

**Wednesday 24 May 2017 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4763/01** Mechanics 3

**QUESTION PAPER**

A 58

B 50

C 43

D 36

E 29

Candidates answer on the Printed Answer Book.

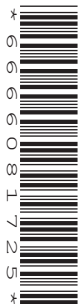
**OCR supplied materials:**

- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ ms}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 A particle P of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point O. The particle P is moving, with negligible air resistance, in a complete vertical circle with centre O. When P is at its highest point the speed of P is  $V$ . The horizontal line CD lies in the plane of the motion and passes through the lowest point of the circular path of P. Fig. 1 shows the particle at a point where OP makes an angle  $\theta$  with the upward vertical.

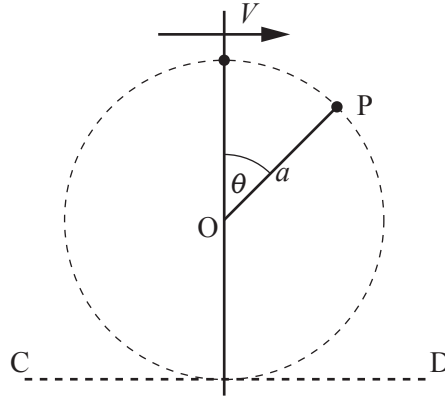


Fig. 1

- (i) Show that the least possible value of  $V$  is  $\sqrt{ag}$ . [2]

- (ii) Given that  $V = \sqrt{ag}$ , find an expression, in terms of  $m$ ,  $g$  and  $\theta$ , for the tension in the string when P is in the position shown in Fig. 1. [6]

Now consider the case  $V = \sqrt{3ag}$ .

- (iii) Find the vertical height of P above CD when the tension in the string is equal to twice its minimum value. [6]

Suppose now that  $V = \sqrt{kag}$ , where  $k$  is a positive constant.

The string breaks if the tension in it exceeds  $12mg$ .

- (iv) Find the set of values that  $k$  can take so that P is able to complete vertical circles. [3]

- 2 (a) A moving car experiences a force  $F$  due to air resistance. It is known that  $F$  depends on a product of powers of its velocity  $v$ , its cross-sectional area  $A$  and the air density  $\rho$ , and is given by

$$F = \frac{1}{2} C \rho^\alpha v^\beta A^\gamma,$$

where  $C$  is a dimensionless constant known as the drag coefficient.

- (i) Write down the dimensions of force and density. [2]

- (ii) Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ . [5]

(b)

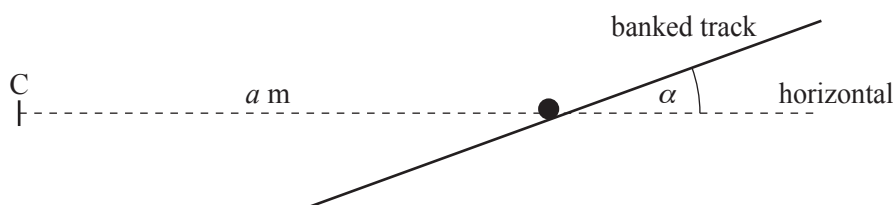


Fig. 2

A motorcyclist is riding his motorcycle around a circular banked track. The track is banked at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{1}{4}$ . The combined mass of the motorcycle and rider is  $M$  kg. The motion of the motorcycle and rider is modelled as a particle travelling at constant speed in a horizontal circle, with centre  $C$  and radius  $a$  m, on the banked track, as shown in Fig. 2.

- (i) Given that there is no tendency for the motorcyclist to slip up or down the slope when his speed is  $5\sqrt{g} \text{ ms}^{-1}$ , show that  $a = 100$ . [4]

Suppose now that the coefficient of friction between the motorcyclist and the track is  $\mu$ .

- (ii) Given that the maximum constant speed for which motion in the horizontal circle centre  $C$  is possible is  $28 \text{ ms}^{-1}$ , find the value of  $\mu$ . [7]

- 3 Fig. 3 shows a smooth plane inclined at an angle of  $30^\circ$  to the horizontal. A particle P of mass 3 kg lies on the plane. One end of a light elastic string, of natural length 2 m, is attached to P and the other end is fixed to a point A. One end of a second light elastic string, of natural length 1 m, is attached to P and the other end is fixed to a point B. Both strings are made from material with modulus of elasticity 12.25 N. APB is parallel to the plane on a line of greatest slope, and the distance AB is 6 m.

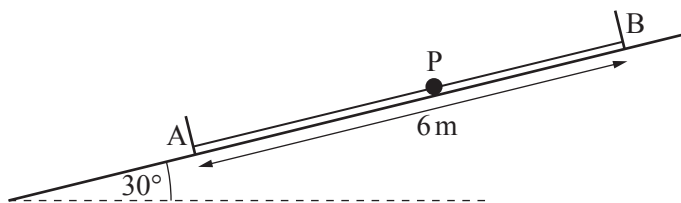


Fig. 3

The particle P moves along part of the line AB with both strings taut throughout the motion.

- (i) Show that, when the extension of the string AP is  $x$  m, the tension in the string BP is  $12.25(3-x)$  N. Show also that the value of  $x$  for which the system is in equilibrium is 1.2. [4]

The particle P is released from rest when  $AP = 3.35$  m. At time  $t$  s, the displacement of P from its equilibrium position is  $y$  m, measured in the direction AB.

- (ii) Show that the motion of P is simple harmonic with equation

$$\frac{d^2y}{dt^2} = -6.125y.$$

State the period of the motion.

[8]

The point C is on the line AB, between A and B, such that  $AC = 3.1$  m.

- (iii) Find the speed of P when it is at C. [2]
- (iv) Find the time elapsed after its release from rest until P is at C moving **up** the plane for the first time. [5]

- 4 Fig. 4.1 shows the shaded region  $R$  bounded by the curve  $y = 2x^{-\frac{1}{2}}$  for  $1 \leq x \leq 4$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 4$ .

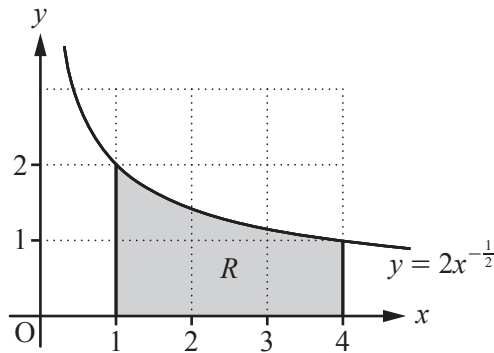


Fig. 4.1

- (i) Find the exact coordinates of the centre of mass of a uniform lamina occupying the region  $R$ . [6]

Fig. 4.2 shows the shaded region  $S$  bounded by the curve  $y = 2x^{-\frac{1}{2}}$  for  $1 \leq x \leq 4$ , the  $x$ -axis and the lines  $x = 4$  and  $y = 2x$ . The line  $y = 2x$  meets the curve  $y = 2x^{-\frac{1}{2}}$  at the point  $A$  with coordinates  $(1, 2)$ .

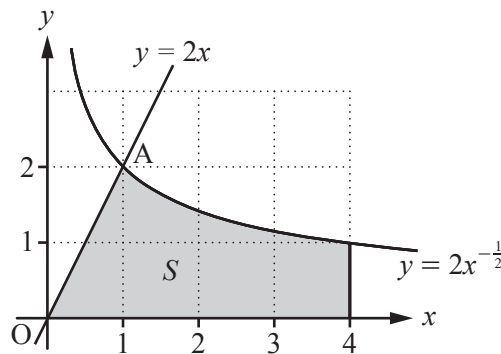


Fig. 4.2

The region  $S$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a uniform solid of revolution.

- (ii) Show that the  $x$ -coordinate of the centre of mass of this solid is  $\frac{39}{4(1+6\ln 2)}$ .

(You may assume the standard results for the volume and the position of the centre of mass of a uniform solid cone.) [8]

- (iii) The solid is suspended from a point on the circle described by  $A$  when  $S$  is rotated about the  $x$ -axis. Find the angle between  $AO$  and the vertical. [4]

END OF QUESTION PAPER

1.	(i)	<p>Tension (<math>T</math>) at highest point = 0</p> <p><math>(T+)mg = mV^2 / a : V^2 = ag</math></p>	<p>B1</p> <p>E1</p> <p>[2]</p>	<p>May be implied</p> <p><i>Without further explanation, <math>g = V^2 / a</math> is B0E0</i></p> <p><i>But <math>mg = mV^2 / a</math> can earn B1E1</i></p>	
	(ii)	<p><math>T + mg \cos \theta = \frac{mv^2}{a}</math></p> <p><math>\frac{1}{2}mv^2 - \frac{1}{2}mV^2 = mga(1 - \cos \theta)</math></p> <p><math>T = 3mg(1 - \cos \theta).</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>N2L at P <i>Allow sign errors, sin/cos interchanged</i></p> <p><i>Condone use of V for M1A1</i></p> <p>Energy equation [ <math>v^2 = ag(3 - 2\cos \theta)</math> ]</p> <p>Eliminate <math>v^2</math> and use <math>V = \sqrt{ag}</math> <i>Dependent on M1M1</i></p>	
	(iii)	<p><math>\frac{1}{2}mv^2 - \frac{1}{2}m(3ag) = mga(1 - \cos \theta)</math></p> <p>Minimum <math>T</math> at highest point: <math>T = 2mg</math></p> <p>Using <math>T = 2 \times 2mg</math> in <math>T + mg \cos \theta = \frac{mv^2}{a}</math></p> <p><math>4mg + mg \cos \theta = mg(5 - 2\cos \theta)</math></p> <p><math>\cos \theta = \frac{1}{3}</math></p> <p>Height = <math>a + a \cos \theta = \frac{4a}{3}</math></p>	<p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>F1</p> <p>[6]</p>	<p>[ <math>v^2 = ag(5 - 2\cos \theta)</math> ]</p> <p>Or <math>\frac{1}{2}mag(4 + \cos \theta) - \frac{1}{2}m(3ag) = mga(1 - \cos \theta)</math></p> <p><i>Dependent on M1M1; provided <math>\cos \theta \neq 0, \pm 1</math></i></p>	
	(iv)	<p>Maximum tension, at lowest point, is <math>12mg</math></p> <p><math>T - mg = \frac{mv^2}{a}</math> and <math>\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + 2mga</math></p> <p><math>V^2 \leq 7ag : k \leq 7</math></p> <p>So <math>1 \leq k \leq 7</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Seen or implied</p> <p>[ <math>v^2 = 11ag</math> , <math>v^2 = (k + 4)ag</math> ]</p>	

2.	(a)	<p>(i) Dimensions of force: <math>MLT^{-2}</math> Dimensions of density: <math>ML^{-3}</math></p> <p>(ii) <math>MLT^{-2} = (ML^{-3})^{\alpha} (LT^{-1})^{\beta} (L^2)^{\gamma}</math></p> <p>Compare powers for at least one dimension  <math>1 = \alpha</math>  <math>1 = -3\alpha + \beta + 2\gamma</math>  <math>-2 = -\beta</math>  <math>\alpha = 1, \beta = 2, \gamma = 1</math></p>	<p>B1 B1 [2]</p> <p>M1 A1 ft M1</p> <p>A1 cao</p> <p>A1 cao [5]</p>	<p>All parts present, dimensions of at least <math>v</math> or <math>A</math> correct</p> <p>At least two equations correct</p> <p>All correct</p>	
	(b)	<p>(i) Let <math>R</math> = normal reaction. For vertical equilibrium, <math>R \cos \alpha = Mg</math></p> <p>Horizontally, <math>R \sin \alpha = \frac{Mv^2}{a}</math> [ <math>= \frac{25Mg}{a}</math> ]</p> <p>Divide: <math>\tan \alpha = \frac{25g}{ag}</math></p> <p><math>a = 25 / \tan \alpha = 100</math> AG</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>E1 [4]</p>	<p>Or B2 for <math>mg \sin \alpha = m \frac{v^2}{a} \cos \alpha</math></p> <p>Or B2 for a correct triangle showing <math>mg</math>, <math>R</math> and <math>mv^2/a</math></p> <p>Dependent on B2</p>	
		<p>(ii) Let <math>F</math> = frictional force Vertically: <math>R \cos \alpha = Mg + F \sin \alpha</math></p> <p>Horizontally: <math>R \sin \alpha + F \cos \alpha = \frac{Mv^2}{a}</math></p> <p><math>F = \mu R</math></p> <p>Eliminate <math>R</math> and <math>F</math>: <math>\frac{v^2}{ag} = \frac{\mu + \tan \alpha}{1 - \mu \tan \alpha}</math></p> <p>Solving to obtain <math>\mu</math></p> <p><math>\mu = \frac{11}{24}</math> (= 0.458(3))</p>	<p>M1</p> <p>M1 A1 M1 M1</p> <p>M1</p> <p>A1 cao [7]</p>	<p>All terms present, allow <math>F</math> in wrong direction, sin/cos interchange</p> <p>All terms present, allow <math>F</math> in wrong direction, sin/cos interchange Both resolutions correct</p> <p>OR: solve for <math>F</math> and <math>R</math> This M1 is dependent on first two M1M1  [ <math>F = \frac{Mv^2}{a} \cos \alpha - Mg \sin \alpha</math> , <math>R = \frac{Mv^2}{a} \sin \alpha + Mg \cos \alpha</math> ]</p> <p>OR: use <math>\mu = \frac{F}{R} = \frac{7.84 - 9.8 \tan \alpha}{7.84 \tan \alpha + 9.8}</math> Dependent on M4</p>	

3.	(i)	<p>Length of BP = <math>4 - x</math>: extension <math>3 - x</math></p> $T_{BP} = \frac{12.25(3 - x)}{1} = 12.25(3 - x) \quad \text{AG}$ <p>In equilibrium: <math>T_{BP} = T_{AP} + 3g \sin 30^\circ</math></p> $12.25(3 - x) = \frac{12.25}{2}x + \frac{3g}{2}$ $x = 1.2 \quad \text{AG}$	<p>E1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p><b>[4]</b></p>	Solve (or evaluate terms)	
	(ii)	<p>Tension in BP = <math>12.25(1.8 - y)</math></p> <p>Tension in AP = <math>6.125(1.2 + y)</math></p> $T_{BP} - T_{AP} - mg \sin 30^\circ = m\ddot{y}$ $12.25(1.8 - y) - 6.125(1.2 + y) - \frac{3g}{2} = 3\ddot{y}$ $\ddot{y} = -6.125y$ $\text{Period} = \frac{2\pi}{\sqrt{6.125}} = 2.53879 \dots \text{ (s)}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 cao</p> <p>E1</p> <p>B1</p> <p><b>[8]</b></p>	<p>Hooke's law for one string</p> <p>Use N2L with at least 3 terms present</p> <p>Allow <math>-m\ddot{y}</math> or <math>ma</math> on RHS</p> <p>Allow <math>\frac{4\sqrt{2}}{7}\pi</math></p> <p><i>Working with <math>x</math> earns max M0M1A1A0E0B1 unless <math>x = 1.2 + y</math> is used at some stage</i></p>	
	(iii)	$v^2 = 6.125(0.15^2 - 0.1^2)$ $v^2 = 0.07656: \quad v = 0.276699 \dots$	<p>M1</p> <p>A1</p> <p><b>[2]</b></p>	<p>Using <math>v^2 = \omega^2(a^2 - y^2)</math> or <math>y = a \cos \omega t</math> and <math>v = -a\omega \sin \omega t</math></p> <p>Or energy equation with all terms present</p>	
	(iv)	$y = a \cos \omega t$ $-0.1 = 0.15 \cos \sqrt{6.125}t$ $t = 0.929(55)$ <p>When P is moving <i>up</i> the plane,</p> <p>time = period - <math>t</math></p> $1.60924 \dots \text{ (s)}$	<p>M1</p> <p>A1 ft</p> <p>A1</p> <p>M1</p> <p>A1 cao</p> <p><b>[5]</b></p>	<p>For <math>y = a \cos \omega t</math>, <math>y = a \sin \omega t</math>, <math>v = (-)a\omega \sin \omega t</math> or <math>v = a\omega \cos \omega t</math></p> <p>Allow <math>\pm 0.1</math> and sin or cos; o.e. for <math>v = \pm 0.277</math></p> <p>Any solution; e.g. 0.295, 0.340, 1.564, 1.61, 2.20 seen</p> <p>o.e. e.g. <math>3T/4 - 0.295</math>, <math>T/2 + 0.340</math>, <math>5T/4 - 1.564</math>, <math>3T/2 - 2.20</math></p>	

4.	(i)	$A\bar{x} = \int_1^4 x \cdot 2x^{-0.5} dx = \left[ \frac{4}{3} x^{1.5} \right]_1^4$ $= \frac{28}{3}$ $A = \int_1^4 2x^{-0.5} dx = \left[ 4x^{0.5} \right]_1^4 = 4$ $\bar{x} = \frac{7}{3}$ $A\bar{y} = \frac{1}{2} \int_1^4 (2x^{-0.5})^2 dx = \left[ 2 \ln x \right]_1^4 = 2 \ln 4$ $\bar{y} = 2 \ln 4 \div 4 = \ln 2$	M1 A1 B1 A1 M1 A1 [6]	For $\int x \left( 2x^{-\frac{1}{2}} \right) dx$     For $\int \frac{1}{2} \left( 2x^{-\frac{1}{2}} \right)^2 dx$ Accept $\frac{1}{2} \ln 4$	
	(ii)	<p>Consider solid formed by rotating R:</p> $V = \pi \int_1^4 y^2 dx = 8\pi \ln 2$ $V\bar{x} = \pi \int_1^4 x \cdot 4x^{-1} dx$ $= \pi [4x]_1^4$ $= 12\pi$ $\bar{x} = \frac{3}{2 \ln 2}$ <p>Consider solid formed by rotating line <math>y = 2x</math> as cone.</p> <p>Volume = <math>\frac{4\pi}{3}</math> : com <math>\frac{3}{4}</math> from y-axis</p>	B1  M1 M1 A1  B1	Ft result obtained in (i) $\times 2\pi$   For $\int x \left( 2x^{-\frac{1}{2}} \right)^2 dx$ For $[4x]$ For $12\pi$ Implied by correct $\bar{x}$	

			<table><tr><td></td><td><i>Cone</i></td><td><i>Solid from R</i></td><td><i>Whole</i></td></tr><tr><td>Volume</td><td><math>\frac{4}{3}\pi</math></td><td><math>8\pi \ln 2</math></td><td><math>\frac{4}{3}\pi + 8\pi \ln 2</math></td></tr><tr><td>Dist. of com from vertex</td><td><math>\frac{3}{4}</math></td><td><math>\frac{3}{2\ln 2}</math></td><td><math>\bar{X}</math></td></tr></table> $\bar{X}\left(\frac{4\pi}{3} + 8\pi \ln 2\right) = \frac{4}{3}\pi \times \frac{3}{4} + 8\pi \ln 2 \times \frac{3}{2\ln 2}$ $\bar{X} = \frac{39}{4(1 + 6\ln 2)} \quad \text{AG}$		<i>Cone</i>	<i>Solid from R</i>	<i>Whole</i>	Volume	$\frac{4}{3}\pi$	$8\pi \ln 2$	$\frac{4}{3}\pi + 8\pi \ln 2$	Dist. of com from vertex	$\frac{3}{4}$	$\frac{3}{2\ln 2}$	$\bar{X}$	M1 A1 ft  E1  [8]	Formula for com of composite body	
	<i>Cone</i>	<i>Solid from R</i>	<i>Whole</i>															
Volume	$\frac{4}{3}\pi$	$8\pi \ln 2$	$\frac{4}{3}\pi + 8\pi \ln 2$															
Dist. of com from vertex	$\frac{3}{4}$	$\frac{3}{2\ln 2}$	$\bar{X}$															
		OR	Consider as a single object $V = \pi \int_0^1 (2x)^2 \mathrm{d}x + \pi \int_1^4 (2x^{-\frac{1}{2}})^2 \mathrm{d}x = \frac{4}{3}\pi + 8\pi \ln 2$ $V\bar{X} = \pi \int_0^1 x(2x)^2 \mathrm{d}x + \pi \int_1^4 x(2x^{-\frac{1}{2}})^2 \mathrm{d}x = \pi + 12\pi$ $\bar{X} = \frac{13\pi}{\frac{4}{3}\pi + 8\pi \ln 2}$ $\bar{X} = \frac{39}{4(1 + 6\ln 2)} \quad \text{AG}$		B1 ft for $8\pi \ln 2$  M1M1A1 for $12\pi$ (as above) B1 for $\frac{4}{3}\pi$ and $\pi$  M1A1 ft  E1													
(iii)		Find angle between AG and line $x = 1$ $\tan \theta = \frac{\bar{X} - 1}{2}$ $\theta = 23.987724\dots^\circ$ Identify required angle as $\theta + \tan^{-1} \frac{1}{2}$ Angle between AO and vertical = $50.552775\dots^\circ$	M1  A1  M1  A1 cao [4]	Award M2A2 for other methods (e.g. triangle OAG)     Accept 0.882 rad														