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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI) 4776/01
Numerical Methods

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Show that the equation

$$\frac{1}{x} = 3 - x^2 \qquad (*)$$

has a root, α, between $x = 1$ and $x = 2$.

Show that the iteration

$$x_{r+1} = \frac{1}{3 - x_r^2},$$

with $x_0 = 1.5$, converges, but not to α. **[5]**

(ii) By rearranging (*), find another iteration that does converge to α. You should demonstrate the convergence by carrying out several steps of the iteration. **[3]**

2 A function f(x) has the values shown in the table.

x	2.8	3	3.2
f(x)	0.9508	0.9854	0.9996

(i) Taking the values of f(x) to be exact, use the forward difference method and the central difference method to find two estimates of f′(3). State which of these you would expect to be more accurate. **[5]**

(ii) Now suppose that the values of f(x) have been rounded to the four significant figures shown. Find, for each method used in part **(i)**, the largest possible value it gives for the estimate of f′(3). **[2]**

3 **(i)** X is an approximation to the number x such that $X = x(1 + r)$. State what r represents.

Show that, provided r is small, $X^n \approx x^n(1 + nr)$. **[4]**

(ii) The number $G = 0.577$ is an approximation to the number g. G is about 0.04% smaller than g. State, in similar terms, relationships between

(A) G^2 and g^2,

(B) $\sqrt{G}$ and $\sqrt{g}$. **[3]**

4 The expression, $\sin x + \tan x$, where x is in radians, can be approximated by $2x$ for values of x close to zero.

(i) Find the absolute and relative errors in this approximation when $x = 0.2$ and $x = 0.1$. **[4]**

(ii) A better approximation is $\sin x + \tan x \approx 2\left(x + \frac{x^3}{k}\right)$, where k is an integer.

Use your results from part **(i)** to estimate k. **[3]**

5 A quadratic function, $f(x)$, is to be determined from the values shown in the table.

x	1	3	6
$f(x)$	–10	–12	30

Explain why Newton's forward difference formula would not be useful in this case.

Use Lagrange's interpolation formula to find $f(x)$ in the form $ax^2 + bx + c$. **[7]**

Section B (36 marks)

6 The integral

$$I = \int_1^{1.8} \sqrt{x^3 + 1}\, dx$$

is to be estimated numerically. You are given that, correct to 6 decimal places, the mid-point rule estimate with $h = 0.8$ is 1.547 953 and that the trapezium rule estimate with $h = 0.8$ is 1.611 209.

(i) Find the mid-point rule and trapezium rule estimates with $h = 0.4$ and $h = 0.2$.

Hence find three Simpson's rule estimates of I. **[7]**

(ii) Write down, with a reason, the value of I to the accuracy that appears to be justified. **[2]**

(iii) Taking your answer in part **(ii)** to be exact, show in a table the errors in the mid-point rule and trapezium rule estimates of I.

Explain what these errors show about

(*A*) the relative accuracy of the mid-point rule and the trapezium rule,

(*B*) the rates of convergence of the mid-point rule and the trapezium rule. **[8]**

7 **(i)** Show that the equation

$$x^5 - 8x + 5 = 0 \qquad (*)$$

has a root in the interval (0, 1).

Find this root, using the Newton-Raphson method, correct to 6 significant figures.

Show, by considering the differences between successive iterates, that the convergence of the Newton-Raphson iteration is faster than first order. **[11]**

(ii) You are now given that equation (*) has a root in the interval (1.4, 1.5). Find this root, correct to 3 significant figures, using the secant method. Determine whether or not the secant method is faster than first order. **[8]**

THERE ARE NO QUESTIONS ON THIS PAGE

GCE

Mathematics (MEI)

Advanced GCE **4776**

Numerical Methods

Mark Scheme for June 2010

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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1(i)	x	LHS		RHS		
	1	1	<	2	(Change of sign implies root.)	
	2	0.5	>	-1	*(or equivalent)*	**[M1A1]**

r	0	1	2	3	4	5	6	
x_r	1.5	1.333333	0.818182	0.429078	0.355127	0.347961	0.347352	**[M1A1]**

State or clearly imply convergence outside the interval (1, 2) **[E1]**

(ii) E.g. $x_{r+1} = \sqrt{(3 - 1/x)}$

r	0	1	2	3	4	5
x_r	1.5	1.527525	1.531452	1.532	1.532077	1.532087

E.g. $x_{r+1} = 3/x - 1/x^2$ **[B1]**

r	0	1	2	3	4	5
x_r	1.5	1.555556	1.515306	1.544287	1.523326	1.538438

[M1A1]
[TOTAL 8]

2(i)	Forward difference:	(0.9996 - 0.9854)/0.2 = 0.071	**[M1A1]**
	Central difference:	(0.9996 - 0.9508)/0.4 = 0.122	**[M1A1]**
	Central difference expected to be more accurate.		**[E1]**
(ii)	Forward difference maximum:	(0.99965 - 0.98535)/0.2 = 0.0715	**[B1]**
	Central difference maximum:	(0.99965 - 0.95075)/0.4 = 0.12225	**[B1]**
			[TOTAL 7]

3(i) r is the relative error (in X as an approximation to x) **[E1]**

$X^n = x^n (1 + r)^n$ $(1 + r)^n = 1 + nr$ (provided r is small) **[M1M1A1]**

(ii) G^2 (= 0.332 929, not required) is about 0.08% smaller than g^2

$\sqrt{G}$ (= 0.795 605, not required) is about 0.02% smaller than $\sqrt{g}$ **[M1A1A1]**

[TOTAL 7]

4(i)

x	sin + tan	$2x$	error	rel error	*accept:*	
					+ve, +ve	
0.2	0.401379	0.4	-0.00138	-0.00344	*-ve, +ve*	**[M1A1A1A1]**
0.1	0.200168	0.2	-0.00017	-0.00084	*-ve, -ve*	

(ii) $2 \times 0.2^3 / k = 0.00138$ gives $k = 11.59$ Either of these (or other methods) **[M1A1]**

$2 \times 0.1^3 / k = 0.00017$ gives $k = 11.76$ to suggest $k = 12$ **[B1]**

[TOTAL 7]

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5 Data not equally spaced in x **[E1]**

$f(x) = -10(x - 3)(x - 6) / (1 - 3)(1 - 6) - 12(x - 1)(x - 6) / (3 - 1)(3 - 6) + 30(x - 1)(x - 3) / (6 - 1)(6 - 3)$

[M1A1A1A1]

$f(x) = -(x^2 - 9x + 18) + 2(x^2 - 7x + 6) + 2(x^2 - 4x + 3)$ **[A1]**

$= 3x^2 - 13x$ **[A1]**

[TOTAL 7]

6(i)

h	M	T	S		
0.8	1.547953	1.611209	**1.569038**	*M:*	**[M1A1A1]**
0.4	**1.563639**	**1.579581**	**1.568953**	*T:*	**[M1A1]**
0.2	**1.567619**	**1.571610**	**1.568949**	*S:*	**[M1A1]**

[subtotal 7]

(ii) 1.56895 appears justified Comparison of last two S values, e.g.: **[B1]**
last change in S is -0.000004; next change negligible **[E1]**
[subtotal 2]

(iii)

h	M error	T error		
0.8	-0.02100	0.04226	*accept consistent*	
0.4	-0.00531	0.01063	*use of other sign*	
0.2	-0.00133	0.00266	*convention*	**[M1A1A1]**

(A) M errors are about half the T errors so M is twice as accurate as T **[E1A1]**
(B) Errors for both T and M reduce by a factor of 4 as h is halved so **[E1]**
the rates of convergence are the same, both second order **[A1A1]**
[subtotal 8]
[TOTAL 17]

7(i) f(0) = 5, f(1) = -2. (Change of sign implies root.) **[M1A1]**

f '(x) = $5x^4$ - 8 hence N-R formula **[M1A1]**

r	0	1	2	3	4	
x_r	0.5	0.634146	0.638232	0.638238	0.638238	**[M1A1A1]**
differences		0.134146	0.004086	5.98E-06	1.29E-11	**[A1]**
ratios			0.030457	0.001462	2.17E-06	**[M1A1]**

The ratios of differences are decreasing (fast) so process is faster than first order **[E1]**
[subtotal 11]

(ii)

r	0	1	2	3	4
x_r	1.4	1.5	1.458054	1.462741	1.46312
f(x_r)	-0.82176	0.59375	-0.0747	-0.00559	5.99E-05

[M1A1A1]

root is 1.46 correct to 3 sf **[A1]**

differences	0.1	-0.04195	0.004687	0.000379	**[A1]**
ratios		-0.41946	-0.11175	0.080876	**[M1A1]**

The ratios of differences are decreasing (fast) so process is faster than first order **[E1]**
accept 'second order' ***[subtotal 8]***
[TOTAL 19]

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4776 Numerical Methods (Written Examination)

General comments
There was a lot of good work seen, but as ever there were some candidates who appeared to be unready for the examination. Routine numerical calculations were generally carried out accurately, though it is yet again disappointing that so many candidates set their work out badly. This is an algorithmic subject and good work will reflect that. A poor layout is difficult to follow for the examiner – and difficult for the candidate to check.

Interpretation is still a weak area, with quite a number of candidates simply omitting such parts of questions – or writing vaguely and at length in the hope that they will produce something worthy of a mark.

Comments on individual questions

1) *(Fixed point iteration to solve an equation)*
This proved an easy starter for most candidates. There were many solutions gaining full marks, though a significant minority failed to find a convergent iteration in part (ii).

2) *(Numerical differentiation)*
The first part of this question proved easy for most candidates. The second part, however, was a little more challenging for some. A curious error, seen quite a few times, involved saying that if 0.9996 is correct to 4 decimal places then its maximum possible value is 0.99964. Presumably the reasoning was that 0.99965 would round to 0.9997. This is of course incorrect.

3) *(Relative error)*
The relationship $X = x(1 + r)$ proved troublesome once again. Candidates are just not happy with errors analysed this way. For certain sorts of problem – such as the one in this question – it is by far the easiest approach. In part (ii), candidates were asked to 'state, in similar terms, a relationship …'. An algebraic result without a suitable form of words did not gain full marks.

4) *(Numerical approximation)*
The first four marks were obtained easily by most candidates with only a few making errors in the numerical work or the signs. (There are two conventions for the meaning of the word 'absolute' in the term 'absolute error'. Some books take 'absolute' to be a contrast with 'relative'; others take it to mean the positive value. Either interpretation, used consistently, is acceptable.) Part (ii) was not done well. In quite a number of cases the idea was understood well enough, but the calculation of k involved algebraic errors. Many candidates appeared to ignore the information that k is an integer.

5) (*Lagrange's interpolation formula)*
This was an easy source of marks for many, but as usual some got the x and $f(x)$ values muddled. There were some algebraic errors in the simplification, but perhaps fewer than usual.

6) *(Numerical integration)*
Part (i) was an easy source of marks for most, though there was a lot of inefficient work poorly set out. In part (ii), the correct approach is to compare the Simpson's rule estimates and, noting how small the change is between the second and third values, to conclude that 1.56895 is justified. In part (iii), candidates were mostly able to calculate the errors in the mid-point and trapezium rules. The interpretation of those errors was less well done however, with a lot of rather vague statements being made.

7) *(Newton-Raphson and secant methods)*
In part (i), the root was generally found successfully using the Newton-Raphson method. Candidates were then required to find differences and ratios of differences to assess the rate of convergence. Quite a number of candidates said that because the ratios of differences are not constant the process is faster than first order. This was not enough: they needed to say that ratios of differences are decreasing (fast).

In part (ii), some candidates seemed less secure in their use of the secant method. The conclusion about the rate of convergence was handled much as in part (i).