

Tuesday 17 January 2012 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Duration: 1 hour 30 minutes

Other materials required:

- Scientific or graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1** (i) Show that the equation

$$\tan x - \cos x = 1,$$

where x is in radians, has a root in the interval $[0.9, 1.1]$

[2]

- (ii) Use the bisection method to find an estimate of this root with maximum possible error 0.025.

[4]

- 2** The function $f(x)$ is a polynomial with exact values as shown in the table.

x	0	1	2	3	4
$f(x)$	-6	-7	-4	15	62

- (i) Find the degree of $f(x)$.

[3]

- (ii) Use Newton's forward difference formula to obtain an expression for $f(x)$. (You are not required to simplify your answer.)

[3]

- (iii) Find $f(2.5)$.

[2]

- 3** In this question you should work entirely in radians.

- (i) Write down the values of $\cos(0.11)$ and $\cos(0.12)$ correct to 4 significant figures. Use these values to write down a value for $\cos(0.11) - \cos(0.12)$.

[3]

- (ii) You are now given that $\cos(0.11) - \cos(0.12) = 2 \sin(0.115) \sin(0.005)$.

Write down values of $\sin(0.115)$ and $\sin(0.005)$ correct to 4 significant figures. Use these values to obtain another value for $\cos(0.11) - \cos(0.12)$.

[2]

- (iii) Check your answer using the full accuracy of your calculator. Comment on your answers to parts (i) and (ii) indicating what gives rise to the outcome in part (i).

[3]

- 4** In an opinion poll, the percentage levels of support for the five political parties contesting an election were reported, to the nearest whole number, as follows.

Party	Red	Blue	Orange	Green	Yellow
% support	36	25	17	13	10

- (i) These percentages do not sum to 100. Explain how this can occur.

[2]

- (ii) A second opinion poll is taken a month later. The five percentages are again reported to the nearest whole number. State the largest and smallest totals that the reported percentages could have, showing by means of an example how each could arise.

[4]

- 5 Two estimates of the integral $\int_{0.2}^1 \left(\frac{1}{2}\right)^x dx$ are given in the table below.

h	Mid-point	Trapezium
0.8	0.527 803 16	0.548 220 23
0.4		

(i) Obtain the two missing values in the table. [4]

(ii) Hence obtain two Simpson's rule estimates of the integral.

Give the value of the integral to the accuracy that you consider to be justified from your working. [4]

Section B (36 marks)

- 6 A function $g(x)$ has the values, correct to 5 decimal places, shown in the table.

x	1.8	1.9	1.95	2	2.05	2.1	2.2
$g(x)$	2.359 60	2.485 71	2.551 17	2.618 28	2.687 10	2.757 65	2.904 13

(i) Find three estimates of $g'(2)$, using the forward difference method with decreasing values of h .

Letting these three estimates be denoted by α, β, γ , respectively, find the value of $\frac{\gamma - \beta}{\beta - \alpha}$. What does this value indicate about the rate of convergence of the forward difference method? [6]

(ii) Find three estimates of $g'(2)$, using the central difference method.

Show that the central difference method converges more quickly than the forward difference method. [6]

(iii) Suppose that D_1 and D_2 are estimates of a quantity d , and that the error in D_2 is approximately $\frac{1}{4}$ of the error in D_1 . Show that

$$d \approx \frac{4D_2 - D_1}{3}. \quad [3]$$

(iv) Use the result in part (iii) to find two improved estimates of $g'(2)$. Give a value for $g'(2)$ to the accuracy that appears justified. [4]

- 7 (i) Show that the equation

$$x^6 - 3x + 1 = 0 \quad (*)$$

has a root, α , in the interval $[0, 1]$, and a root, β , in the interval $[1, 2]$. [2]

- (ii) Determine how many turning points the curve $y = x^6 - 3x + 1$ has. What can you conclude about the number of real roots of $(*)$? [4]

- (iii) Show that the iteration $x_{r+1} = \frac{1}{3}(x_r^6 + 1)$, with a suitable starting value, converges to α . Find α correct to 5 significant figures. [4]

- (iv) Show, by considering an appropriate derivative, that the iteration in part (iii) will not converge to β . [3]

- (v) Find another iteration, based on a rearrangement of $(*)$, that converges to β . Find β correct to 5 significant figures. [4]



RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

5 (i)

h	Mid-point	Trapezium
0.8	0.527 803 16	0.548 220 23
0.4		

5 (ii)

Question			Answer	Marks	Guidance
1	(i)		$ \begin{array}{ccccc} x & \text{LHS} & & \text{RHS} \\ 0.9 & 0.638548 & < & 1 \\ 1.1 & 1.511164 & > & 1 \end{array} $	M1 A1 [2]	Or comparisons of $f(0.9)$ and $f(1.1)$ with zero
1	(ii)		$ \begin{array}{ccccccc} a & b & f(a) & f(b) & m & f(m) & \text{mpe} \\ 0.9 & 1.1 & -\text{ve} & +\text{ve} & 1 & 0.017105 & 0.1 \\ 0.9 & 1 & & & 0.95 & -0.1833 & 0.05 \\ 0.95 & 1 & & & 0.975 & & 0.025 \end{array} $	M1A1 A1 A1 [4]	This mark requires stopping at 0.975 Accept $0.95 < x < 1$ provided 0.975 seen
2	(i)		$ \begin{array}{ccccc} x & f(x) & \Delta & \Delta^2 & \Delta^3 \\ 0 & -6 & & & \\ 1 & -7 & -1 & & \\ 2 & -4 & 3 & 4 & \\ 3 & 15 & 19 & 16 & 12 \\ 4 & 62 & 47 & 28 & 12 \end{array} $ <p>It's a cubic</p>	M1 A1 B1 [3]	Method For all correct Cao dependent on equal third differences
2	(ii)		$f(x) = -6 - x + 4x(x-1)/2 + 12x(x-1)(x-2)/6$	M1A1A1 [3]	A1 for 1 st , 2 nd terms; A1 3 rd , 4th
2	(iii)		$f(2.5) = 2.75$	M1A1 [2]	M1 substitution in their $f(x)$, A1 cao
3	(i)		$ \begin{array}{lll} \cos(0.11) & \cos(0.12) & \cos(0.11) - \cos(0.12) \\ 0.9940 & 0.9928 & 0.0012 \end{array} $	M1 A1A1 [3]	Use of 4 sf A1 two cos values, A1 difference
3	(ii)		$ \begin{array}{lll} \sin(0.115) & \sin(0.005) & \text{other value for } \cos(0.11) - \cos(0.12) \\ 0.1147 & 0.005000 & 0.001147 \end{array} $	A1A1 [2]	A1 two sin values, A1 other value

Question		Answer	Marks	Guidance
3	(iii)	Using calculator. 0.0011475 or 0.00114746 (or better) First answer much less precise / accurate than the second Subtraction of nearly equal quantities	A1 E1 E1 [3]	Either
4	(i)	Rounding. Different no. of figures rounded up and rounded down	E1E1 [2]	
4	(ii)	Maximum 102: eg 19.6, 19.6, 19.6, 19.6, 21.6 Minimum 98: eg 19.4, 19.4, 19.4, 19.4, 22.4	A1E1 A1E1 [4]	The 5 values must sum to 100 Allow 0.5 to round up or down
5	(i)	$\begin{array}{ll} x & f(x) \\ 0.4 & 0.757858 \\ 0.8 & 0.574349 \end{array}$ hence $M_2 = 0.532883$ $T_2 = 0.5(M_1 + T_1) = 0.538012$	M1A1 M1A1 [4]	
5	(ii)	$S_1 = (2M_1 + T_1) / 3 = 0.534609$ $S_2 = (2M_2 + T_2) / 3 = 0.534593$ Hence integral is 0.53459	M1A1 A1 B1 [4]	M1 for either S_1 or S_2 (5sf is justified by convergence of S)
6	(i)	$\begin{array}{cccc} h & g'(2) & \text{diffs} & \text{ratio} \\ 0.2 & 1.42925 & & \\ 0.1 & 1.39370 & -0.03555 & \\ 0.05 & 1.37640 & -0.01730 & 0.486639 \end{array}$ (approx 0.5) indicates 1 st order method	M1A1A1A1 B1 E1 [6]	Estimates of $g'(2)$ ratio Must see 'first order' Use of Backward Difference Sc 4
6	(ii)	$\begin{array}{cccc} h & g'(2) & & \\ 0.2 & 1.361325 & & \\ 0.1 & 1.359700 & -0.00163 & \\ 0.05 & 1.359300 & -0.00040 & 0.246154 \end{array}$ (approx 0.25) indicates 2nd order method (Hence faster convergence than forward difference)	M1A1A1A1 B1 E1 [6]	Estimates of $g'(2)$ Ratio Identify ratio as indicating 2 nd order
6	(iii)	Errors are $D_1 - d$ and $D_2 - d$ $D_2 - d \approx \frac{1}{4}(D_1 - d)$ Convincing algebra to given result	M1 M1 A1 [3]	

Question		Answer	Marks	Guidance
6	(iv)	1.361325 improved estimates 1.359700 1.359158 1.359300 1.359167 1.3592 seems secure	M1A1A1 A1 [4]	M1: use formula on central diff values Improved estimates cao cao
7	(i)	x 0 1 2 f(x) 1 -1 59	M1A1 [2]	Argument to roots may be implied
7	(ii)	$f'(x) = 6x^5 - 3 = 0$ at only one value of x , hence one turning point hence there are only 2 real roots	B1B1A1 E1 [4]	B1 for $f'(x)$, B1 setting to zero A1 only 1 tp, E1 conclusion
7	(iii)	r 0 1 2 3 4 5 x_r 0.5 0.338542 0.333835 0.333795 0.333794 0.333794 Root is 0.33379 to 5 significant figures	M1A1A1 A1 [4]	M1 using iterative formula A1 up to $r = 2$ A1 up to $r = 5$ or better cao
7	(iv)	Required derivative is $2x^5$ Throughout [1, 2] this is greater than 1; hence iteration does not converge	M1 A1E1 [3]	A0E1 for selected values only in interval
7	(v)	Eg (inverse of previous iteration) $x_{r+1} = \frac{1}{(3x_r - 1)^6}$ r 0 1 2 3 4 5 6 x_r 1.5 1.232191 1.179783 1.168029 1.16531 1.164677 1.164529 r 7 8 9 10 11 x_r 1.1645 1.164494 1.164486 1.164484 1.164484 Root is 1.1645 to 5 significant figures	M1A1A1 A1 [4]	Accept any correct convergent it'n M1 using a convergent iteration A1 first few terms A1 terms to convergence cao

4776 Numerical Methods (Written Examination)

General Comments

The standard of work exhibited by the candidates was rather better than the average over recent years. It was pleasing that, on the whole, candidates were thinking numerically; that is, they showed an appreciation of both the power and the limitations of numerical methods.

Comments on Individual Questions

1 Solution of an equation (bisection)

Virtually everyone was able to locate the root using change of sign in part (i). The bisection method in part (ii) was usually done successfully, though some were not careful enough about the maximum possible error (mpe). The mpe initially is 0.1, but some candidates took it to be 0.5. A minority of candidates made no reference to mpe at all, and just iterated until they were happy with their solution. Such attempts did not receive full marks.

2 Newton's forward difference interpolation formula

Apart from some errors in signs, the difference table in part (i) was done well. Most were able to find an expression for $f(x)$, though there were the usual confusions between values of x and values of $f(x)$ from a few. The substitution in part (iii) was usually, but not invariably, done correctly.

3 Errors and accuracy

This was a pretty routine question about working to limited precision. Those who lost marks generally did so because they did not follow the instructions carefully enough. A small number of candidates lost marks through not rounding correctly to the required number of significant figures. In part (iii) some reference to subtracting nearly equal quantities was expected.

4 Errors and accuracy

Though the idea of rounded percentages not summing to 100 can hardly be new to candidates, quite a few of them found this question difficult. In part (i) it was not enough to say that rounding has occurred: that, in itself, would not necessarily give the wrong total. In this case it must be that the number rounded up exceeds the number rounded down. In part (ii), The maximum and minimum figures are 102 and 98, but candidates produced arguments for figures as high as 104 and as low as 96. It was quite common to see an asymmetrical and curious pair of figures such as 103 and 98.5 – the latter not even being an integer!

5 Numerical integration

The routine calculations were generally done well. The final answer, however, was often based on the agreement between the two Simpson's rule values rather than on a consideration of the change. The two values are 0.534609 and 0.534593; the difference is 16 points and so the next difference is likely to be 1, giving 0.53459 as secure to 5 significant figures.

6 Numerical differentiation

The numerical values in parts (i) and (ii) were generally obtained correctly. It was expected that candidates would say that the ratios of differences indicate that the forward difference method has first order convergence and the central difference method has second order convergence. The algebra in part (iii) was disappointing; the main problem appeared to be that candidates could not express the given information algebraically. Part (iv) required the formula in part (iii) to be applied to the answers from part (ii), these being for the second order method. A majority wrongly applied the formula to some or all of the answers from part (i).

7 Solution of an equation (fixed point iteration)

Part (i), locating the roots, was very easy. Part (ii) required candidates to determine, by differentiation, that the graph has only one turning point and hence (using the information from part (i)) that the equation has exactly two roots. This reasoning defeated those who were convinced that sextics must have six roots and/or five turning points. The iteration in part (iii) was generally well done, but in part (iv) the reasoning was often sloppy. Some argument to the effect that the derivative is greater than 1 *throughout* an interval containing the root was required. The majority were able to find a convergent iteration in part (v); most chose the inverse of the iteration in part (iii) which is, of course, guaranteed to converge.