OCR

RECOGNISING ACHIEVEMENT

Friday 17 May 2013 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Show by sketching two curves on the same axes that the equation

$$x^2 = \cos x,$$

where x is in radians, has exactly one positive root. Give a rough initial estimate of the root. **[3]**

(ii) By re-arranging the equation, find an iterative formula for x_{r+1} in terms of x_r. Use this iterative formula to find the root correct to 2 decimal places. **[5]**

2 This question concerns binomial coefficients of the form $\binom{2n}{n}$, where $\binom{2n}{n} = \frac{(2n)!}{(n!)^2}$.

An approximate formula for $\binom{2n}{n}$ is $\frac{4^n}{\sqrt{n\pi}}$.

(i) Calculate the absolute and relative errors in the approximate formula for $n = 5$ and $n = 10$. Comment briefly on how the absolute errors and relative errors appear to change with n. **[5]**

(ii) It can be shown that the relative errors in part **(i)** are approximately equal to $\frac{1}{kn}$ for some integer k. Use the values calculated in part **(i)** to determine k. **[2]**

3 The function $f(x)$ has the values shown in the table.

x	0.1	0.2	0.3	0.4
$f(x)$	1.641	1.990	1.840	1.192

(i) Show by means of a difference table that $f(x)$ can be closely approximated by a quadratic function. **[3]**

(ii) Use Newton's forward difference interpolation formula to obtain an estimate of $f(0.15)$. **[4]**

4 **(i)** Show, graphically or otherwise, that the equation

$$2^x + 3^x = 4 \qquad (*)$$

has exactly one root.

Show that the root lies in the interval [0.7, 0.8]. **[4]**

(ii) Use the method of false position to find the root of (*) correct to 2 decimal places. **[4]**

5 The values of the function $g(x)$ in the table are correct to 4 decimal places.

x	–0.2	–0.15	–0.1	–0.05	0	0.05	0.1	0.15	0.2
$g(x)$	1.1292	1.1540	1.1766	1.1974	1.2163	1.2335	1.2489	1.2625	1.2745

(i) Use the central difference formula with suitable values of h to obtain a sequence of three estimates of $g'(0)$. **[4]**

(ii) Hence give a value for $g'(0)$ to an appropriate degree of accuracy, explaining your reasoning. **[2]**

Section B (36 marks)

6 In this question, $I = \int_0^{0.5} \sqrt{1 + \tan x}\,dx$, where x is in radians. Estimates of I should be given correct to 6 decimal places.

(i) Obtain the trapezium rule and mid-point rule estimates of I with $h = 0.5$.

Use these two values to obtain a Simpson's rule estimate of I. **[3]**

(ii) Find, as efficiently as possible, two further trapezium rule estimates, two further mid-point rule estimates, and two further Simpson's rule estimates.

Give the value of I to the accuracy that is justified. **[7]**

(iii) Find the differences and the ratio of differences for the trapezium rule estimates and also for the mid-point rule estimates.

What do the ratios of differences indicate?

State, with a reason, whether either of the mid-point and trapezium rules gives more accurate estimates than the other. **[8]**

[Question 7 is printed overleaf]

7 The series $S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ is summed, for various values of n, using a spreadsheet. The spreadsheet gives the answers $S_{100} = 18.5896$ and $S_{200} = 26.8593$. For the purposes of this question, these values may be regarded as exact.

(i) The same calculations are now carried out with each term in the series rounded to 4 decimal places. The answers obtained are 18.5897 and 26.8589 respectively.

Explain how it arises that one sum is too large and the other is too small. **[2]**

(ii) Now suppose that the same calculations were carried out with each term in the series chopped to 4 decimal places. Estimate the answers that would be obtained, explaining your reasoning. **[4]**

(iii) Show, by using the mid-point rule on the integral $\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}} \mathrm{d}x$, that

$$\frac{1}{\sqrt{k}} \approx 2\left(\sqrt{k+0.5} - \sqrt{k-0.5}\right).$$ **[4]**

(iv) It follows from the result in part **(iii)** that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} \approx 2\left(\sqrt{n+0.5} - \sqrt{0.5}\right).$$

Use this result to find approximations for S_{100} and S_{200}. Find the errors in these approximations. What do you notice about the values of these errors? **[5]**

(v) Making a suitable assumption about the error, obtain as accurate an estimate of S_{1000} as you can. **[3]**

Question			Answer	Marks	Guidance	
1	(i)		Convincing sketches of x^2 and cos x. Single intersection. Estimate of root in [0.5, 1]	G2 B1 **[3]**	G1 for each graph Accept π/4. Accept an interval in [0.5, 1]	
1	(ii)		Iteration $x_{r+1} = (\cos x_r)^{0.5}$ r: 0, 1, 2, 3, 4, 5 x_r: 0.8, 0.83469, 0.819395, 0.826235, 0.823195, 0.82455 0.82 correct to 2 dp	M1 A1 M1 A1 A1 **[5]**	For any valid rearrangement For writing it as an iteration (soi) Max 1 for a diverging iteration A1 requires agreement to 2 dp Dependent on previous A1	
2	(i)		n, exact, approx, error, rel error 5, 252, 258.3688, 6.36877, 0.025273 10, 184756, 187079, 2322.973, 0.012573 errors increase but relative errors decrease with n	M1 B1 B1 B1 B1 **[5]**	Requires method for abs and rel error Approximations Errors Relative errors	
2	(ii)		$10k = \dfrac{1}{0.01257} = 79.5548$ $k = 8$ to nearest integer OR $5k = \dfrac{1}{0.0257} = 39.5726$ $k = 8$ to nearest integer	M1 A1 M1 A1 **[2]**	 Must be an integer Must be an integer	
3	(i)		x, f(x), Δ, Δ^2 0.1, 1.641 0.2, 1.990, 0.349 0.3, 1.840, -0.150, -0.499 these almost equal 0.4, 1.192, -0.648, -0.498 (so approx qdratic)	M1 A1 E1 **[3]**		

Question			Answer	Marks	Guidance	
3	(ii)		$f(1.5)=1.641+\dfrac{0.349(0.15-0.1)}{0.1}-\dfrac{0.499(0.15-0.1)(0.15-0.2)}{2(0.1)^2}$ = 1.878 to 3dp	M1 A1 A1 A1 **[4]**	For recognizable attempt at correct formula Either second or third term correct All three terms correct. Accept cubic. Accept any awrt 1.878	2 out 3 for formula with x and no 0.15
4	(i)		Sketch or convincing argument to an increasing function hence a single root x function: 0.7 3.782174 < 4; 0.8 4.149326 > 4 (Hence root)	G2/E2 M1 A1 **[4]**	If comparing with zero: -0.2178, 0.1493 Max 1 if function sign but not values	
4	(ii)		a f(a) b f(b) x f(x) 0.7 -0.21783 0.8 0.149326 0.759329 -0.00431 0.759329 -0.00431 0.8 0.149326 0.760469 0.76 to 2dp	M1 A1 M1 A1 **[4]**	Allow a maximum of 3 out 4 for a solution which goes wrong but self corrects For correct interval and calculating x Must follow from false position	
5	(i)		h g(-h) g(h) g '(0) 0.2 1.1292 1.2745 0.36325 0.1 1.1766 1.2489 0.3615 0.05 1.1974 1.2335 0.361	M1 A1 A1 A1 **[4]**	Full marks for h = 0.15, 0.1, 0.05. Max 3 if other values of h used	h = 0.15 gives 0.361667
5	(ii)		0.36 because last figure still changing and so unreliable Or 0.361 if some argument about convergence or extrapolation is used	A1 E1 **[2]**	Any sensible comment or attempt to analyse errors	
6	(i)		x f(x) T M S 0 1 0.5 1.243504 0.25 1.12042 **0.560876** **0.560210** **0.560432**	M1 M1 M1 **[3]**	M Award these marks for a correct answer S or a correct method with wrong answer T Do not penalise no. of sf	

Question			Answer	Marks	Guidance	
6	(ii)		x f(x) T M S 0.125 1.060969 0.375 1.18052 **0.560543** **0.560372** **0.560429** 0.0625 1.030816 0.1875 1.090747 0.3125 1.150255 0.4375 1.211499 **0.560458** **0.560415** **0.560429** 0.560429 is justified (for information only: 0.5604289 is justified if more sf used)	A6 A1 **[7]**	Values Lose 1 for each error Lose 1 overall if no. of sf is not 6 FT sensible but incorrect M and/or T to S	
6	(iii)		T diffs ratio M diffs ratio 0.560876 0.560210 0.560543 -0.000333 0.560372 0.000162 0.560458 -0.000085 0.256788 0.560415 0.000043 0.262091 Ratios about 0.25 in each case; indicates both have 2nd order convergence But M is more accurate than T; smaller differences so nearer the correct answer	M1 A1 M1 A1 E1 E1 E1 E1 **[8]**	T Allow small errors that still give ratios … M … of approximately 0.25 Allow correct explanations using 0.25 if the ratios come out wrong Allow correct statements about M and T even if not supported by the numbers	
7	(i)		In the first 100 terms the positive rounding errors exceed the negative rounding errors The opposite occurs in the first 200 terms.	E1 E1 **[2]**	Allow E1 for an incomplete explanation that shows some understanding	
7	(ii)		Chopping will reduce the sum by an average of 0.00005 per term ie by 0.005 and 0.01 in S_{100} and S_{200} Hence estimate as 18.5846 (18.585) and 26.8493 (26.85)	E1 M1A1 A1 **[4]**	M1 for 0.00005, A1 rest	

Question			Answer	Marks	Guidance	
7	(iii)		$\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}}\,\mathrm{d}x = \left[2\sqrt{x}\right]_{k-0.5}^{k+0.5}$	M1		
			= RHS Midpoint rule Gives LHS	A1 M1 A1 **[4]**	Answer given Must be convincing Answer given	
7	(iv)		approx exact error S_{100} 18.63572 18.5896 0.046124 S_{200} 26.90539 26.8593 0.046091 Errors almost exactly equal	B1 B1 M1 A1 E1 **[5]**	Approximations Errors	
7	(v)		assumed approx error estimate S_{1000} 61.84715 0.046 61.80115 (61.801 or 61.80) (For information, correct sum is 61.80101 to 5dp)	B1 M1 A1 **[3]**	Approx Correction using 0.046 (or similar) Penalize more dp	