# OXFORD CAMBRIDGE AND RSA EXAMINATIONS 

## Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

## 4766

## Statistics 1

Thursday 9 JUNE $2005 \quad$ Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- There is an insert for use in Questions 2,5 and 6.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.


## Section A (36 marks)

1 At a certain stage of a football league season, the numbers of goals scored by a sample of 20 teams in the league were as follows.
$\begin{array}{llllllllllllllllllll}22 & 23 & 23 & 23 & 26 & 28 & 28 & 30 & 31 & 33 & 33 & 34 & 35 & 35 & 36 & 36 & 37 & 46 & 49 & 49\end{array}$
(i) Calculate the sample mean and sample variance, $s^{2}$, of these data.
(ii) The three teams with the most goals appear to be well ahead of the other teams. Determine whether or not any of these three pieces of data may be considered outliers.

2 Answer part (i) of this question on the insert provided.
A taxi driver operates from a taxi rank at a main railway station in London. During one particular week he makes 120 journeys, the lengths of which are summarised in the table.

| Length <br> $(x$ miles $)$ | $0<x \leqslant 1$ | $1<x \leqslant 2$ | $2<x \leqslant 3$ | $3<x \leqslant 4$ | $4<x \leqslant 6$ | $6<x \leqslant 10$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> journeys | 38 | 30 | 21 | 14 | 9 | 8 |

(i) On the insert, draw a cumulative frequency diagram to illustrate the data.
(ii) Use your graph to estimate the median length of journey and the quartiles.

Hence find the interquartile range.
(iii) State the type of skewness of the distribution of the data.

3 Jeremy is a computing consultant who sometimes works at home. The number, $X$, of days that Jeremy works at home in any given week is modelled by the probability distribution

$$
\begin{equation*}
\mathrm{P}(X=r)=\frac{1}{40} r(r+1) \quad \text { for } r=1,2,3,4 . \tag{1}
\end{equation*}
$$

(i) Verify that $\mathrm{P}(X=4)=\frac{1}{2}$.
(ii) Calculate $\mathrm{E}(X)$ and $\operatorname{Var}(X)$.
(iii) Jeremy works for 45 weeks each year. Find the expected number of weeks during which he works at home for exactly 2 days.

4 An examination paper consists of three sections.

- Section A contains 6 questions of which the candidate must answer 3
- Section B contains 7 questions of which the candidate must answer 4
- Section C contains 8 questions of which the candidate must answer 5
(i) In how many ways can a candidate choose 3 questions from Section A?
(ii) In how many ways can a candidate choose 3 questions from Section A, 4 from Section B and 5 from Section C?

A candidate does not read the instructions and selects 12 questions at random.
(iii) Find the probability that they happen to be 3 from Section A, 4 from Section B and 5 from Section C.

## 5 Answer part (i) of this question on the insert provided.

The lowest common multiple of two integers, $x$ and $y$, is the smallest positive integer which is a multiple of both $x$ and $y$. So, for example, the lowest common multiple of 4 and 6 is 12 .
(i) On the insert, complete the table giving the lowest common multiples of all pairs of integers between 1 and 6 .

|  |  | Second integer |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| First | 1 | 1 | 2 | 3 | 4 | 5 | 6 |  |
|  | 2 | 2 | 2 | 6 | 4 | 10 | 6 |  |
|  | 3 | 3 | 6 | 3 | 12 | 15 | 6 |  |
|  | 4 | 4 | 4 | 12 |  |  | 12 |  |
|  | 5 | 5 | 10 | 15 |  |  |  |  |
|  | 6 | 6 | 6 | 6 | 12 |  |  |  |

Two fair dice are thrown and the lowest common multiple of the two scores is found.
(ii) Use the table to find the probabilities of the following events.
(A) The lowest common multiple is greater than 6.
(B) The lowest common multiple is a multiple of 5 .
(C) The lowest common multiple is both greater than 6 and a multiple of 5 .
(iii) Use your answers to part (ii) to show that the events "the lowest common multiple is greater than 6 " and "the lowest common multiple is a multiple of 5 " are not independent.

Section B (36 marks)
6 Answer part (i) of this question on the insert provided.
Mancaster Hockey Club invite prospective new players to take part in a series of three trial games. At the end of each game the performance of each player is assessed as pass or fail. Players who achieve a pass in all three games are invited to join the first team squad. Players who achieve a pass in two games are invited to join the second team squad. Players who fail in two games are asked to leave. This may happen after two games.

- The probability of passing the first game is 0.9
- Players who pass any game have probability 0.9 of passing the next game
- Players who fail any game have probability 0.5 of failing the next game
(i) On the insert, complete the tree diagram which illustrates the information above.

(ii) Find the probability that a randomly selected player
(A) is invited to join the first team squad,
$(B)$ is invited to join the second team squad.
(iii) Hence write down the probability that a randomly selected player is asked to leave.
(iv) Find the probability that a randomly selected player is asked to leave after two games, given that the player is asked to leave.

Angela, Bryony and Shareen attend the trials at the same time. Assuming their performances are independent, find the probability that
(v) at least one of the three is asked to leave,
(vi) they pass a total of 7 games between them.

7 A game requires 15 identical ordinary dice to be thrown in each turn.
Assuming the dice to be fair, find the following probabilities for any given turn.
(i) No sixes are thrown.
(ii) Exactly four sixes are thrown.
(iii) More than three sixes are thrown.

David and Esme are two players who are not convinced that the dice are fair. David believes that the dice are biased against sixes, while Esme believes the dice to be biased in favour of sixes.

In his next turn, David throws no sixes. In her next turn, Esme throws 5 sixes.
(iv) Writing down your hypotheses carefully in each case, decide whether
(A) David's turn provides sufficient evidence at the $10 \%$ level that the dice are biased against sixes,
(B) Esme's turn provides sufficient evidence at the $10 \%$ level that the dice are biased in favour of sixes.
(v) Comment on your conclusions from part (iv).

RECOGNISING ACHIEVEMENT

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## MEI STRUCTURED MATHEMATICS

4766
Statistics 1
INSERT
Thursday
9 JUNE 2005
Morning
1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- This insert should be used in Questions 2 part (i), 5 part (i) and 6 part (i).
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

2 (i)


5 (i)

|  |  | Second integer |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| First <br> integer | 1 | 1 | 2 | 3 | 4 | 5 | 6 |  |
|  | 2 | 2 | 2 | 6 | 4 | 10 | 6 |  |
|  | 3 | 3 | 6 | 3 | 12 | 15 | 6 |  |
|  | 4 | 4 | 4 | 12 |  |  | 12 |  |
|  | 5 | 5 | 10 | 15 |  |  |  |  |
|  | 6 | 6 | 6 | 6 | 12 |  |  |  |

6 (i)


Statistics 1 (4766)
Mark Scheme

\begin{tabular}{|c|c|c|c|}
\hline Qn \& Answer \& Mk \& Comment \\
\hline \begin{tabular}{l}
1 \\
(i) \\
(ii)
\end{tabular} \& \begin{tabular}{l}
\[
\text { Mean }=657 / 20=32.85
\]
\[
\text { Variance }=\frac{1}{19}\left(22839-\frac{657^{2}}{20}\right)=66.13
\] \\
Standard deviation \(=8.13\)
\[
32.85+2(8.13)=49.11
\] \\
none of the 3 values exceed this so no outliers
\end{tabular} \& \begin{tabular}{l}
B1 cao \\
M1 \\
A1 cao \\
M1 ft \\
A1 ft
\end{tabular} \& Calculation of 49.11 \\
\hline \begin{tabular}{l}
\[
2
\] \\
(i)
\end{tabular} \& Length of journey \& \[
\begin{aligned}
\& \text { G1 } \\
\& \text { G1 } \\
\& \text { G1 }
\end{aligned}
\] \& \begin{tabular}{l}
For calculating \\
38,68,89,103,112,120 \\
Plotting end points \\
Heights inc \((0,0)\)
\end{tabular} \\
\hline (ii)

(iii) \& | Median $=1.7$ miles |
| :--- |
| Lower quartile $=0.8$ miles |
| Upper quartile $=3$ miles |
| Interquartile range $=2.2$ miles |
| The graph exhibits positive skewness | \& \[

$$
\begin{aligned}
& \mathrm{B} 1 \\
& \text { M1 } \\
& \text { M1 } \\
& \text { A1 ft } \\
& \text { E1 }
\end{aligned}
$$
\] \& <br>

\hline
\end{tabular}




| $6$ <br> (i) |  | $\begin{aligned} & \text { G1 } \\ & \text { G1 } \end{aligned}$ | Probabilities Outcomes |
| :---: | :---: | :---: | :---: |
| (ii) |  | M1 |  |


| (A) | $\mathrm{P}($ First team $)=0.9^{3}=0.729$ | A1 |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{P}($ Second team $)=$ | M1 | 1 correct triple |
| (B) | $0.9 \times 0.9 \times 0.1+0.9 \times 0.1 \times 0.5+0.1 \times 0.9 \times 0.5$ | M1 | 3 correct triples added |
|  | $=0.081+0.045+0.045=0.171$ | A1 |  |
| (iii) | $\mathrm{P}($ asked to leave $)=1-0.729-0.171$ |  |  |
|  | $=0.1$ | B1 |  |
| (iv) | P (Leave after two games given leaves) |  |  |
|  | $=\frac{0.1 \times 0.5}{0.1}=\frac{1}{2}$ | M1 ft A1 cao | Denominator |
| (v) | P (at least one is asked to leave) $=1-0.9^{3}=0.271$ | M1 ft M1 <br> A1 cao | $\begin{aligned} & \text { Calc'n of } 0.9 \\ & 1-()^{3} \end{aligned}$ |
| (vi) | P (Pass a total of 7 games) |  |  |
|  | $\begin{aligned} & =\mathrm{P}(\text { First, Second, Second) }+\mathrm{P}(\text { First, First, } \\ & \text { Leave after three games) } \end{aligned}$ | M1 <br> M1 ft | Attempts both $0.729(0.171)^{2}$ |
|  | $=3 \times 0.729 \times 0.171^{2}+3 \times 0.729^{2} \times 0.05$ | M1 ft | $0.05(0.729)^{2}$ |
|  | $\begin{aligned} & =0.064+0.080 \\ & =0.144 \end{aligned}$ | M1 <br> A1 cao | multiply by 3 |


| 7 <br> (i) | $X \sim B\left(15, \frac{1}{6}\right)$ |  |  |
| :--- | :--- | :--- | :--- |
|  | $P(X=0)=\left(\frac{5}{6}\right)^{15}=0.065$ | M1 | $\left(\frac{5}{6}\right)^{15}$ |
| (ii) | $P(X=4)=\binom{15}{4} \times\left(\frac{1}{6}\right)^{4} \times\left(\frac{5}{6}\right)^{11}$ | M1 cao | $\left(\frac{1}{6}\right)^{4}\left(\frac{5}{6}\right)^{11}$ |
|  | $=0.142$ (or 0.9102-0.7685) | M1 | multiply by $\left.\begin{array}{l}15 \\ 4\end{array}\right)$ |


| (iii) | $\begin{aligned} P(X>3) & =1-P(X \leq 3) \\ & =1-0.7685=0.232 \end{aligned}$ | M1 <br> A1 |  |
| :---: | :---: | :---: | :---: |
| (iv) |  | B1 | Definition of p |
| (A) | Let $\mathrm{p}=$ probability of a six on any throw $\begin{array}{ll} H_{0}: p=\frac{1}{6} & H_{1}: p<\frac{1}{6} \\ X \sim B\left(15, \frac{1}{6}\right) & \end{array}$ | B1 | Both hypotheses |
|  | 6 | M1 | 0.065 |
|  | $\begin{aligned} & P(X=0)=0.065 \\ & 0.065<0.1 \text { and so reject } H_{0} \end{aligned}$ | M1 dep | Comparison |
|  | Conclude that there is sufficient evidence at the $10 \%$ level that the dice are biased against sixes. | B1 | Both hypotheses |
| (B) | Let $\mathrm{p}=$ probability of a six on any throw $H_{0}: p=\frac{1}{6} \quad H_{1}: p>\frac{1}{6}$ |  |  |
|  | $\begin{aligned} & X \sim B\left(15, \frac{1}{6}\right) \\ & P(X \geq 5)=1-P(X \leq 4)=1-0.910=0.09 \\ & 0.09<0.1 \text { and so reject } H_{0} \end{aligned}$ | M1 <br> M1 dep <br> E1 dep | 0.09 <br> Comparison |
|  | Conclude that there is sufficient evidence at the $10 \%$ level that the dice are biased in favour of sixes. | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \end{aligned}$ | Contradictory By chance |
| (v) | Conclusions contradictory. <br> Even if null hypothesis is true, it will be rejected $10 \%$ of the time purely by chance. Or other sensible comments. |  |  |

