OCR
Oxford Cambridge and RSA

Wednesday 25 May 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01   Mechanics 3

QUESTION PAPER

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

* The Question Paper will be found inside the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
* **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a scientific or graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.
* The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

* The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **72**.
* The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

* Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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1 **(a)** In an investigation, small spheres are dropped into a long column of a viscous liquid and their terminal speeds measured. It is thought that the terminal speed V of a sphere depends on a product of powers of its radius r, its weight mg and the viscosity η of the liquid, and is given by

$$V = kr^\alpha (mg)^\beta \eta^\gamma ,$$

where k is a dimensionless constant.

(i) Given that the dimensions of viscosity are $\text{ML}^{-1}\text{T}^{-1}$ find α, β and γ. **[6]**

A sphere of mass 0.03 grams and radius 0.2 cm has a terminal speed of $6\,\text{m s}^{-1}$ when falling through a liquid with viscosity η. A second sphere of radius 0.25 cm falling through the same liquid has a terminal speed of $8\,\text{m s}^{-1}$.

(ii) Find the mass of the second sphere. **[4]**

(b) A manufacturer is testing different types of light elastic ropes to be used in bungee jumping. You may assume that air resistance is negligible.

A bungee jumper of mass 80 kg is connected to a fixed point A by one of these elastic ropes. The natural length of this rope is 25 m and its modulus of elasticity is 1600 N. At one instant, the jumper is 30 m directly below A and he is moving vertically upwards at $15\,\text{m s}^{-1}$. He comes to instantaneous rest at a point B, with the rope slack.

(i) Find the distance AB. **[5]**

The same bungee jumper now tests a second rope, also of natural length 25 m. He falls from rest at A. It is found that he first comes instantaneously to rest at a distance 54 m directly below A.

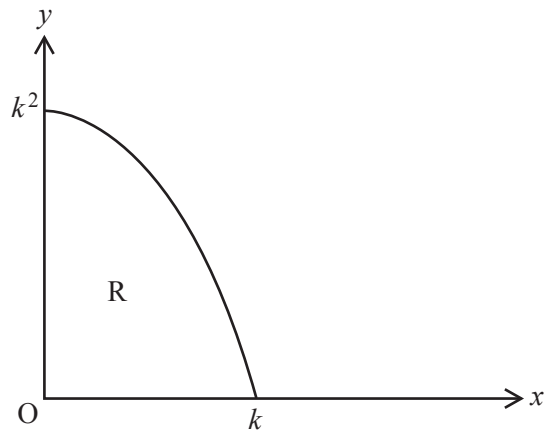**(ii)** Find the modulus of elasticity of this second rope. **[4]**

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Fig. 2.1

The region R shown in Fig. 2.1 is bounded by the curve $y = k^2 - x^2$, for $0 \leqslant x \leqslant k$, and the coordinate axes. The x-coordinate of the centre of mass of a uniform lamina occupying the region R is 0.75.

(i) Show that $k = 2$. **[4]**

A uniform solid S is formed by rotating the region R through 2π radians about the x-axis.

(ii) Show that the centre of mass of S is at (0.625, 0). **[7]**

Fig. 2.2 shows a solid T made by attaching the solid S to the base of a uniform solid circular cone C. The cone C is made of the same material as S and has height 8 cm and base radius 4 cm.
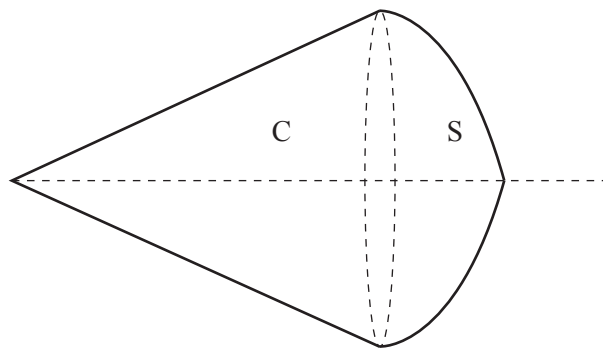

Fig. 2.2

(iii) Show that the centre of mass of T is at a distance of 6.75 cm from the vertex of the cone. [You may quote the standard results that the volume of a cone is $\frac{1}{3}\pi r^2 h$ and its centre of mass is $\frac{3}{4}h$ from its vertex.] **[4]**

(iv) The solid T is suspended from a point P on the circumference of the base of C.
Find the acute angle between the axis of symmetry of T and the vertical. **[3]**
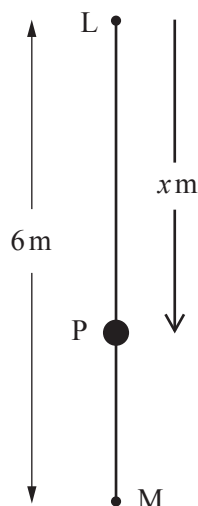
3

L •

x m

6 m

P ● ↓

• M

Fig. 3

One end of a light elastic string, of natural length 2.7 m and modulus of elasticity 54 N, is attached to a fixed point L. The other end of the string is attached to a particle P of mass 2.5 kg. One end of a second light elastic string, of natural length 1.7 m and modulus of elasticity 8.5 N, is attached to P. The other end of this second string is attached to a fixed point M, which is 6 m vertically below L. This situation is shown in Fig. 3.

The particle P is released from rest when it is 4.2 m below L. Both strings remain taut throughout the subsequent motion. At time *t* s after P is released from rest, its displacement below L is *x* m.

   (i) Show that $\dfrac{d^2x}{dt^2} = -10(x-4)$. **[7]**

   (ii) Write down the value of *x* when P is at the centre of its motion. **[1]**

   (iii) Find the amplitude and the period of the oscillations. **[4]**

   (iv) Find the velocity of P when *t* = 1.2. **[5]**

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Particle P is projected so that it moves in complete vertical circles with centre O; there is no air resistance. A and B are two points on the circle, situated on opposite sides of the vertical through O. The lines OA and OB make angles α and β with the upward vertical as shown in Fig. 4.
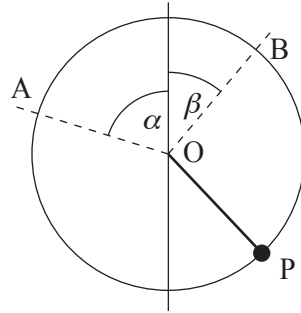

Fig. 4

The speed of P at A is $\sqrt{\dfrac{17ag}{3}}$. The speed of P at B is $\sqrt{5ag}$ and $\cos\beta = \dfrac{2}{3}$.

  (i) Show that $\cos\alpha = \dfrac{1}{3}$.   **[3]**

On one occasion, when P is at its lowest point and moving in a clockwise direction, it collides with a stationary particle Q. The two particles coalesce and the combined particle continues to move in the same vertical circle. When this combined particle reaches the point A, the string becomes slack.

  (ii) Show that when the string becomes slack, the speed of the combined particle is $\sqrt{\dfrac{ag}{3}}$.   **[2]**

The mass of the particle Q is km.

  (iii) Find the value of k.   **[9]**

  (iv) Find, in terms of m and g, the instantaneous change in the tension in the string as a result of the collision.   **[4]**

END OF QUESTION PAPER

1	(a)	(i)	Units of weight are MLT^{-2}	B1	
			$LT^{-1} = L^\alpha \left(MLT^{-2} \right)^\beta \left(ML^{-1}T^{-1} \right)^\gamma$	M1	
			Compare powers for at least one dimension	M1	
			$0 = \beta + \gamma$		
			$1 = \alpha + \beta - \gamma$	A1	One equation correct
			$-1 = -2\beta - \gamma$	A1	Another equation correct
			$\alpha = -1,\ \beta = 1,\ \gamma = -1$	A1	All correct
				[6]	
		(ii)	**EITHER:** $V = \dfrac{kmg}{r\eta}:\quad 600 = \dfrac{0.03kg}{0.2\eta}$	M1	Put $m = 0.(000)03$, $r = 0.(00)2$, $V = 6(00)$ into equation *Allow one error*
			$\dfrac{kg}{\eta} = 4000$	A1	FT correct substitution *Condone m/cm mix*
			Use $V = 800$ and $r = 0.25$	M1	Put $r = 0.(00)25$, $V = 8(00)$ into equation *Allow one error*
			Mass is 0.05 (grams) [0.000 05 kg]	A1	CAO *Correct answer implies full marks A0 for 0.05 kg*
				[4]	
			OR: $0.03 \times \dfrac{800}{600}$	M1	Use of $(8/6)^{1/\beta}$ or $(6/8)^{1/\beta}$
				A1	FT for $0.03 \times (8/6)^{1/\beta}$ or $0.03 \times (0.25/0.2)^{-\alpha/\beta}$
			$\times \dfrac{0.25}{0.2}$	M1	Use of $(0.25/0.2)^{-\alpha/\beta}$ or $(0.2/0.25)^{-\alpha/\beta}$
			Mass is 0.05	A1	CAO
				[4]	
	(b)	(i)	Loss in KE + loss in EPE = Gain in GPE	M1	Equation involving KE, EPE and GPE
			$KE = \dfrac{1}{2}.80.15^2 = 9000$ and $GPE = 80gH$	B1	
			$EPE = \dfrac{1}{2} \times \dfrac{1600}{25} \times (30-25)^2 = 800$	B1	
			$9000 + 800 = 80gH$	A1	
			$AB = 30 - H = 17.5\,\text{m}$	A1	
				[5]	

		(ii)	Loss in GPE $= 80g \times 54$ $(= 42336)$	B1	
			Gain in EPE $= \dfrac{1}{2} \times \dfrac{\lambda}{25} \times (54-25)^2$ $(= 16.82\lambda)$	B1	
				M1	Equate and solve to obtain a value of λ
			Modulus of elasticity is 2517 N (to 4 s.f.)	A1 **[4]**	
2	**(i)**		$A\bar{x} = \displaystyle\int_0^k x\left(k^2 - x^2\right)\mathrm{d}x = \left[\dfrac{1}{2}k^2 x^2 - \dfrac{1}{4}x^4\right]_0^k$	M1	For $\displaystyle\int_0^k x\left(k^2 - x^2\right)\mathrm{d}x$ [or $\displaystyle\int_0^{k^2} \tfrac{1}{2}\left(k^2 - y\right)\mathrm{d}y$]
			$= \dfrac{1}{4}k^4$	A1	
			$A = \displaystyle\int_0^k \left(k^2 - x^2\right)\mathrm{d}x = \left[k^2 x - \dfrac{1}{3}x^3\right]_0^k = \dfrac{2}{3}k^3$	B1	
			$\bar{x} = 0.375k$ and hence $k = 2$	E1 **[4]**	Correctly shown
	(ii)		$V = \pi \displaystyle\int_0^2 \left(4 - x^2\right)^2 \mathrm{d}x =$		
			$= \pi \displaystyle\int_0^2 (16 - 8x^2 + x^4)\mathrm{d}x = \pi\left[16x - \dfrac{8}{3}x^3 + \dfrac{1}{5}x^5\right]_0^2$	M1	Integrate expression to obtain volume
			$= \dfrac{256}{15}\pi$ $\left[\text{ or } \dfrac{8k^5}{15}\pi\right]$	A1	
			$V\bar{x} = \pi \displaystyle\int_0^2 x(16 - 8x^2 + x^4)\mathrm{d}x = \pi\left[8x^2 - 2x^4 + \dfrac{1}{6}x^6\right]_0^2$	M1	For $\displaystyle\int_0^k x(k^2 - x^2)^2\mathrm{d}x$
				M1	Integrate
			$= \dfrac{32}{3}\pi$ $\left[\text{ or } \dfrac{k^6}{6}\pi\right]$	A1	
			$\bar{x} = 0.625$	E1	Correctly shown
			$\bar{y} = 0$ by symmetry	E1 **[7]**	*Accept 'com is on the axis'*

			C	S	T		
(iii)		Volume	$\dfrac{128}{3}\pi$	$\dfrac{256}{15}\pi$	$\dfrac{896}{15}\pi$	B1	FT Volume of T is $\dfrac{1}{3}\pi \times 4^2 \times 8 + \dfrac{256}{15}\pi$
		Dist of com from vertex	6	$8 + 0.625 = \dfrac{69}{8}$	$\bar{x}$	B1	com of C and S relative to the same origin

$$\frac{896}{15}\pi\bar{x} = \frac{128}{3}\pi \times 6 + \frac{256}{15}\pi \times \frac{69}{8}$$

M1   Take moments

$$\bar{x} = \frac{27}{4} = 6.75$$

E1   Correctly shown

[4]

(iv)	Identify required angle	B1		
	$\tan\theta = \dfrac{4}{8 - 6.75}$	M1	Correct trigonometry in the relevant triangle	
	$\theta = 72.6°$ (to 3 s.f.)	A1	*If working unclear, give B3 for 72.6 or B1 for 17.4*	
		[3]		

3	**(i)**	Tension in LP = $\dfrac{54}{2.7}(x - 2.7)$ [$= 20(x - 2.7)$]	M1 A1	Use Hooke's law
		Tension in MP = $\dfrac{8.5}{1.7}(6 - x - 1.7)$ [$= 5(4.3 - x)$]	A1	
			M1	N2L with at least 3 terms
			A1	For $mg + T_{\text{MP}} - T_{\text{LP}} = (\pm)ma$
		$2.5\ddot{x} = 5(4.3 - x) + 2.5g - 20(x - 2.7)$	A1	*Must have $\ddot{x}$ here or in final answer*
		$\qquad = 100 - 25x$		
		$\ddot{x} = 40 - 10x = -10(x - 4)$	E1	Correctly shown
			[7]	

	(ii)	$x = 4$	B1	
			[1]	

	(iii)		M1	Amplitude is 4.2 – answer to (ii)
		Amplitude is $4.2 - 4 = 0.2$ m	A1	CAO
		$\omega = \sqrt{10}$	B1	
		Period is $\dfrac{2\pi}{\omega} = 1.99$ s (to 3.s.f.)	B1	CAO *Accept* $\dfrac{2\pi}{\sqrt{10}}$ *o.e.*
			[4]	

	(iv)		**EITHER:** $x = 4 + 0.2\cos\omega t$	B1	FT For $0.2\cos\sqrt{10}\,t$
			$v = -0.2\omega\sin\omega t$	M1	Differentiate
				M1	Substituting $t = 1.2$
			When $t = 1.2$, $v = 0.3844...$, speed is 0.384 m s^{-1}	A1**	CAO
			Moving downwards (since $v > 0$)	A1	*Dep A1***
				[5]	
			OR: $x = 4 + 0.2\cos\omega t$ [or $y = 0.2\cos\omega t$]	B1	FT For $0.2\cos\sqrt{10}\,t$
			When $t = 1.2$, $x = 3.84(12)$	M1	Substituting $t = 1.2$
			Use $v^2 = \omega^2\left(a^2 - y^2\right)$: $v^2 = 10(0.2^2 - \left(4 - 3.84\right)^2)$	M1	*M0 if 3.84 used for y in formula*
			$v^2 = 0.148$ Speed is 0.384 m s^{-1}	A1**	CAO
			Moving downwards	A1	*Dep A1** Must be justified, by e.g. 1/2 period $< 1.2 <$ period*
				[5]	
4	**(i)**			M1	Energy equation A to B
			$\dfrac{1}{2}m\left(\dfrac{17ag}{3} - 5ag\right) = mga\left(\cos\beta - \cos\alpha\right)$	A1	
			$\cos\alpha = \dfrac{1}{3}$	E1	Correctly shown
				[3]	
	(ii)		V is speed of combined particle after collision; u is speed when string goes slack; v is speed of P before collision , T is the tension in the string		
			When string goes slack at A: Use $T = 0$	M1	*May be implied*
			$Mg\cos\alpha = \dfrac{Mu^2}{a}$ and so $u = \sqrt{\dfrac{ag}{3}}$	E1	Correctly shown (N2L at A) *Condone use of m or km for M ($= (k+1)m$)*
				[2]	

(iii)	$\dfrac{1}{2}m\left(v^2 - 5ag\right) = mg\dfrac{5a}{3}$	M1*	Energy equation B (or A) to lowest point
	$v^2 = \dfrac{25ag}{3}$	A1	
		M1*	Energy equation: lowest point to when string goes slack at A
	$\dfrac{1}{2}M\left(V^2 - u^2\right) = Mg\dfrac{4a}{3}$	A1	*Condone use of m or km for M ($= (k+1)m$)*
	$V^2 = \dfrac{8ag}{3} + \dfrac{ag}{3} = 3ag$	A1	*The first 5 marks can be earned in (iv)* *SC B2 for $V^2 = 3ag$ seen (or clearly implied) after M0*
		M1*	PCLM at collision
	$(k+1)mV = mv$	A1	
	$(k+1)\sqrt{3ag} = \sqrt{\dfrac{25ag}{3}}$	M1	*Dep M3* Substitute into momentum equation*
	$(k+1)^2 = \dfrac{25}{9}$		
	$k = \dfrac{2}{3}$	A1	Must be positive
		[9]	
(iv)	$T_1 - mg = \dfrac{mv^2}{a}$	M1	N2L at lowest point before collision
	$T_1 = \dfrac{28}{3}mg$	A1	
	$T_2 - (k+1)mg = \dfrac{m(k+1)V^2}{a}$	M1	N2L at lowest point immediately after collision *Must use $(k+1)m$ in both terms*
	$T_2 = \dfrac{20}{3}mg$		
	Change in tension is $\dfrac{8}{3}mg$	A1	CAO
		[4]	

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General Comments:

Most candidates were able to demonstrate a good working knowledge of the topics being examined. The first three questions (on dimensional analysis, elasticity, centres of mass and simple harmonic motion) were well answered; but the last two items on the paper, Q.4(iii) and Q.4(iv), were found to be considerably more challenging.

Comments on Individual Questions:

Q.1(a)(i) The method for finding powers in a formula was very well understood; although some candidates started with the wrong dimensions for velocity or weight. There were a few slips in solving the equations.

Q.1(a)(ii) Most candidates used the first set of values to obtain an expression for k, then used this with the new values of V and r to calculate the mass. This was very often carried out accurately; but careless errors such as forgetting to change one of the variables, or omitting g from the final equation, were fairly common. The answer was sometimes given as 0.05 kg instead of 0.05 grams. The more efficient approach, noting that m is proportional to Vr, was adopted by some candidates.

Q.1(b)(i) This application of the conservation of energy was well answered, although many candidates gave the distance travelled (12.5 m) instead of the required AB (17.5 m).

Q.1(b)(ii) This was also answered well, using conservation of energy. Just a few candidates assumed that the jumper was in equilibrium at the lowest point.

Q.2(i)-(ii) The use of integration to find centres of mass was well understood, and most candidates obtained the given results correctly.

Q.2(iii) Most candidates knew how to find the centre of mass of the composite body. A very common error was to take the distance of the centre of mass of S from the vertex to be 0.625 cm instead of 8.625 cm.

Q.2(iv) Most candidates realised that the centre of mass was vertically below the point of suspension, although very many found the complementary angle (17.4° instead of 72.6°).

Q.3(i) Candidates were expected to show explicit expressions for the tension in each string and to form the equation of motion. This was quite well done and the given result was very often obtained convincingly.

Q.3(ii)-(iii) These parts were usually answered correctly.

Q.3(iv) Most candidates formed a displacement-time equation. Many used x to represent a different quantity in this part (such as the displacement upwards from the centre of motion); this in itself was not penalised, although it is of course not a practice to be recommended. A fairly common error was to put ω equal to the period (even when $\omega = \sqrt{10}$ had been used correctly in the previous part). Those who differentiated their equation to obtain the velocity were usually able to find the speed and direction of motion correctly. Those who calculated the displacement first and then used $v^2 = \omega^2(A^2 - y^2)$ often failed to determine the direction of motion.

Q.4(i) Most candidates used the conservation of energy correctly in this part.

Q.4(ii) Most candidates formed a radial equation of motion, with zero tension in the string, and obtained the given result correctly.

Q.4(iii) There were three stages to consider: using conservation of energy to find the speed immediately before the collision; using the result from part (ii) and conservation of energy to find the speed immediately after the collision; using conservation of linear momentum in the collision. Unfortunately this was rarely seen. A very common strategy was to use conservation of energy from the single particle at A (or B) to the combined particle at A, in effect assuming that energy was conserved in the collision. Most candidates scored 2 marks or fewer (out of 9) in this part.

Q.4(iv) This was omitted altogether by about one fifth of the candidates, and most scored 2 marks or fewer (out of 4). The tension in the string immediately before the collision was quite often found correctly, but previous errors usually prevented a successful conclusion.