

**ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)**

Numerical Methods

4776/01



Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

**Friday 14 January 2011
Afternoon**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

- 1 (i) Show that the equation $1 + x = \tan x$, where x is in radians, has a root in the interval $[1, 1.2]$. [2]
- (ii) Show numerically that the iteration $x_{r+1} = \tan x_r - 1$ with $x_0 = 1.1$ diverges. [2]
- (iii) Use another iteration to find the root correct to 3 decimal places. [4]
- 2 The table shows some estimates of an integral, $\int_2^4 f(x) dx$, using the mid-point rule (M) and the trapezium rule (T), for given values of h .

h	M	T
2	1.987467	1.354440
1	1.830595	
0.5		

Copy the table and fill in the additional estimates that can be found.

Obtain the Simpson's rule estimates that can be found.

Give the value of the integral to the accuracy that appears justified. [8]

- 3 The table shows values of $g(x)$ correct to 4 decimal places.

x	0	0.5	1
$g(x)$	1.4509	1.6799	2.0100

- (i) Use the forward difference method to find two estimates of $g'(0)$. State, with a reason, which of these is likely to be more accurate. [4]
- (ii) Use the central difference method to find an estimate of $g'(0.5)$. Comment on the likely accuracy of this estimate compared to those in part (i). [2]

- 4 A bank's computer system calculates the interest payable on each savings account every day. A running total is kept of the daily amounts of interest, and accounts are credited with this interest at the end of each year. The bank used to *round* the daily amounts of interest payable to the nearest 0.01 of a penny, but they decide to *chop* to the nearest 0.01 of a penny instead.
- (i) Find the maximum possible loss in a year to a savings account because of the chopping, and explain how this loss could occur. State, with a reason, what the average loss will be. [4]
- (ii) The bank calculates that chopping in this way will generate an additional profit of about £150 000 per year. Estimate the number of savings accounts the bank has. [2]
- 5 The function $P(x)$ is known to be a polynomial. Some values of $P(x)$ are given in the table.

x	1	3	5	7	9
$P(x)$	-10	3	44	129	274

- (i) Use a difference table to determine, with a reason, the least possible degree of polynomial that will fit all the data points. [4]
- (ii) Assuming that $P(x)$ is of this degree, extend your table to find the values of $P(-1)$ and $P(11)$. [4]

Section B (36 marks)

- 6 In this question,

$$f(x) = \frac{x}{\sin x} - \frac{\sin x}{x},$$

where x is in radians. For small non-zero values of x , $f(x)$ may be approximated by $g(x)$ or by $h(x)$, where

$$g(x) = \frac{1}{3}x^2 \quad \text{and} \quad h(x) = \frac{2x^2}{6-x^2}.$$

- (i) Find the absolute and relative errors in $g(x)$ and $h(x)$ as approximations to $f(x)$ for
 (A) $x = 0.2$,
 (B) $x = 0.1$ [9]
- (ii) A third approximation to $f(x)$ is given by $\frac{4g(x) + h(x)}{5}$. Explain by reference to part (i) why this would be expected to be a good approximation.
 Find the absolute and relative errors when this third approximation is used to estimate $f(0.2)$ and $f(0.1)$. [6]
- (iii) Use your calculator to evaluate $\frac{x}{\sin x}$ when $x = 10^{-4}$.
 When $x = 10^{-4}$, a cheap calculator evaluates $f(x)$ as zero. Use an approximate formula to find a better value for $f(10^{-4})$. Explain why the cheap calculator makes an error. [3]

- 7 (i) Show that the equation $f(x) = 0$, where

$$f(x) = x^7 + x^5 - 1, \quad (*)$$

has a root in the interval $[0, 1]$.

By considering $f'(x)$ show that there are no other roots.

Sketch the graph of $y = f(x)$ for $x \geq 0$. [7]

- (ii) Obtain the Newton-Raphson iteration based on (*). Starting with $x_0 = 0.6$, find x_1 and x_2 . Illustrate this iteration on your sketch of $y = f(x)$. [7]

- (iii) Use the Newton-Raphson iteration to find x_1 and x_2 in the cases

(A) $x_0 = 0.3$,

(B) $x_0 = 0.9$.

Comment on your results in each case. [4]



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Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4776**: Numerical Methods

Mark Scheme for January 2011

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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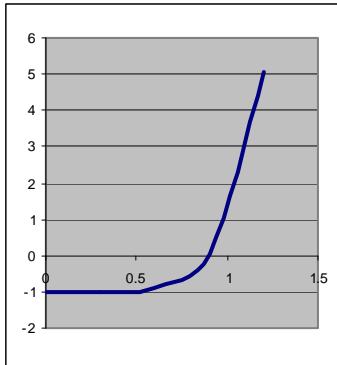
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Question		Answer	Marks	Guidance												
1	(i)	$\begin{array}{ccccc} x & \text{LHS} & & \text{RHS} \\ 1 & 2 & > & 1.557408 \\ 1.2 & 2.2 & < & 2.572152 \end{array}$	M1 A1 [2]	no explicit explanation required												
	(ii)	$\begin{array}{ccccccc} r & 0 & 1 & 2 & 3 & 4 \\ x_r & 1.1 & 0.96476 & 0.442927 & -0.52564 & -1.58007 \end{array}$	M1 A1 [2]	$r = 3$ required												
	(iii)	e.g. re-arrange to $x = \arctan(1 + x)$ $\begin{array}{ccccccc} r & 0 & 1 & 2 & 3 & 4 & 5 \\ x_r & 1.1 & 1.126377 & 1.131203 & 1.132076 & 1.132233 & 1.132261 \\ & & & & & & 1.132 \end{array}$	B1 M1 A1 A1 [4]													
2		<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <th>h</th> <th>M</th> <th>T</th> </tr> <tr> <td>2</td> <td>1.987467</td> <td>1.354440</td> </tr> <tr> <td>1</td> <td>1.830595</td> <td>1.670954</td> </tr> <tr> <td>0.5</td> <td></td> <td>1.750774</td> </tr> </table> <p style="display: inline-block; vertical-align: middle;">Simpson's rule $(2M + T) / 3$ 1.776458 1.777381</p> <p>Reference to justification/accuracy : 1.777 or 1.78</p>	h	M	T	2	1.987467	1.354440	1	1.830595	1.670954	0.5		1.750774	T: M1A1A1 S: M1A1A1 E1 A1 [8]	Lose 1 for any additional 'answer'(s) but do not penalise extrapolation
h	M	T														
2	1.987467	1.354440														
1	1.830595	1.670954														
0.5		1.750774														
3	(i)	$h = 1 \quad g'(0) = (2.0100 - 1.4509)/1 = 0.5591$ $h = 0.5 \quad g'(0) = (1.6799 - 1.4509)/0.5 = 0.458$ Estimate with smaller h (0.458) likely to be more accurate: smaller h is more accurate (provided there is no great loss of significant figures)	B1 B1 B1 E1 [4]													
	(ii)	$h = 0.5 \quad g'(0.5) = (2.0100 - 1.4509)/1 = 0.5591$ This estimate, central diff, likely to be more accurate than either of the forward diffs	M1 E1 [2]													

Question		Answer	Marks	Guidance
4	(i)	Max poss loss: 365 (or 366) times 0.01 pence: = 3.65 (or 3.66) pence Arises if each daily amount would round up but gets chopped down Average loss 1.825 (or 1.83) pence, because average is half of max.	B1 E1 B1 E1 [4]	
	(ii)	£150 000 divided by 1.825 pence: about 8.2 million (8 million) accounts	M1 A1 [2]	
5		$ \begin{array}{cccccc} x & P(x) & \Delta P(x) & \Delta^2 P(x) & \Delta^3 P(x) \\ -1 & -11 & & & & \\ 1 & -10 & 1 & & & \\ 3 & 3 & 13 & 12 & & \\ 5 & 44 & 41 & 28 & 16 & \\ 7 & 129 & 85 & 44 & 16 & \\ 9 & 274 & 145 & 60 & 16 & \\ 11 & 495 & 221 & 76 & 16 & \end{array} $ <p style="text-align: center;">(i) bold: Diff table 3rd diffs constant so cubic</p> <p style="text-align: center;">(ii) italic: working forwards working backwards</p>	M1 A1 E1 B1 M1 A1 M1 A1 [4] + [4]	
6	(i)	$ \begin{array}{cccccccc} x & f & g & h & \text{abs err } g & \text{rel err } g & \text{abs err } h & \text{rel err } h \\ 0.2 & 0.013351 & 0.013333 & 0.013423 & 0.0000179 & -0.0013424 & 0.0000716 & 0.0053600 \\ 0.1 & 0.003334 & 0.003333 & 0.003339 & 0.0000011 & -0.0003339 & 0.0000045 & 0.0013350 \end{array} $ <p style="text-align: center;">A1 A1 A1 A1 A1 A1 A1</p>	abs M1 rel M1	f, g, h values may be implied
	(ii)	Errors in g and h are of opposite sign; g is about 4 times as accurate as h. $ \begin{array}{ccccc} x & f & (4g+h)/5 & \text{abs err} & \text{rel err} \\ 0.2 & 0.013351 & 0.013351 & -2.5E-08 & -1.9E-06 \\ 0.1 & 0.003334 & 0.003334 & -4E-10 & -1.2E-07 \end{array} $ <p style="text-align: center;">A1 A1 A1</p>	E1 E1 M1	
	(iii)	$x / \sin x \approx 1.000\ 000\ 002 \approx 1$ $g(10^{-4}) = 3.33 \times 10^{-9}$ Subtraction of nearly equal quantities	B1 B1 E1 [3]	

Question		Answer	Marks	Guidance																
7	(i)	<p>$f(0) = -1$, $f(1) = 1$ (hence root) $f'(x) = 7x^6 + 5x^4$ which is zero only at $x = 0$. Convincing argument that this is not a turning point No turning points implies no other roots.</p> 	B1 M1 A1 B1 E1 G2																	
	(ii)	<p>NR iteration: $x_{r+1} = x_r - (x_r^7 + x_r^5 - 1) / (7x_r^6 + 5x_r^4)$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>r</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>x_r</td> <td>0.6</td> <td>1.51756</td> <td>1.289164</td> </tr> </table> <p>On graph: tangent at 0.6, intersection at 1.5, ordinate & tangent, intersection at 1.3</p>	r	0	1	2	x_r	0.6	1.51756	1.289164	B1 A1 A1 G4 [7]									
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