$  and  $\sum x^2 = 5\,206\,937$ , calculate the sample mean and sample standard deviation of these weights. [3]

- 5 A school athletics team has 10 members. The table shows which competitions each of the members can take part in.

		Competiton				
		100 m	200 m	110 m hurdles	400 m	Long jump
Athlete	Abel	✓	✓			✓
	Bernoulli		✓		✓	
	Cauchy	✓		✓		✓
	Descartes	✓	✓			
	Einstein		✓		✓	
	Fermat	✓		✓		
	Galois				✓	✓
	Hardy	✓	✓			✓
	Iwasawa		✓		✓	
	Jacobi			✓		

An athlete is selected at random. Events  $A, B, C, D$  are defined as follows.

$A$ : the athlete can take part in exactly 2 competitions.

$B$ : the athlete can take part in the 200 m.

$C$ : the athlete can take part in the 110 m hurdles.

$D$ : the athlete can take part in the long jump.

- (i) Write down the value of  $P(A \cap B)$ . [1]
- (ii) Write down the value of  $P(C \cup D)$ . [1]
- (iii) Which two of the four events  $A, B, C, D$  are mutually exclusive? [1]
- (iv) Show that events  $B$  and  $D$  are not independent. [2]

- 6** A band has a repertoire of 12 songs suitable for a live performance. From these songs, a selection of 7 has to be made.
- (i) Calculate the number of different selections that can be made. [2]
- (ii) Once the 7 songs have been selected, they have to be arranged in playing order. In how many ways can this be done? [2]

**Section B** (36 marks)

- 7 At East Cornwall College, the mean GCSE score of each student is calculated. This is done by allocating a number of points to each GCSE grade in the following way.

Grade	A*	A	B	C	D	E	F	G	U
Points	8	7	6	5	4	3	2	1	0

- (i) Calculate the mean GCSE score,  $\bar{X}$ , of a student who has the following GCSE grades:

A\*, A\*, A, A, A, B, B, B, B, C, D. [2]

60 students study AS Mathematics at the college. The mean GCSE scores of these students are summarised in the table below.

Mean GCSE score	Number of students
$4.5 \leq X < 5.5$	8
$5.5 \leq X < 6.0$	14
$6.0 \leq X < 6.5$	19
$6.5 \leq X < 7.0$	13
$7.0 \leq X \leq 8.0$	6

- (ii) Draw a histogram to illustrate this information. [3]

- (iii) Calculate estimates of the sample mean and the sample standard deviation. [5]

The scoring system for AS grades is shown in the table below.

AS Grade	A	B	C	D	E	U
Score	60	50	40	30	20	0

The Mathematics department at the college predicts each student's AS score,  $Y$ , using the formula  $Y = 13X - 46$ , where  $X$  is the student's average GCSE score.

- (iv) What AS grade would the department predict for a student with an average GCSE score of 7.4? [2]
- (v) What do you think the prediction should be for a student with an average GCSE score of 5.5? Give a reason for your answer. [3]
- (vi) Using your answers to part (iii), estimate the sample mean and sample standard deviation of the predicted AS scores of the 60 students in the department. [3]

- 8** Jane buys 5 jam doughnuts, 4 cream doughnuts and 3 plain doughnuts.

On arrival home, each of her three children eats one of the twelve doughnuts. The different kinds of doughnut are indistinguishable by sight and so selection of doughnuts is random.

Calculate the probabilities of the following events.

- (i) All 3 doughnuts eaten contain jam. [3]
- (ii) All 3 doughnuts are of the same kind. [3]
- (iii) The 3 doughnuts are all of a different kind. [3]
- (iv) The 3 doughnuts contain jam, given that they are all of the same kind. [3]

On 5 successive Saturdays, Jane buys the same combination of 12 doughnuts and her three children eat one each. Find the probability that all 3 doughnuts eaten contain jam on

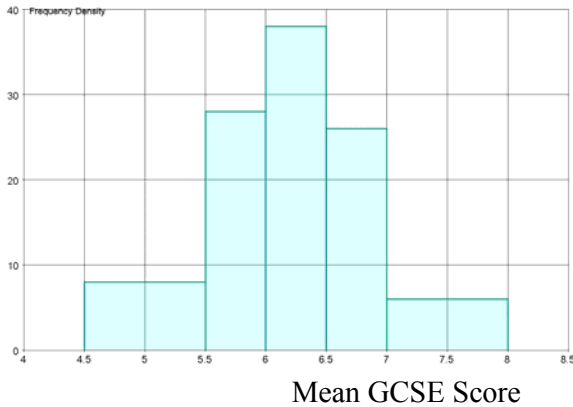
- (v) exactly 2 Saturdays out of the 5, [3]
- (vi) at least 1 Saturday out of the 5. [3]

## Mark Scheme

<b>Q 1</b>	The range = $55 - 15 = 40$	B1 CAO	
<b>(i)</b>	The interquartile range = $35 - 26 = 9$	B1 CAO	<b>2</b>
<b>(ii)</b>	$35 + 1.5 \times 9 = 48.5$ $26 - 1.5 \times 9 = 12.5$ Any value $> 48.5$ is an outlier (so 55 will be an outlier),	M1 for 48.5 oe M1 for 12.5 oe  A1 (FT their IQR in (i))	<b>3</b>
<b>(iii)</b>	One valid comment such as eg: Positively skewed Middle 50% of data is closely bunched	E1	<b>1</b>
		<b>TOTAL</b>	<b>6</b>
<b>2</b>			
<b>(i)</b>	Impossible because if 3 letters are correct, the fourth must be also.	E1	<b>1</b>
<b>(ii)</b>	There is only one way to place letters correctly. There are $4! = 24$ ways to arrange 4 letters. OR: $\frac{1}{4} \times \frac{1}{3} \times \frac{1}{2}$ NOTE: <b>ANSWER GIVEN</b>	E1 E1  B1 for $\frac{1}{4} \times \frac{1}{3}$ B1 for $\times \frac{1}{2}$	<b>2</b>
<b>(iii)</b>	$E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{4} + 4 \times \frac{1}{24} = 1$  $E(X^2) = 1 \times \frac{1}{3} + 4 \times \frac{1}{4} + 16 \times \frac{1}{24} = 2$  So $\text{Var}(X) = 2 - 1^2 = 1$	M1 For $\sum xp$ (at least 2 non-zero terms correct) A1 CAO  M1 for $\sum x^2 p$ (at least 2 non-zero terms correct) M1dep for – their $E(X)^2$  A1 FT their $E(X)$ provided $\text{Var}(X) > 0$	<b>5</b>
		<b>TOTAL</b>	<b>8</b>

<b>3</b> <b>(i)</b>	$X \sim B(10, 0.2)$ $P(X < 4) = P(X \leq 3) = 0.8791$ OR attempt to sum $P(X = 0, 1, 2, 3)$ using $X \sim B(10, 0.2)$ can score M1, A1	M1 for $X \leq 3$ A1	<b>2</b>
<b>(ii)</b>	Let $p$ = the probability that a bowl is imperfect  $H_0 : p = 0.2 \quad H_1 : p < 0.2$  $X \sim B(20, 0.2)$ $P(X \leq 3) = 0.2061$ $0.2061 > 5\%$ Cannot reject $H_0$ and so insufficient evidence to claim a reduction.  OR using critical region method: CR is $\{0\}$ B1, 2 not in CR M1, A1 as above	B1 Definition of $p$  B1, B1  B1 for 0.2061 seen M1 for this comparison  A1 <i>dep</i> for comment <u>in context</u>	<b>3</b>
		<b>TOTAL</b>	<b>8</b>
<b>4</b> <b>(i)</b>	The company could increase the mean weight. The company could decrease the standard deviation.	B1 CAO B1	<b>2</b>
<b>(ii)</b>	Sample mean = $11409/25 = 456.36$  $S_{xx} = 5206937 - \frac{11409^2}{25} = 325.76$  Sample s.d = $\sqrt{\frac{325.76}{24}} = 3.68$	B1  M1 for $S_{xx}$  A1	<b>3</b>
		<b>TOTAL</b>	<b>5</b>
<b>5</b> <b>(i)</b>	$P(A \cap B) = 0.4$	B1 CAO	<b>1</b>
<b>(ii)</b>	$P(C \cup D) = 0.6$	B1 CAO	<b>1</b>
<b>(iii)</b>	Events B and C are mutually exclusive.	B1 CAO	<b>1</b>
<b>(iv)</b>	$P(B) = 0.6, P(D) = 0.4$ and $P(B \cap D) = 0.2$  $0.6 \times 0.4 \neq 0.2$ (so B and D not independent)	B1 for $P(B \cap D) = 0.2$ soi  E1	<b>2</b>
		<b>TOTAL</b>	<b>5</b>
<b>6</b> <b>(i)</b>	Number of selections = $\binom{12}{7} = 792$	M1 for $\binom{12}{7}$ A1 CAO	<b>2</b>
<b>(ii)</b>	Number of arrangements = $7! = 5040$	M1 for $7!$ , A1 CAO	<b>2</b>
		<b>TOTAL</b>	<b>4</b>



7 (i)	Mean score = $(2 \times 8 + 3 \times 7 + 4 \times 6 + 5 + 4) / 11 = 6.36$	M1 for $\sum fx / 11$ A1 CAO	2																												
(ii)		<p>G1 Linear sensible scales</p> <p>G1 fds of 8, 28, 38, 26, 6 or <math>4k</math>, <math>14k</math>, <math>19k</math>, <math>13k</math>, <math>3k</math> for sensible values of <math>k</math> either on script or on graph.</p> <p>G1 (dep on reasonable attempt at fd) Appropriate label for vertical scale eg 'Frequency density', 'frequency per <math>\frac{1}{2}</math> unit', 'students per mean GCSE score'. (allow Key)</p>	3																												
(iii)	<table border="1" data-bbox="349 835 922 1123"> <thead> <tr> <th>Mid point, <math>x</math></th><th><math>f</math></th><th><math>fx</math></th><th><math>fx^2</math></th></tr> </thead> <tbody> <tr> <td>5</td><td>8</td><td>40</td><td>200</td></tr> <tr> <td>5.75</td><td>14</td><td>80.5</td><td>462.875</td></tr> <tr> <td>6.25</td><td>19</td><td>118.75</td><td>742.1875</td></tr> <tr> <td>6.75</td><td>13</td><td>87.75</td><td>592.3125</td></tr> <tr> <td>7.5</td><td>6</td><td>45</td><td>337.5</td></tr> <tr> <td></td><td>60</td><td>372</td><td>2334.875</td></tr> </tbody> </table> <p>Sample mean = <math>372 / 60 = 6.2</math></p> $S_{xx} = 2334.875 - \frac{372^2}{60} = 28.475$ <p>Sample s.d = <math>\sqrt{\frac{28.475}{59}} = 0.695</math></p>	Mid point, $x$	$f$	$fx$	$fx^2$	5	8	40	200	5.75	14	80.5	462.875	6.25	19	118.75	742.1875	6.75	13	87.75	592.3125	7.5	6	45	337.5		60	372	2334.875	<p>B1 mid points</p> <p>B1FT <math>\sum fx</math> and <math>\sum fx^2</math></p> <p>B1 CAO</p> <p>M1 for their <math>S_{xx}</math></p> <p>A1 CAO</p>	5
Mid point, $x$	$f$	$fx$	$fx^2$																												
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(iv)	<p>Prediction of score = <math>13 \times 7.4 - 46 = 50.2</math></p> <p>So predicted AS grade would be B</p>	<p>M1 For <math>13 \times 7.4 - 46</math></p> <p>A1 dep on 50.2 (or 50) seen</p>	2																												
(v)	<p>Prediction of score = <math>13 \times 5.5 - 46 = 25.5</math></p> <p>So predicted grade would be D/E (allow D or E)</p> <p>Because score roughly halfway from 20 to 30, OR (for D) closer to D than E OR (for E) past E but not up to D boundary</p>	<p>M1 For <math>13 \times 5.5 - 46</math></p> <p>A1 dep on 25.5 (or 26 or 25) seen</p> <p>E1 For explanation of conversion – logical statement/argument that supports their choice.</p>	3																												
(vi)	<p>Mean = <math>13 \times 6.2 - 46 = 34.6</math></p> <p>Standard deviation = <math>13 \times 0.695 = 9.035</math></p>	<p>B1 FT their 6.2</p> <p>M1 for <math>13 \times</math> their 0.695</p> <p>A1 FT</p>	3																												
		<b>TOTAL</b>	<b>18</b>																												

<b>8</b> <b>(i)</b>	$P(\text{all jam})$ $= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$ $= \frac{1}{22} = 0.04545$	M1 $5 \times 4 \times 3$ or $\binom{5}{3}$ in numerator M1 $12 \times 11 \times 10$ or $\binom{12}{3}$ in denominator A1 CAO	<b>3</b>
<b>(ii)</b>	$P(\text{all same})$ $= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} + \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} + \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10}$ $= \frac{1}{22} + \frac{1}{55} + \frac{1}{220} = \frac{3}{44} = 0.06818$	M1 Sum of 3 reasonable triples or combinations M1 Triples or combinations correct A1 CAO	<b>3</b>
<b>(iii)</b>	$P(\text{all different})$ $= 6 \times \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$ $= \frac{3}{11} = 0.2727$	M1 5,4,3 M1 $6 \times$ three fractions or $\binom{12}{3}$ denom. A1 CAO	<b>3</b>
<b>(iv)</b>	$P(\text{all jam given all same}) = \frac{\frac{1}{22}}{\frac{3}{44}} = \frac{2}{3}$	M1 Their (i) in numerator M1 Their (ii) in denominator A1 CAO	<b>3</b>
<b>(v)</b>	$P(\text{all jam exactly twice})$ $= \binom{5}{2} \times \left(\frac{1}{22}\right)^2 \times \left(\frac{21}{22}\right)^3 = 0.01797$	M1 for $\binom{5}{2} \times \dots$ M1 for their $p^2 q^3$ A1 CAO	<b>3</b>
<b>(vi)</b>	$P(\text{all jam at least once})$ $= 1 - \left(\frac{21}{22}\right)^5 = 0.2075$	M1 for their $q^5$ M1 indep for $1 - 5^{\text{th}}$ power A1 CAO	<b>3</b>
<b>TOTAL</b>			<b>18</b>