

# Wednesday 16 May 2012 – Morning

## AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

**Duration:** 1 hour 30 minutes

**Other materials required:**

- Scientific or graphical calculator

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

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**Section A (36 marks)**

- 1 Use Lagrange's method to find the equation of the quadratic curve  $y = f(x)$  that passes through the following data points.

|     |    |   |    |
|-----|----|---|----|
| $x$ | -1 | 0 | 2  |
| $y$ | 3  | 6 | -4 |

Hence find the value of  $x$  for which  $f(x)$  is a maximum.

[7]

- 2 The number  $X$  is an approximation to an exact value  $x$ , and  $X = x(1 + r)$ .

(i) Show that  $r$  is the relative error in  $X$ .

[2]

(ii) Use the binomial theorem to show that  $X^n \approx x^n(1 + nr)$  provided  $r$  is small.

[2]

(iii) The number  $Y$  is an approximation to an exact value  $y$ . The relative error in  $Y$  is 2%. State the approximate relative errors in

(A)  $Y^3$  as an approximation to  $y^3$ ,

(B)  $\frac{1}{Y}$  as an approximation to  $\frac{1}{y}$ .

[3]

- 3 (i) Show that the equation  $x^5 = x^4 + 2$  has a root in the interval  $[1, 2]$ .

[2]

(ii) Use the Newton-Raphson method to find this root correct to 6 decimal places.

[6]

- 4 The function  $g(x)$  has the values shown in the table.

|        |          |          |          |          |
|--------|----------|----------|----------|----------|
| $x$    | 5        | 5.1      | 5.2      | 5.4      |
| $g(x)$ | 0.820 86 | 0.780 82 | 0.742 73 | 0.672 05 |

(i) Find three estimates of  $g'(5)$  using the forward difference method with  $h = 0.4, 0.2, 0.1$ .

[3]

(ii) Use these estimates to show that the forward difference method has first order convergence.

[3]

(iii) Give the value of  $g'(5)$  to the accuracy that is justified, explaining your reasoning.

[2]

- 5 The cells of a spreadsheet have the formulae shown in Fig. 5a. The values displayed by the spreadsheet are shown in Fig. 5b.

|   | A       | B     | C     |
|---|---------|-------|-------|
| 1 | 0.6     |       |       |
| 2 | =A1-0.2 |       |       |
| 3 | =A2-0.2 |       |       |
| 4 | =A3-0.2 | =A4+1 | =B4-1 |

Fig. 5a

|   | A        | B | C |
|---|----------|---|---|
| 1 | 0.6      |   |   |
| 2 | 0.4      |   |   |
| 3 | 0.2      |   |   |
| 4 | -5.5E-17 | 1 | 0 |

Fig. 5b

- (i) State what the entry in cell A4 of Fig. 5b means. Explain why it is not zero. [3]
- (ii) What can you deduce about the way the spreadsheet stores and displays numbers from the values shown in cells B4 and C4? [3]

### Section B (36 marks)

- 6 The table below gives some values of a function  $f(x)$ .

| x      | 0        | 0.25     | 0.5      | 0.75     | 1        | 1.25     | 1.5      | 1.75     | 2        |
|--------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $f(x)$ | 1.000 00 | 1.060 51 | 1.116 87 | 1.168 88 | 1.216 32 | 1.259 01 | 1.296 78 | 1.329 49 | 1.357 07 |

In this question, you are required to find estimates of the integral  $\int_0^2 f(x)dx$ .

- (i) Find trapezium rule estimates  $T_1, T_2, T_4, T_8$  with  $h = 2, 1, 0.5, 0.25$  respectively.

Find the values of  $\frac{T_4 - T_2}{T_2 - T_1}$  and  $\frac{T_8 - T_4}{T_4 - T_2}$ . State what these values indicate about the trapezium rule. [7]

- (ii) Use your trapezium rule estimates from part (i) to find three Simpson's rule estimates of the integral.

Calculate the ratio of differences for these estimates. What does this value indicate about Simpson's rule? [6]

- (iii) State, with reasons, the value of the integral to the accuracy that is justified if the given values of  $f(x)$  are exact.

Hence give a range within which the value of the integral lies if the given values of  $f(x)$  had been rounded to 5 decimal places. [5]

- 7 In this question you are asked to find the roots of the equation  $x^2 - 1 = \sin x$ , where  $x$  is in radians.
- (i) Show that the equation has a root in the interval  $[-1, 0]$  and another in the interval  $[1, 2]$ . [3]
- (ii) Starting with the interval  $[-1, 0]$ , find the initial estimate of the negative root as given by the method of false position. Apply this method to find two further estimates of the negative root. Discuss briefly the accuracy to which the root has been found. [7]
- (iii) Starting with  $x_0 = 1$  and  $x_1 = 2$ , find the first estimate of the positive root as given by the secant method. Apply this method to find two further estimates of the positive root. Discuss briefly the accuracy with which the root has been found. [6]
- (iv) Comment briefly on the relative merits of the method of false position and the secant rule in this case. [2]



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| Question |            | Answer  | Marks                             | Guidance  |           |   |   |   |       |     |          |          |           |           |                                     |   |
|----------|------------|---|-----------------------------------|---|-----------|---|---|---|-------|-----|----------|----------|-----------|-----------|-------------------------------------|---|
| 1        |            | $y = 3x(x-2)/(-1)(-3) + 6(x+1)(x-2)/(1)(-2) - 4(x+1)x/(3)(2)$<br>$y = (x^2 - 2x) - 3(x^2 - x - 2) - \frac{2}{3}(x^2 + x)$<br>$y = \frac{1}{3}(-8x^2 + x + 18)$<br>$y' = \frac{1}{3}(-16x + 1) = 0 \text{ when } x = 1/16$   | B1B1B1<br>A1<br>A1<br>M1A1<br>[7] | M1 for diffn, A1 cao  |           |   |   |   |       |     |          |          |           |           |                                     |   |
| 2        | (i)        | rearrange convincingly to $r = (X - x) / x$   | M1A1<br>[2]                       | No further explanation required                               |           |   |   |   |       |     |          |          |           |           |                                     |   |
| 2        | (ii)       | $(1+r)^n = 1 + nr + n(n-1)r^2/2 + \dots$<br>result given  | M1<br>E1<br>[2]                   | Need to see correct $r^2$ term<br>For saying $r^2$ negligible |           |   |   |   |       |     |          |          |           |           |                                     |   |
| 2        | (iii)      | (A) $n=3$ 6%<br>(B) $n=1$ -2%   | B1<br>B2<br>[3]                   | B1 for 2%   |           |   |   |   |       |     |          |          |           |           |                                     |   |
| 3        | (i)        | $x$ LHS RHS<br>1 1 < 3<br>2 32 > 18<br>Or equivalent  | M1A1<br>[2]                       | No explanation required                                       |           |   |   |   |       |     |          |          |           |           |                                     |   |
| 3        | (ii)       | $f(x) = x^5 - x^4 - 2$ $f'(x) = 5x^4 - 4x^3$ hence N-R formula<br><table style="margin-left: auto; margin-right: auto;"> <tr> <td><math>r</math></td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td> </tr> <tr> <td><math>x_r</math></td><td>1.5</td><td>1.455026</td><td>1.451113</td><td>1.4510851</td><td>1.4510851</td> </tr> </table><br>hence root is 1.451085 to 6 decimal places | $r$                               | 0   | 1         | 2 | 3 | 4 | $x_r$ | 1.5 | 1.455026 | 1.451113 | 1.4510851 | 1.4510851 | M1A1<br><br>M1A1A1<br><br>A1<br>[6] | May be implied by subsequent work<br><br>M1 1 <sup>st</sup> application, A1 2 <sup>nd</sup> , A1 for 2 successive values agreeing to 6 dp |
| $r$      | 0          | 1   | 2                                 | 3   | 4         |   |   |   |       |     |          |          |           |           |                                     |   |
| $x_r$    | 1.5        | 1.455026  | 1.451113                          | 1.4510851   | 1.4510851 |   |   |   |       |     |          |          |           |           |                                     |   |
| 4        | (i) & (ii) | $h$ $g'(5)$<br>0.4 -0.37203<br>0.2 -0.39065 -0.01863<br>0.1 -0.40040 -0.00975 0.52349 (or 'the differences approx. halve')<br>approx 0.5 (as $h$ is halved) so first order  | M1A1A1<br>[3]<br><br>M1A1<br>E1   | Estimates<br><br>M1 diffs, A1 ratio (may be implied)          |           |   |   |   |       |     |          |          |           |           |                                     |   |

| Question |          | Answer   | Marks                         | Guidance   |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
|----------|----------|--|-------------------------------|--|-------|--------|----|----------|----------|--|----|----------|----------|----------|---|--|----------|----------|---|--|
|          |          |  | [3]                           |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 4        | (iii)    | Some way off convergence, so accept -0.4<br>(or an argument by extrapolation to -0.41)   | E1A1<br>[2]                   | A1 for -0.4 or for -0.41 with extrapolation, E1 for a reason |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 5        | (i)      | It means $-5.5 \times 10^{-17}$<br>It is not zero because of rounding errors<br>in the representation of numbers like 0.6, 0.4, 0.2 (or in the calculations)   | E1<br>E2,1,0<br>[3]           | (Or 0.6 was not exact)                                       |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 5        | (ii)     | Cell B4 displays 1 either because the spreadsheet does not show enough dp<br>or because adding 1 pushes the error beyond the sf the spreadsheet stores.<br>The zero in C4 shows that the error must have been lost in B4   | E1<br>E1<br>E1<br>[3]         |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 6        | (i)      | <table style="margin-left: auto; margin-right: auto;"> <tr> <td>T1</td> <td>2.357070</td> <td>diffs</td> <td>ratios</td> </tr> <tr> <td>T2</td> <td>2.394855</td> <td>0.037785</td> <td></td> </tr> <tr> <td>T4</td> <td>2.404253</td> <td>0.009397</td> <td>0.248710</td> </tr> <tr> <td>T8</td> <td>2.406599</td> <td>0.002346</td> <td>0.249667</td> </tr> </table> <p>both approximately 0.25<br/>indicates 2nd order method</p> | T1                            | 2.357070   | diffs | ratios | T2 | 2.394855 | 0.037785 |  | T4 | 2.404253 | 0.009397 | 0.248710 | T8  | 2.406599   | 0.002346 | 0.249667 | M1A1<br>A1<br>A1<br>B1<br><br>E1<br>E1<br>[7] | Any T value<br>2 further T values<br>All T values<br>Ratios<br><br>explanation |
| T1       | 2.357070 | diffs  | ratios                        |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| T2       | 2.394855 | 0.037785   |                               |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| T4       | 2.404253 | 0.009397   | 0.248710                      |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| T8       | 2.406599 | 0.002346   | 0.249667                      |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 6        | (ii)     | Using $\frac{1}{3} (4*T_{2n} - T_n)$ to obtain Simpson's rule estimates<br><table style="margin-left: auto; margin-right: auto;"> <tr> <td>S1</td> <td>2.407450</td> <td>diffs</td> <td>ratios</td> </tr> <tr> <td>S2</td> <td>2.407385</td> <td>-6.5E-05</td> <td></td> </tr> <tr> <td>S4</td> <td>2.407381</td> <td>-4.2E-06</td> <td>0.064103</td> </tr> </table> <p>approximately 1/16<br/>indicates 4th order method</p>        | S1                            | 2.407450   | diffs | ratios | S2 | 2.407385 | -6.5E-05 |  | S4 | 2.407381 | -4.2E-06 | 0.064103 | M1<br>A1<br>A1<br>B1<br><br>E1<br>E1<br>[6] | (Or via M values)<br>Any S value<br>All S values<br>Ratio<br><br>Explanation |          |          |   |  |
| S1       | 2.407450 | diffs  | ratios                        |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| S2       | 2.407385 | -6.5E-05   |                               |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| S4       | 2.407381 | -4.2E-06   | 0.064103                      |  |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |
| 6        | (iii)    | Convergence of the Simpson's rule estimates suggests 2.40738<br>(or even 2.407381 by extrapolation)<br>If data are rounded to 5 dp, there is an error in the range $\pm 0.000\ 005$ in each value Since the integral is over a range of 2 units the correct value will lie within $\pm 0.000\ 01$ of the value given previously.   | A1E1<br>B1<br><br>A1E1<br>[5] | Either answer<br>Sight of $f(x) +$ or $- 0.000\ 005$         |       |        |    |          |          |  |    |          |          |          |   |  |          |          |   |  |

| Question |          | Answer  | Marks           | Guidance                          |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
|----------|----------|---|-----------------|-----------------------------------|----------|----------|----------|------------|----------|----|----------|----------|----------|------------|---------|----------|---------------------------------|---------------|----------|----------|----|----------|----------|----------|----------|--|---|------------------------------|
| 7        | (i)      | <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th><math>x</math></th> <th>LHS</th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>0</td> <td>&gt; -0.84147</td> </tr> <tr> <td>0</td> <td>-1</td> <td>&lt; 0</td> </tr> <tr> <td>1</td> <td>0</td> <td>&lt; 0.841471</td> </tr> <tr> <td>2</td> <td>3</td> <td>&gt; 0.909297</td> </tr> </tbody> </table> <span style="margin-left: 20px;">or equivalent<br/>no explicit explanation reqd.</span>  | $x$             | LHS                               | RHS      | -1       | 0        | > -0.84147 | 0        | -1 | < 0      | 1        | 0        | < 0.841471 | 2       | 3        | > 0.909297                      | M1A1A1<br>[3] |          |          |    |          |          |          |          |  |   |                              |
| $x$      | LHS      | RHS   |                 |                                   |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| -1       | 0        | > -0.84147  |                 |                                   |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 0        | -1       | < 0   |                 |                                   |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 1        | 0        | < 0.841471  |                 |                                   |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 2        | 3        | > 0.909297  |                 |                                   |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 7        | (ii)     | <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th><math>a</math></th> <th><math>b</math></th> <th><math>f(a)</math></th> <th><math>f(b)</math></th> <th><math>x</math></th> <th><math>f(x)</math></th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>0</td> <td>0.841471</td> <td>-1</td> <td>-0.54304</td> <td>-0.18836</td> </tr> <tr> <td>-1</td> <td>-0.54304</td> <td>0.841471</td> <td>-0.18836</td> <td>-0.62662</td> <td>-0.02093</td> </tr> <tr> <td>-1</td> <td>-0.62662</td> <td>0.841471</td> <td>-0.02093</td> <td>-0.63568</td> <td></td> </tr> </tbody> </table> <p>The sequence of values of <math>x</math> has not converged: -0.6 is the only safe estimate.</p> | $a$             | $b$                               | $f(a)$   | $f(b)$   | $x$      | $f(x)$     | -1       | 0  | 0.841471 | -1       | -0.54304 | -0.18836   | -1      | -0.54304 | 0.841471                        | -0.18836      | -0.62662 | -0.02093 | -1 | -0.62662 | 0.841471 | -0.02093 | -0.63568 |  | M1A1<br>M1A1<br>A1<br><br>E1A1<br><br>[7] | M1 is for correct end-points |
| $a$      | $b$      | $f(a)$  | $f(b)$          | $x$                               | $f(x)$   |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| -1       | 0        | 0.841471  | -1              | -0.54304                          | -0.18836 |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| -1       | -0.54304 | 0.841471  | -0.18836        | -0.62662                          | -0.02093 |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| -1       | -0.62662 | 0.841471  | -0.02093        | -0.63568                          |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 7        | (iii)    | <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th><math>x</math></th> <th>1</th> <th>2</th> <th><math>f(x)</math></th> <th>1.286979</th> <th>1.377411</th> <th>1.412046</th> </tr> </thead> <tbody> <tr> <td></td> <td>-0.84147</td> <td>2.090703</td> <td></td> <td>-0.30368</td> <td>-0.0841</td> <td>0.006448</td> </tr> </tbody> </table> <p>The sequence of values of <math>x</math> has not converged, but 1.4 appears safe</p>  | $x$             | 1                                 | 2        | $f(x)$   | 1.286979 | 1.377411   | 1.412046 |    | -0.84147 | 2.090703 |          | -0.30368   | -0.0841 | 0.006448 | M1A1A1A1<br><br>E1A1<br><br>[6] |               |          |          |    |          |          |          |          |  |   |                              |
| $x$      | 1        | 2   | $f(x)$          | 1.286979                          | 1.377411 | 1.412046 |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
|          | -0.84147 | 2.090703  |                 | -0.30368                          | -0.0841  | 0.006448 |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |
| 7        | (iv)     | <p>The rules converge at about the same rate<br/>     The rules converge slowly<br/>     Secant rule involves less calculation at each step (but may be harder to implement on a calculator)</p>  | E2,1<br><br>[2] | Reward other appropriate comments |          |          |          |            |          |    |          |          |          |            |         |          |                                 |               |          |          |    |          |          |          |          |  |   |                              |

# 4776 Numerical Methods (Written Examination)

## General Comments

The purely computational parts of this paper were found straightforward by most candidates. Theoretical parts were found more challenging, and interpreting results was difficult for all but the very best candidates.

The standard of presentation of work, and in particular the systematic setting out of numerical algorithms, seems to have improved somewhat. However some candidates frequently resort to scattering calculations on the page, making it difficult for examiners to detect and reward any correct work.

## Comments on Individual Questions

### 1) Lagrange's interpolation formula

This question attracted many correct solutions. However marks were lost by those who confused the  $x$  and  $f(x)$  values, and by those who could not simplify the quadratic.

### 2) Relative errors

The algebra required in parts (i) proved very straightforward, but only a minority were able to use the binomial theorem correctly in part (ii). Part (iii) required the understanding of relative error but was not well answered.

### 3) Solution of an equation, Newton-Raphson method

This was a very straightforward question, with very many candidates scoring full marks.

### 4) Numerical differentiation

Parts (i) and (ii), finding the estimates and demonstrating that the forward difference method is first order, were done well by most candidates. Part (iii), giving the answer to the accuracy that is justified, proved tricky for quite a few. One surprisingly common error was to drop the negative sign.

### 5) Errors in the representation and storage of numbers

Almost all candidates were able to interpret the spreadsheet notation in part (i). The explanation expected for the ‘dirty zero’ result in cell A4 was that the spreadsheet does not store numbers such as 0.6 exactly; hence a calculation which would give exactly zero on paper may not give zero when carried out on a computer. The most common answer, however, was that the value entered in cell A1 is not 0.6. This is a possibility, of course, but it rather misses the point. Very few candidates were able to make the required inferences in part (ii). Adding 1 causes the significant figures shown in cell A4 to be hidden in cell B4. Subtracting 1 again to get a clean zero shows that these figures were lost altogether.

### 6) Numerical integration

The numerical work in first two parts was done well by the vast majority of candidates. It was pleasing to see so many correctly dealing with the orders of the trapezium rule and Simpson’s rule. Part (iii) was found more difficult: in particular, very few could estimate the effect of the values of  $f(x)$  being approximate. If each  $f(x)$  may be in error by  $\pm 0.000\ 005$ , and the range of integration is of length 2, then a sensible estimate of the error in the integral is the product of these two numbers, i.e.  $\pm 0.000\ 01$ . (A few resolute candidates arrived at this conclusion by re-working all their calculations.)

7) **Solution of an equation, false position and secant methods**

Locating the roots in part (i) was well answered, but the modal score for each of the other three parts was zero. It seems that the false position and secant methods are not as well known as they should be – and indeed that they are frequently confused. (Of course the two methods use essentially the same formula to get from two estimates of the root to a third: the difference lies in what is done with the third estimate. It is an important feature of the false position method that, at every stage, we have two estimates which bracket the root; it is an important feature of the secant method that we pay no attention to whether the current two estimates do or do not bracket the root.)

| <b>GCE Mathematics (MEI)</b> |  |     | <b>Max Mark</b> | <b>a</b> | <b>b</b> | <b>c</b> | <b>d</b> | <b>e</b> | <b>u</b> |
|------------------------------|--|-----|-----------------|----------|----------|----------|----------|----------|----------|
| 4751/01                      | (C1) MEI Introduction to Advanced Mathematics  | Raw | 72              | 57       | 50       | 44       | 38       | 32       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4752/01                      | (C2) MEI Concepts for Advanced Mathematics   | Raw | 72              | 54       | 48       | 42       | 36       | 31       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4753/01                      | (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper                   | Raw | 72              | 60       | 53       | 47       | 41       | 34       | 0        |
| 4753/02                      | (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework                      | Raw | 18              | 15       | 13       | 11       | 9        | 8        | 0        |
| 4753/82                      | (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark | Raw | 18              | 15       | 13       | 11       | 9        | 8        | 0        |
| 4753                         | (C3) MEI Methods for Advanced Mathematics with Coursework                                  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4754/01                      | (C4) MEI Applications of Advanced Mathematics  | Raw | 90              | 65       | 57       | 50       | 43       | 36       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4755/01                      | (FP1) MEI Further Concepts for Advanced Mathematics  | Raw | 72              | 63       | 56       | 49       | 42       | 35       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4756/01                      | (FP2) MEI Further Methods for Advanced Mathematics   | Raw | 72              | 61       | 53       | 46       | 39       | 32       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4757/01                      | (FP3) MEI Further Applications of Advanced Mathematics                                     | Raw | 72              | 54       | 47       | 40       | 34       | 28       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4758/01                      | (DE) MEI Differential Equations with Coursework: Written Paper                             | Raw | 72              | 63       | 57       | 51       | 45       | 39       | 0        |
| 4758/02                      | (DE) MEI Differential Equations with Coursework: Coursework                                | Raw | 18              | 15       | 13       | 11       | 9        | 8        | 0        |
| 4758/82                      | (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark           | Raw | 18              | 15       | 13       | 11       | 9        | 8        | 0        |
| 4758                         | (DE) MEI Differential Equations with Coursework  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4761/01                      | (M1) MEI Mechanics 1   | Raw | 72              | 58       | 50       | 42       | 34       | 27       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4762/01                      | (M2) MEI Mechanics 2   | Raw | 72              | 58       | 51       | 44       | 38       | 32       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4763/01                      | (M3) MEI Mechanics 3   | Raw | 72              | 63       | 56       | 50       | 44       | 38       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4764/01                      | (M4) MEI Mechanics 4   | Raw | 72              | 56       | 49       | 42       | 35       | 29       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4766/01                      | (S1) MEI Statistics 1  | Raw | 72              | 54       | 46       | 38       | 30       | 23       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4767/01                      | (S2) MEI Statistics 2  | Raw | 72              | 61       | 55       | 49       | 43       | 38       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4768/01                      | (S3) MEI Statistics 3  | Raw | 72              | 58       | 51       | 44       | 38       | 32       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4769/01                      | (S4) MEI Statistics 4  | Raw | 72              | 56       | 49       | 42       | 35       | 28       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4771/01                      | (D1) MEI Decision Mathematics 1  | Raw | 72              | 53       | 47       | 42       | 37       | 32       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4772/01                      | (D2) MEI Decision Mathematics 2  | Raw | 72              | 56       | 50       | 44       | 39       | 34       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4773/01                      | (DC) MEI Decision Mathematics Computation  | Raw | 72              | 46       | 40       | 34       | 29       | 24       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4776/01                      | (NM) MEI Numerical Methods with Coursework: Written Paper                                  | Raw | 72              | 50       | 44       | 38       | 33       | 27       | 0        |
| 4776/02                      | (NM) MEI Numerical Methods with Coursework: Coursework                                     | Raw | 18              | 14       | 12       | 10       | 8        | 7        | 0        |
| 4776/82                      | (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark                | Raw | 18              | 14       | 12       | 10       | 8        | 7        | 0        |
| 4776                         | (NM) MEI Numerical Methods with Coursework   | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |
| 4777/01                      | (NC) MEI Numerical Computation   | Raw | 72              | 55       | 47       | 39       | 32       | 25       | 0        |
|                              |  | UMS | 100             | 80       | 70       | 60       | 50       | 40       | 0        |