Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Show that the equation

$$x = (\cos x)^3 \qquad (*)$$

where x is in radians, has a root in the interval $(0, 1)$. **[2]**

This root is denoted by α.

(ii) Show numerically that the iteration

$$x_{r+1} = (\cos x_r)^3$$

with $x_0 = 0.6$ does not converge to α. **[2]**

(iii) Show that

$$x = \sqrt{x(\cos x)^3}$$

is a rearrangement of (*). Use the corresponding iteration to find α correct to 4 decimal places. **[4]**

2 An estimate is required of I, where

$$I = \int_0^{0.6} \mathrm{f}(x)\,\mathrm{d}x.$$

The only available values of $\mathrm{f}(x)$ are as follows.

x	0	0.1	0.2	0.4
$\mathrm{f}(x)$	2.3063	2.2769	2.1883	1.8308

(i) Obtain the best possible estimates of $\int_0^{0.2} \mathrm{f}(x)\,\mathrm{d}x$ and $\int_{0.2}^{0.6} \mathrm{f}(x)\,\mathrm{d}x$. Hence give an estimate of I. **[5]**

(ii) State what you would do differently if $\mathrm{f}(0.6)$ became available. **[1]**

3 A computer program is used to calculate values of f where

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}.$$

One possible formula for f is

$$f = \frac{1}{\left(\frac{1}{u} - \frac{1}{v}\right)}. \qquad (*)$$

(i) Show that another formula for f is

$$f = \frac{uv}{v-u}. \qquad (**)$$ **[1]**

The program stores and calculates all numbers rounded to 5 significant figures.

(ii) Find the values of f given by the program using (*) and (**) when $u = 11$ and $v = 11.05$.

Show that one of these values is exact and find the relative error in the other. **[6]**

(iii) State what process gives rise to the error in the inexact value. **[1]**

4 The table shows values of a function $\mathrm{f}(x)$ correct to 5 decimal places.

x	1	1.1	1.01	1.001	1.0001	1.00001
$\mathrm{f}(x)$	0.94404	0.96771	0.94641	0.94428	0.94406	0.94404

(i) Find five estimates of $\mathrm{f}'(1)$ using the forward difference method. **[3]**

(ii) Comment on the accuracy of these estimates.

Give a value for $\mathrm{f}'(1)$ to the accuracy that you consider appropriate. Justify your answer. **[3]**

5 An approximate formula for $\sqrt{x}$ of the form

$$\sqrt{x} \approx a + bx \qquad (*)$$

is required for values of x near to 1.

(i) Find the values of a and b for which (*) is exact when $x = 1$ and $x = 1.21$. **[4]**

(ii) With these values of a and b, find the absolute and relative errors in (*) when $x = 0.81$. **[4]**

Section B (36 marks)

6 The variables p and q are known to take the following values.

p	1	2	3
q	2.2	2.8	5.2

(i) Plot these points and draw, by eye, a smooth curve through them. **[2]**

(ii) Use Newton's forward difference interpolation formula to obtain a quadratic expression for q in terms of p. Give your answer in simplified form. **[6]**

(iii) Estimate q when $p = 2.5$. Let this estimate be denoted by α. **[2]**

(iv) Now suppose that a quadratic expression for p in terms of q is required. Explain why Newton's formula could not be used for this purpose. Use Lagrange's method to write down an expression for this quadratic. (You are not required to simplify this expression.) **[5]**

(v) Estimate p when q is equal to the value α found in part **(iii)**. Comment on your answer. **[3]**

7 **(i)** Show that the equation

$$3x^4 + x - 2 = 0 \qquad (*)$$

has a root, α, in the interval $(0, 1)$.

Show that (*) does not have any other positive roots. **[3]**

(ii) Sketch the curve $y = 3x^4 + x - 2$ for $0 \leqslant x \leqslant 1$. **[2]**

The secant method, with $x_0 = 0$ and $x_1 = 1$ is used to find α.

(iii) Find x_2 exactly, and calculate x_3 and x_4 correct to 6 significant figures.

Show, by means of appropriate lines on your graph, how the secant method produces these values. **[8]**

(iv) Iterate the secant method further to find α correct to 3 significant figures. Show that you have obtained the required accuracy. **[5]**

END OF QUESTION PAPER

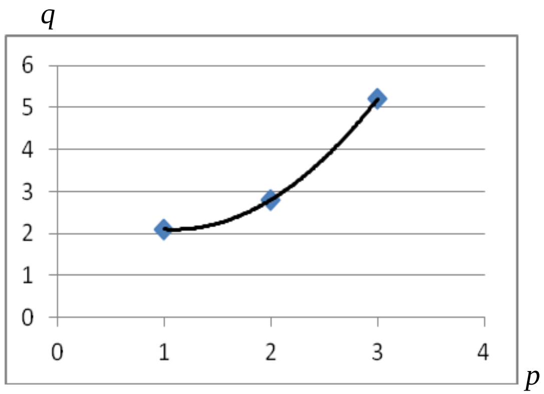
Section B (36 marks)

6 (i) **There is a spare copy of this graph paper below.**

Spare copy of graph paper for question 6 (i)

Question			Answer	Marks	Guidance	
1	(i)		x LHS RHS 0 0 < 1 1 1 > 0.1577	M1A1 **[2]**	Or equivalent	±0.8423
1	(ii)		First few iterates are: 0.6 0.562201 0.605676 0.555651 0.613166 (clearly not converging)	M1A1 **[2]**	At least 4 values required No explanation required	
1	(iii)		Convincing algebra: e.g. multiply by x and take square root First few iterates are: 0.6 0.580793 0.582561 0.582431 0.582441 Root is 0.5824 to 4 dp	B1 M1A1 A1 **[4]**	Accept reverse argument Cao. SC: A1 for correct answer with no iterations shown	
2	(i)		Use Simpson's rule for (0, 0.2), mid-point rule for (0.2, 0.6) S = 0.45341, M = 0.73232, I = 1.18573	M1M1 A1A1A1 **[5]**	Stated or used Accept 4 or more dp for I	
2	(ii)		Use a second Simpson's rule	E1 **[1]**		
3	(i)		Convincing algebra: eg common denominator then reciprocal	B1 **[1]**	Accept reverse argument	
3	(ii)		To 5 dp, 1/11 = 0.090909, 1/11.05 = 0.090498 Hence (*) gives2433.1 And (**) gives 2431, which is the correct answer The relative error in (*) is 0.00086 (or 0.086%)	M1A1 A1 B1E1 B1 **[6]**	 It's exact because >5dp not required	
3	(iii)		Subtraction of nearly equal rounded quantities	B1 **[1]**	Must say rounding + nearly equal	

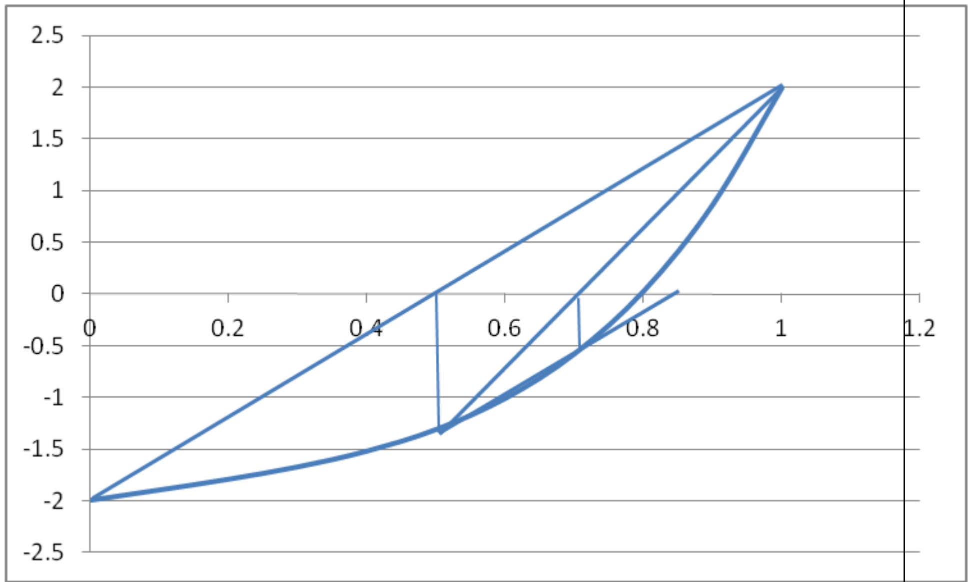
Question			Answer	Marks	Guidance	
4	(i)		h f '(1) 0.1 0.2367 0.01 0.237 0.001 0.24 0.0001 0.2 0.00001 0	M1A1A1 **[3]**	M1 for first value A1 two more A1 all	
	(ii)		Estimates become less precise as h reduces so it is difficult to comment on accuracy. But 0.237 or 0.24 seem best - agreement before precision lost	E1 B1E1 **[3]**	Must refer to reducing h for 2nd E1 Accept either	
5	(i)		$x = 1$: $1 = a + b$ $x = 1.21$ $1.1 = a + 1.21\ b$ Hence $a = 11/21 = 0.523\,81$ and $b = 10/21 = 0.476\,19$	B1 B1 M1A1 **[4]**	Either equation Other equation Cao	
5	(ii)		$x = 0.81$: exact 0.9, approximate 0.909 52 Absolute error 0.009 52 Relative error 0.010 58	B1B1 B1 B1 **[4]**		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>6</td><td>(i)</td><td></td><td>
</td><td>G1
G1

[2]</td><td>Points
Smooth curve
Accept axes interchanged</td><td></td></tr>
<tr><td>6</td><td>(ii)</td><td></td><td>1   2.2   1st diffs   2nd diff
2   2.8   0.6
3   5.2   2.4   1.8
$q = 2.2 + 0.6\,(p - 1) + 1.8\,(p - 1)(p - 2) / 2$
$q = 0.9\,p^2 - 2.1\,p + 3.4$</td><td>M1A1

B1B1B1
B1
[6]</td><td>Table

Each term
Cao</td><td></td></tr>
<tr><td>6</td><td>(iii)</td><td></td><td>Substitute $p = 2.5$ into $q = \ldots$ from part (ii)
Obtain $q = 3.775$</td><td>M1
A1
[2]</td><td>FT sub'n into incorrect quadratic
Cao</td><td></td></tr>
<tr><td>6</td><td>(iv)</td><td></td><td>Unequal differences in values of q
$p = 1\,(q - 2.8)\,(q - 5.2) / (2.2 - 2.8)\,(2.2 - 5.2)$
$+ 2\,(q - 2.2)\,(q - 5.2) / (2.8 - 2.2)\,(2.8 - 5.2)$
$+ 3\,(q - 2.2)\,(q - 2.8) / (5.2 - 2.2)\,(5.2 - 2.8)$</td><td>E1
M1A1
A1
A1
[5]</td><td>
One term correct
Two terms
Three terms</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
6	(v)		Substitute q = 3.775 into p = from part **(iii)** Obtain p = 2.985... Not 2.5 (or more sophisticated comments about inverse interpolation!)	M1 A1 E1 **[3]**	Allow sub'n of their q Cao Must be from correct p, q	
7	(i)		x = 0, LHS = –2, x = 1, LHS = 2 (hence root) Gradient is $12x^3 + 1$ which is positive for x positive (hence no more positive roots)	B1 M1 E1 **[3]**	Accept +ve, -ve Or, eg, for $x > 1$ f(x) > 0	
7	(ii)		Sketch of curve (see below)	G2 **[2]**	G1 points G1 smooth curve	
7	(iii)		$x_2 = 0.5$, $x_3 = 0.698\ 113$, $x_4 = 0.859\ 557$ 	M1A1A1A1 A1 A1 A1 A1 **[8]**	M1 for secant rule A for cao Use of f-p gets M0A1A1M0 (sc) each correct construction line accurate location of roots	

Question			Answer	Marks	Guidance	
7	**(iv)**		Further iterates: 0.785 679, 0.794 930, 0.795959, 0.795 942, ...	M1A1	Must be using secant method	
			Root is 0.796 to 3 dp	A1	Cao. SC M0A0A1 for other methods	
			Check: f(0.7955) = –0.003 12, f(0.7965) = 0.003 94	M1 A1		
				[5]		

4776 Numerical Methods (Written Examination)

General Comments:

As usual, there was a lot of good work seen from candidates who have clearly engaged with the specification and who understand the techniques of numerical mathematics. Routine numerical work with standard algorithms was generally carried out accurately – though not always efficiently.

It was notable on this paper that elementary algebra is difficult for some candidates. Q1(iii), Q3(i) and Q5(i) all revealed weaknesses in some candidates' algebraic skills.

Poor presentation continues to be an issue, with illegible hand-writing and a propensity to scatter numerical work haphazardly on the page being far too common. Examiners will always do their best to decipher candidates' work, but sometimes the challenges are insurmountable.

Comments on Individual Questions:

Question 1

The numerical solution of equations is generally done well and this question was no exception. Locating the root in part (i) and showing that the iteration fails to converge in part (ii) presented no difficulties. The algebra in part (iii) defeated some candidates, but almost everyone was able to use the new iteration to obtain the root.

Question 2

Many found this question difficult. In part (i) the best possible estimates for the two integrals come from using Simpson's rule for the first and the mid-point rule for the second. Common errors were to use the trapezium rule or the mid-point rule for the first integral. And some candidates thought that the second integral required the estimation of f(0.6).

In part (ii) the statements made were often vague: 'I would use f(0.6) to get better estimate' or 'I would use Simpson's rule'. Examiners were looking for a clearly expressed understanding that the second integral could now be found using Simpson's rule.

Question 3

A substantial number of candidates appeared to miss the point of this question – perhaps because they didn't read it carefully enough, perhaps because they were not well-prepared for the topic. The key words in the question were 'The program stores and calculates all numbers rounded to 5 significant figures', but a very common error was to work to the full accuracy of the calculator. There *is* a substantial error when intermediate answers are rounded, but this error does not show when full calculator accuracy is used.

In part (iii) the comment required was that the subtraction of the nearly equal rounded values gives rise to the error. Very few candidates stated this clearly.

Question 4

The first part of this question was done well by most candidates. However, the comments on the accuracy of the estimates were often wrong. Many candidates were still thinking in pure mathematical terms and saying that a smaller *h* will give a better estimate of the derivative. In numerical mathematics – mathematics using limited precision arithmetic – that is just not true. A smaller *h* will always lead eventually to *less* accurate estimates.

Part (ii) required some judgement as to the point at which accuracy is lost. Answers of 0.24 and 0.237 were accepted.

Question 5

Part (i) of this question required candidates to set up and solve a pair of simple linear equations. This was very straightforward for most, but led some into great difficulty. The second part was a pretty routine exercise in absolute and relative error. The only common problem here was confusion between the exact and the approximate values.

Question 6

Though this question explored a somewhat different take on interpolation, many candidates acquitted themselves well. The sketch in part (i) was easy, as was using the Newton formula in parts (ii) and (iii) to express q as a quadratic function of p. In part (iv) candidates were expected to realise that, as the values of q are not equally spaced, the inverse interpolation requires Lagrange's formula. The only problem was confusion, for some candidates, between arguments and function values.

In part (v) the point was that the inverse interpolation does not give the starting value. There were some simple comments to that effect (for which credit was given), and some that were a little more analytical, pointing out that different approximate methods should be *expected* to give different answers.

Question 7

Showing the existence of the root in part (i) was easy, but showing that there are no other positive roots defeated quite a few. The simplest approach is to show that the gradient of the function is always positive for x positive. Another common method was to locate the single turning point and argue from that to a single positive root.

The sketch in part (ii) was simple enough, but in part (iii) the construction lines for the secant method had to be added and that proved a lot more difficult. It was evident that some candidates were using the false position method. This gives the same values for x_2 and x_3 as the secant method, but they give different values for x_4. This difference leads, in turn, to different construction lines on the graph.

In part (iv) most candidates obtained the correct solution, though not always using the required method. Showing that the required accuracy has been obtained involves checking for a change of sign. Though this is not difficult, many candidates omitted it.