OCR

Monday 16 June 2014 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

1239422135

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 You are given that the equation

$$x^2 + \frac{1}{x} - 5 = 0$$

has one root, α, in the interval $(0.2, 0.3)$ and another root, β, in the interval $(2, 2.2)$.

(i) Use the method of false position with starting values 0.2 and 0.3 to find a first estimate of α. Apply false position again to find a second estimate of α.

A third application of the false position method gives 0.201 640 as an estimate of α. Give the value of α to the accuracy that appears to be justified. **[5]**

(ii) Suppose that β is to be found using the bisection method, starting with the interval (2, 2.2).

Write down the initial estimate of β and the associated maximum possible error.

Determine how many further steps of the bisection method would be required to find β with a maximum possible error of less than 0.005. (You are not required to carry out this bisection process.) **[3]**

2 The table shows several values of the mid-point rule (M) and trapezium rule (T) estimates of an integral for different values of h.

h	M	T
0.5	0.536 650	0.594 027
0.25		
0.125	0.555 282	0.558 599

(i) Calculate the missing values in the table. Calculate also three Simpson's rule estimates of the integral. **[7]**

(ii) Give the value of the integral to the accuracy that appears justified. **[1]**

3 In computer science, powers of 2 occur frequently. It is quite common to approximate 2^{10k} as 10^{3k}.

(i) Find the relative error in this approximation for each of $k = 1$, $k = 2$ and $k = 3$. **[4]**

(ii) You are reminded that if $X = x(1 + r)$, where r is small, then $X^n \approx x^n(1 + nr)$.

How does this result relate to your answers in part **(i)**? **[2]**

4 The function $f(x)$ has the values shown in the table.

x	1.9	1.95	2	2.05	2.1
$f(x)$	0.385 570	0.677 625	1	1.356 076	1.749 638

(i) Calculate two estimates of $f'(2)$ using the forward difference method. **[2]**

(ii) Calculate two estimates of $f'(2)$ using the central difference method. **[2]**

(iii) Give the value of $f'(2)$ to the accuracy that appears justified, explaining your reasoning. **[2]**

5 The table gives some values of the function $g(x)$ correct to 6 decimal places.

x	1	1.1	1.2
$g(x)$	1.188 395	1.234 281	1.287 500

(i) Estimate $g(1.14)$ using linear interpolation. **[2]**

(ii) Use Newton's forward difference interpolation formula to write down a quadratic approximation to $g(x)$. You need not simplify your answer. **[4]**

(iii) Hence obtain a second estimate of $g(1.14)$. **[2]**

Section B (36 marks)

6 You are given that the equation

$$6x^5 - 3x^2 - 2x + 1 = 0$$

has exactly 3 real roots. Let these roots be α, β, γ where $\alpha < \beta < \gamma$.

(i) Show that there is one root in the interval $(-1, 0)$ and two roots in the interval $(0, 1)$. **[3]**

(ii) Use the iteration

$$x_{r+1} = \left(\frac{1}{6}\left(3x_r^2 + 2x_r - 1\right)\right)^{\frac{1}{5}}$$

to find α correct to 2 decimal places. **[4]**

(iii) Use the iteration

$$x_{r+1} = \frac{1}{2}\left(6x_r^5 - 3x_r^2 + 1\right) \quad (*)$$

to verify that $\beta \approx 0.34$.

By considering a suitable derivative, show that the convergence of (*) to β will be slow. **[6]**

(iv) Use the Newton-Raphson method to find γ correct to 4 decimal places. **[5]**

7 A function $\mathrm{f}(x)$ is known to have the following values correct to 3 decimal places.

x	0	0.5	1	1.5
$\mathrm{f}(x)$	1.693	1.405	1.288	1.253

The value of the integral $I = \int_0^{1.5} \mathrm{f}(x)\,\mathrm{d}x$ is required.

You should give all numerical answers in this question correct to 5 decimal places.

(i) Find the best estimate possible of I using just the trapezium rule. **[2]**

(ii) By drawing a suitable graph, determine whether your answer in part **(i)** is likely to be an underestimate or an overestimate. **[4]**

(iii) Obtain two further estimates of I, each of them using both the trapezium rule and Simpson's rule. With reference to your graph, explain which of these estimates seems likely to be more accurate. **[6]**

(iv) The integration rule

$$\int_0^{3h} \mathrm{f}(x)\,\mathrm{d}x \approx \frac{3h}{8}\big(\mathrm{f}(0) + 3\,\mathrm{f}(h) + 3\,\mathrm{f}(2h) + \mathrm{f}(3h)\big)$$

is designed to be used when 4 equally spaced values of a function are known.

Find the value of I given by this rule.

You should now assume that the value of I you have just calculated is correct to 3 decimal places. Find the approximate errors in your three earlier estimates of I, and hence determine whether your conclusions in parts **(ii)** and **(iii)** are confirmed. **[6]**

END OF QUESTION PAPER

2 (i)

h	M	T
0.5	0.536 650	0.594 027
0.25		
0.125	0.555 282	0.558 599

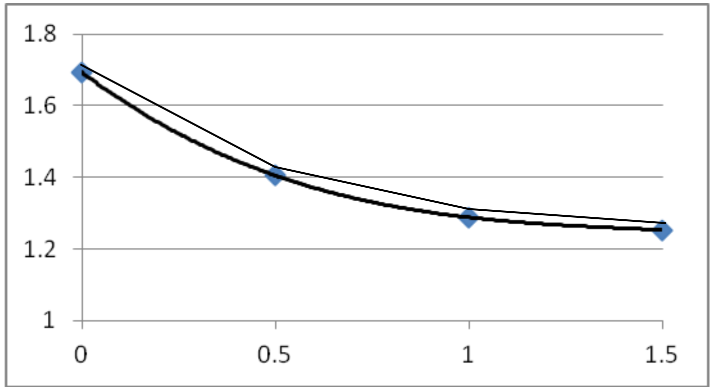
2 (ii)

7 (i)	
7 (ii)	

Question			Answer	Marks	Guidance	
1	(i)		a b f(a) f(b) x f(x) 0.2 0.3 0.04 –1.57667 **0.202474** –0.02010 0.2 0.202474 0.04 –0.02010 **0.201647** 0.201 640 suggests that **0.2016** is secure, accept **0.20164**	M1A1 M1A1 B1 **[5]**	(Root is 0.201640 to 6 dp)	
1	(ii)		Initial estimate 2.1 with mpe 0.1 mpe: 0.05, 0.025, 0.0125, 0.00625, 0.003125 So 5 further steps required	B1 M1 A1 **[3]**	soi cao	
2	(i)		h M T S 0.5 0.536650 0.594027 **0.555776** 0.25 **0.551860** **0.5653385** **0.556353** 0.125 0.555282 0.558599 **0.556388**	B1 M1A1 M1A1A1A1 **[7]**	Missing T Missing M S values	
2	(ii)		Agreement suggests 0.5564; extrapolation gives 0.556 39	B1 **[1]**	Accept either	
3	(i)		k exact approximate relative error 1 1024 1000 –0.0234375 2 1048576 1000000 –0.0463257 3 1073741824 1000000000 –0.0686774	B1 B1B1B1 **[4]**	For exact and approximate values Each relative error Condone consistent missing minus signs on this occasion	soi
3	(ii)		k = 2 and k = 3 represent the square and cube of value when k = 1 The relative errors have doubled and tripled respectively	E1 E1 **[2]**		

Question			Answer	Marks	Guidance	
4	(i)		fwd diff h est 0.1 7.49638 0.05 7.12152	M1 A1 **[2]**	Correct formula for either value Both values correct	
	(ii)		cent diff h est 0.1 6.82034 0.05 6.78451	M1 A1 **[2]**	Correct formula for either value Both values correct	
4	(iii)		6.8 looks secure Using central difference values (as the method is more accurate)	B1 E1 **[2]**	(Extrapolation gives 6.77...) Seen or implied	
5	(i)		$g(1.14) = 1.234\,281 + 0.4\,(1.287\,500 - 1.234\,281) = 1.255\,568\,6$	M1A1 **[2]**	Accept 5-7 dp	
5	(ii)		1 1.188395 1.1 1.234281 0.045886 1.2 1.287500 0.053219 0.007333 $g(x) = 1.188395 + 0.045886\,(x-1) / 0.1$ $+ 0.007333\,(x-1)\,(x-1.1) / ((0.1)^2\,2!)$	B1 M1 A1A1 **[4]**	 formula A1 for 2 terms, A1A1 for all 3	
5	(iii)		Substitute $x = 1.14$ to get $g(1.14) = 1.254\,688\,6$	M1A1 **[2]**	Accept 5-7 dp	

Question			Answer	Marks	Guidance	
6	(i)		Exhibit changes of sign, e.g. x –1 0 0.5 1 $f(x)$ –6 1 –0.5625 2	M1 A1 A1 **[3]**	Accept signs instead of numbers One interval Rest	
6	(ii)		E.g. starting at –0.5 iterates are: –0.73072 –0.67800 –0.69558 –0.69018 –0.69188 Root is –0.69 to 2 dp	M1A1A1 A1 **[4]**	M1 1st term, A1 next 2, A1 rest cao	
6	(iii)		Start at 0.34: 0.340231 0.340042 0.340197 0.340070 converging to a value close to 0.34 Derivative of RHS is $15x^4 – 3x$ At $x = 0.34$ this is –0.82 Approaching 1 in magnitude, hence slow convergence	M1A1 E1 B1 B1 E1 **[6]**		
6	(iv)		Req'd derivative is $30x^4 – 6x – 2$ Hence NR formula NR iterations eg 0.75 1.005222 0.911575 0.876798 0.872091 0.**8720**09 1 0.909091 0.876258 0.872074 0.872009 0.**8720**09	M1A1 A1A1A1 **[5]**	NR formula soi A1 2 it'ns, A1 rest, A1 0.8720	

Question			Answer	Marks	Guidance	
7	(i)		T = 2.08300	M1A1 **[2]**	Accept 2.083 or 2.0830	
7	(ii)		 Draw lines on graph. T will be an overestimate;	G1 G1 G1E1 **[4]**	Points Curve	
7	(iii)		$T + S$ = 0.77450 + 1.30167 = 2.07617 $S + T$ = 1.43350 + 0.63525 = 2.06875 Trapezium rule seems like a better fit over (1, 1.5) than (0, 0.5) So second estimate likely to be more accurate	M1A1 M1A1 E1 A1 **[6]**	A1 each new correct T or S value	
7	(iv)		4-point rule gives 2.06719 Errors: T only: 0.01581 so it is an overestimate $T + S$: 0.00898 $S + T$: 0.00156 so $S + T$ is more accurate than $T + S$	M1A1 A1E1 A1E1 **[6]**	Error soi Accept relative errors to 3dp.	