OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4776

Numerical Methods

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

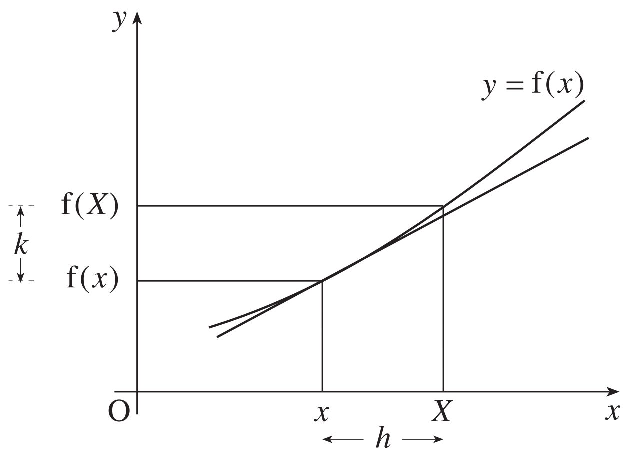
- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

HN/5

Section A (36 marks)

1 In the diagram below, X is an approximation to x with error h. Also $\mathrm{f}(X)$ is an approximation to $\mathrm{f}(x)$ with error k. Show, by considering the tangent at x, that $k \approx h\mathrm{f}'(x)$.



Use this result, with $\mathrm{f}(x) = \sqrt{x}$, to estimate the maximum possible error in $\sqrt{X}$ when $X = 2.5$ correct to 1 decimal place. [7]

2 Show that the equation

$$x^5 - 5x + 1 = 0$$

has a root in the interval [0, 1].

Apply two iterations of the false position method to obtain an estimate of this root. Give your answer to 3 decimal places.

Determine whether or not your answer is correct to 3 decimal places. [8]

3 The integral $I = \int_0^2 \sqrt{1 + 2^x}\, \mathrm{d}x$ is to be found numerically.

Obtain the estimates given by the mid-point rule and the trapezium rule with $h = 2$. *Use these values* to obtain a Simpson's rule estimate of I.

Given that the mid-point rule estimate with $h = 1$ is 3.510 411, obtain as efficiently as possible a second trapezium rule estimate and a second Simpson's rule estimate.

Give the value of I to the accuracy that appears justified. [8]

4 Given the data in the table below, find three estimates of $f'(2)$.

h	0	0.1	0.01	0.001
$f(2+h)$	1.4427	1.3478	1.4324	1.4416

Discuss briefly the likely accuracy of these estimates. [6]

5 Show, by means of a difference table, that the function $g(x)$ tabulated below is approximately but not exactly quadratic.

x	1	2	3	4	5	6
$g(x)$	3.2	12.8	28.4	50.2	77.9	111.6

Use Newton's forward difference formula to estimate the value of $g(1.5)$. [7]

Section B (36 marks)

6 **(i)** Show that the equation

$$x^2 = \tan x \qquad (*)$$

(where x is in radians) has a root in the interval [4.6, 4.7].

Use the bisection method with starting values 4.6 and 4.7 to find this root with maximum possible error 0.0125. [9]

(ii) You are now given that equation (*) also has a root in the interval [7.7, 7.9]. Show that 7.7 and 7.9 are *not* suitable starting points for the bisection method. Explain with the aid of a sketch graph how this situation arises. [5]

(iii) Using only the fact that equation (*) has a root in the interval [7.7, 7.9], write down the best possible estimate of the root. Determine whether or not this estimate is correct to 1 decimal place. [4]

[Turn over

7 The following values of the function $f(x)$ are known.

x	1	2	4
$f(x)$	–3	8	36

It is required to estimate $D = f'(2)$ and $I = \int_1^4 f(x)\,dx$.

(i) Use the forward difference method to estimate D.

Use the trapezium rule to obtain the best possible estimate of I. [4]

(ii) Use Lagrange's method to find the quadratic that passes through the given points.

Hence find new estimates of D and I. [11]

(iii) Comment on the extent to which the estimates in part **(i)** agree with those in part **(ii)**. [3]

1 Sketch, with explanation. **[M1A1E1]**

f '(x) = 1/(2√x) **[M1A1]**

hence mpe is approx 0.05 / (2√2.5) =		0.01581 1	(0.016)	**[M1A1]**
(or 0.05 / 2√2.45 =	0.01597 2	or 0.05 / 2√2.55 =	0.01565 6)	

[TOTAL 7]

2

x	0	1		
f(x)	1	-3	change of sign so root	**[M1A1]**

a	b	f(a)	f(b)	x	f(x)	
0	1	1	-3	0.25	-0.24902	**[M1A1]**
0	0.25	1	-0.24902	0.20015 6	-0.00046	**[A1]**
				0.200 to 3 dp		**[A1]**

x	0.1995	0.2005		
f(x)	0.00281 6	-0.00218	sign change so root is correct to 3 dp	**[M1A1]**

[TOTAL 8]

3

h	M	T	S	
2	**3.46410 2**	**3.65028 2**	**3.52616 2**	**values**
1	3.51041 1	**3.55719 2**	**3.52600 4**	**[A1A1A1A1A1]**

evidence of efficient formulae for T and S
[M1M1]

3.526(0) appears to be justified **[A1]**

[TOTAL 8]

4

h	0	0.1	0.01	0.001	
f(2 + *h*)	1.4427	1.3478	1.4324	1.4416	
est f '(2)		-0.949	-1.03	-1.1	**[M1A1A1A1]**

Clear loss of significant figures as h is reduced **[E1]**
Impossible to know which estimate is most accurate **[E1]**

[TOTAL 6]

5

x	g(*x*)	Δg	Δ^2g		
1	3.2	9.6	6		**table**
2	12.8	15.6	6.2	second differences nearly constant	**[M1A1]**
3	28.4	21.8	5.9	so approximately quadratic	**[E1]**
4	50.2	27.7	6		
5	77.9	33.7			
6	111.6				

g(1.5) = 3.2 + 0.5*9.6 + 0.5*(-0.5)*6/2 = 7.25 **[M1A1A1A1]**

[TOTAL 7]

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6 (i)

x	4.6	4.7		*NB: 3 pi /2 =4.71 (not reqd)*
x^2-tan(x)	12.29983	-58.6228	change of sign, so root	**[M1A1]**

a	b	sign f(a)	sign f(b)	x	sign f(x)	mpe	
4.6	4.7	1	-1	4.65	1	0.05	**[M1A1]**
4.65	4.7	1	-1	4.675	-1	0.025	**[M1A1]**
4.65	4.675	1	-1	4.6625		0.0125	**[M1A1]**

root is 4.6625 with mpe 0.0125 **[A1]**

[subtotal 9]

(ii)

x	7.7	7.9		
x^2-tan(x)	52.84713	84.12511	no change of sign, so no evidence of root	**[M1A1]**

Sketch showing asymptote for tan(x) at 5pi/2 = 7.854 **[G2]**

So x^2 curve is above tan(x) at both end points **[E1]**

[subtotal 5]

(iii) best possible estimate is 7.8 **[A1]**

x	7.75	7.85		**[M1]**
x^2-tan(x)	50.4801	-189.529	change of sign so 7.8 is correct to 1 dp	**[A1E1]**

[subtotal 4]

[TOTAL 18]

7

(i) D = (36 - 8) / (4 - 2) = 14 **[M1A1]**

I = 0.5 (-3 + 8) + (8 + 36) = 46.5 **[M1A1]**

[subtotal 4]

(ii) q(x) = -3 (x-2)(x-4)/(1-2)(1-4) + 8 (x-1)(x-4)/(2-1)(2-4) + 36 (x-1)(x-2)/(4-1)(4-2) **[M1A1A1A1]**

= - (x^2-6x+8) - 4 (x^2-5x+4) + 6 (x^2-3x+2) **[A1]**

= x^2 + 8x - 12 **[A1]**

q'(x) = 2x + 8 so D = 12 **[M1A1]**

∫ q(x) dx = x^3/3 + 4x^2 - 12x so I = 45 **[M1A1A1]**

[subtotal 11]

(iii) Large relative difference between estimates of D **[E1]**

Small relative difference in estimates of I **[E1]**

To be expected as integration is a more stable process than differentiation **[E1]**

[subtotal 3]

[TOTAL 18]