OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4776/01

Numerical Methods

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 18 May 2011
Morning

Duration: 1 hour 30 minutes

4713900611

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 The equation $f(x) = 0$, where $f(x)$ is a continuous function, is known to have a single root in the interval $[0.4, 1.8]$.

(i) Suppose the root is to be found using the bisection method. State the best possible estimate of the root at the start of the process. State also the maximum possible error associated with that estimate.

Determine how many iterations of the bisection process would be required to reduce the maximum possible error to less than 0.05. **[4]**

(ii) Given now that $f(0.4) = -0.2$ and $f(1.8) = 0.5$, find an estimate of the root using the false position method. **[3]**

2 The function $g(x)$ has the values shown in the table.

x	1.80	2.00	2.20
$g(x)$	2.66	2.85	3.02

(i) Taking the data to be exact, use the central difference formula to estimate $g'(2)$. **[2]**

(ii) Suppose instead that the x values are exact but the values of $g(x)$ are rounded to 2 decimal places. Find an appropriate range of estimates of $g'(2)$. **[3]**

(iii) Now suppose that all the values in the table have been rounded to 2 decimal places. Find the appropriate range of estimates of $g'(2)$ in this case. **[3]**

3 The function $Q(x)$ is known to be quadratic and it has the values shown in the table.

x	-1	1	5
$Q(x)$	-4	-12	20

(i) Write down the estimate of $Q(0)$ obtained by linear interpolation. **[1]**

(ii) Use Lagrange's method to write down an expression for $Q(x)$. [You are not required to simplify this expression.] **[5]**

(iii) Find the exact value of $Q(0)$. **[2]**

4 **(i)** Show that the equation $x = 1 - x^4$ has a root in the interval [0.7, 0.8]. **[2]**

(ii) Show, by considering the derivative of $1 - x^4$, that the iteration $x_{r+1} = 1 - x_r^{\,4}$, with a starting value in the interval [0.7, 0.8], will diverge. **[4]**

5 **(i)** Find the absolute error and the relative error when $X = 3.162$ is used as an approximation to $x = \sqrt{10}$. **[3]**

(ii) Find the relative error if X^4 is used as an approximation to x^4. **[3]**

(iii) State, in terms of k, the approximate relative error if X^k is used as an approximation to x^k. **[1]**

Section B (36 marks)

6 The integral $I = \int_2^{2.8} \sqrt{1+x^3}\, \mathrm{d}x$ is to be determined numerically. You should give all your answers to 7 decimal places unless instructed otherwise.

(i) Find mid-point rule and trapezium rule estimates of I, taking $h = 0.8$.

Use these two estimates to find a second trapezium rule estimate and a Simpson's rule estimate of I. **[8]**

(ii) Find the mid-point rule estimate with $h = 0.4$, and hence obtain a second Simpson's rule estimate of I. **[3]**

(iii) You are now given that the mid-point rule estimate of I with $h = 0.2$ is 3.091 429 8, correct to 7 decimal places.

Find a third Simpson's rule estimate. Show by considering ratios of differences that Simpson's rule is of order h^4.

Give the value of I to the accuracy that appears justified. **[7]**

[Question 7 is printed overleaf.]

7 The function $f(x)$ has the exact values shown in the table.

x	1	3	5
$f(x)$	4	-2	10

(i) Use Newton's forward difference interpolation formula to find the quadratic function that fits the data. (There is no need to simplify your answer.) **[6]**

(ii) Hence estimate the values of $f(2)$ and $f(6)$. State, with a reason, which of these estimates is likely to be more accurate. **[3]**

(iii) Now suppose that $f(7) = 11$. Find the cubic function that fits all the data. Use this cubic to estimate $f(2)$ and $f(6)$. **[7]**

(iv) Comment on (*A*) the absolute changes and (*B*) the relative changes in the estimates of $f(2)$ and $f(6)$ from part **(ii)** to part **(iii)**. **[2]**

1(i)	Best estimate: 1.1 Maximum possible error: 0.7		**[B1B1]**
	Bisecting mpe: 0.7 → 0.35 → 0.175 → 0.0875 → 0.04375 so 4 iterations		**[M1A1]**
(ii)	False position: estimate is (0.4 × 0.5 – 1.8 × (–0.2) / (0.5 – (–0.2))	*BoD for 0.8 alone*	**[M1A1]**
	= 0.8		**[A1]**
			[TOTAL 7]

2(i)	(3.02 – 2.66) / (2.20 – 1.80) =		0.9		**[M1A1]**
(ii)	max:	(3.025 – 2.655) / (2.20 – 1.80) =	0.925		**[M1A1]**
	min:	(3.015 – 2.665) / (2.20 – 1.80) =	0.875		**[A1]**
(iii)	max:	(3.025 – 2.655) / (2.195 – 1.805) =	0.94872	*[M1] either denominator*	**[M1A1]**
	min:	(3.015 – 2.665) / (2.205 – 1.795) =	0.85366	*correct. Max [2] if 3 dp*	**[A1]**
					[TOTAL 8]

3(i)	Linear interpolation: Q(0) = –8	*write down or any method*	**[B1]**
(ii)	Lagrange: Q(x) = (–4)(x – 1)(x – 5) / (–1 – 1)(–1 – 5) +	*Lagrange form*	**[M1]**
	(–12)(x + 1)(x – 5) / (1 + 1)(1 – 5) +	*three terms*	**[DM1]**
	20(x + 1)(x – 1) / (5 + 1)(5 – 1)	*terms*	**[A1,1,1]**
(iii)	Hence by substitution Q(0) = –10	*cao*	**[M1A1]**
			[TOTAL 8]

4(i)	x		$1 - x^4$			
	0.7	<	0.7599			
	0.8	>	0.5904	(hence root)		**[M1A1]**
(ii)	Derivative of $1 - x^4$ is $-4x^3$					**[M1]**
	x	abs($-4x^3$)				
	0.7	1.372 > 1			*[M1] for grad anywhere*	
	0.8	2.048 > 1		(so for all [0.7, 0.8] abs gradient or abs RHS > 1)	*in interval*	**[M1A1E1]**
						[TOTAL 6]

5(i)	x	X	abs err	rel err	*do not insist on sign*	
	3.162278	3.162	–0.00028	–0.0000878	*but do require consistency*	**[M1A1A1]**
					between parts (i) and (ii)	
(ii)	x^4	X^4	(abs err	rel err		
	100	99.96488	–0.03512)	–0.0003512	*allow multiplying answer in (i) by 4*	**[M1A1A1]**
(iii)	Relative error will be approximately k × (–)0.0000878					**[B1]**
						[TOTAL 7]

6(i)

x	f(x)
2	3
2.4	3.850195
2.8	4.790825

M1 = 3.0801558 — *Lose A1, 2, 3, 4 if 6, 5, 4, 3 dp* — **[M1A1]**
T1 = 3.1163298 — **[M1A1]**
T2 = 3.0982428 *(=(M1+T1)/2)* — **[M1A1]**
S1 = 3.0922138 *(=(2M1+T1)/3)* — **[M1A1]**
[subtotal 8]

(ii)

x	f(x)
2.2	3.412917
2.6	4.309988

M2 = 3.0891620 — **[M1A1]**
S2 = 3.0921890 *(=(2M2+T2)/3)* — **[B1]**
or ... 889
[subtotal 3]

(iii)

M4 = 3.0914298
T4 = 3.0937024 — **[B1]**
S4 = 3.0921873 — **[B1]**

		diffs	ratio of diffs
S1 =	3.0922138		
S2 =	3.0921890	–2.5E–05	
S4 =	3.0921873	–1.6E–06	0.0648518

approx 1/16 so fourth order — **[M1A1E1]**
(FT their precision)

Consider rate of convergence of S: conclude I = 3.09219 — *(or 3.092187 if using extrapolation)* — **[E1A1]**
[B2] for 3.09219 alone
[subtotal 7]
[TOTAL 18]

7(i)

x	f(x)	Δf(x)	Δ^2f(x)
1	4		
3	–2	–6	
5	10	12	18

[M1A1]

$f(x) = 4 + (-6)(x-1)/2 + 18(x-1)(x-3)/(2^2 \times 2!)$ — **[M1A1A1A1]**
[subtotal 6]

(ii) f(2) = –1.25 likely to be more accurate (interpolation)
f(6) = 22.75 than this (extrapolation) — **[A1A1E1]**
[subtotal 3]

(iii)

x	f(x)	Δf(x)	Δ^2f(x)	Δ^3f(x)	
1	4				
3	–2	–6			
5	10	12	18		
7	**11**	**1**	**–11**	**–29**	← extend table

[M1A1]

new $f(x) = \text{old } f(x) + (-29)(x-1)(x-3)(x-5)/(2^3 \times 3!)$ — *FT incorrect old f(x) here* — **[M1A1]**

f(2) = –1.25 + (–1.8125) = –3.0625 — *[M1] for substituting* — **[M1A1]**
f(6) = 22.75 + (–9.0625) = 13.6875 — *cao for each [A1]* — **[A1]**
[subtotal 7]

(iv) Absolute change greater in f(6), relative change greater in f(2) — *Must have all 4 values correct* — **[E1E1]**
[E1], [E1] for intelligent comments on absolute, relative changes
[subtotal 2]
[TOTAL 18]

4776/01: Numerical Methods (Written Examination)

General Comments

Most candidates found this a straightforward paper, and there were many high marks. Candidates showed a good grasp of the basic ideas of numerical mathematics, but as in previous years the standard of presentation of work was frequently not good. The point has to be made yet again that numerical mathematics is systematic and algorithmic. Setting down work in a logical order, frequently in tabular form, makes it easier to see whether a solution is correct – easier, that is, for both the candidate and the marker. Jumbles of unidentified numbers scattered across a page are unlikely to receive credit.

Comments on Individual Questions

1 **Solution of an equation**
In part (i), the idea of halving the maximum possible error at each iteration was well understood, but many candidates counted the starting position of the bisection method as an iteration. In part (ii), false position was usually done correctly.

2 **Numerical differentiation**
Parts (i) and (ii) were generally done well, but part (iii) defeated most candidates. The idea in part (iii) is to combine the largest possible numerator with the smallest possible denominator, and vice versa.

3 **Lagrange's interpolation formula**
This was a straightforward question, but quite a number of candidates were not able to apply linear interpolation in part (i). In part (ii) there were the usual confusions between x values and function values. The question said, quite clearly, that no algebraic simplification was required; some candidates chose to simplify anyway.

4 **Fixed point iteration**
Locating the root in part (i) was very easy, but the majority of candidates were unable to use the magnitude of the derivative in a region surrounding the root to show that the iteration will diverge.

5 **Absolute and relative errors**
The numerical work in parts (i) and (ii) was done well by almost all. In part (iii), candidates were expected to know that when a number with a small relative error is raised to a power k the relative error will be increased (approximately) by a factor of k. This part was not done well.

6 **Numerical integration**
The numerical work in first two parts was done well by the vast majority of candidates. Part (iii) was more challenging. Not all candidates appeared to know how to recognise fourth order convergence. Some knew what to look for but arithmetical errors prevented them finding it. The value of I was often given to fewer decimal places than the work warranted. It is *not* correct to look for the number of figures of agreement in the two best Simpson's rule values. Thinking about differences and ratios of differences will show that the last Simpson's rule value will be very much more accurate than the one before.

7 **Newton's forward difference interpolation formula**

Almost all candidates found the quadratic in part (i) successfully, though some insisted on doing algebraic simplification which was not required. Part (i) was found easy too. In part (iii), the best approach is to add on the cubic term to the quadratic already found: this is one of the virtues of Newton's method. Some candidates worked from scratch here. The answers to part (iv) were frequently poor. Either the wrong numerical values found earlier prevented sensible comments, or what candidates said was muddled and unclear. The intended point was that the absolute change is greater in f(6) but the relative change is greater in f(2).

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0