OCR
Oxford Cambridge and RSA

Wednesday 20 May 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Give the dimensions of force, work and power. **[3]**

The force due to air resistance acting on a car is given by λv^2, where v is the speed and λ is a constant for that car.

(ii) Find the dimensions of λ. **[2]**

The power P of the car and its maximum speed U are related by the equation $P = \lambda U^3$.

(iii) Show that this equation is dimensionally consistent. **[2]**

The time t taken for the car to accelerate from speed $\frac{1}{3}U$ to speed $\frac{2}{3}U$ is given by $t = km^{\alpha}P^{\beta}\lambda^{\gamma}$, where m is the mass of the car and k is a dimensionless constant.

(iv) Find α, β and γ. **[4]**

Car C has mass 800 kg, power 35 kW, maximum speed 45 m s^{-1}, and takes 9.18 s to accelerate from 15 m s^{-1} to 30 m s^{-1}.

(v) Find the value of λ for Car C

(A) in SI units (based on kilograms, metres and seconds),

(B) in a system of units based on pounds, miles and hours, given that

1 pound = 0.454 kg, 1 mile = 1609 m, 1 hour = 3600 s. **[3]**

(vi) Car D has mass 1250 kg, power 75 kW and maximum speed 54 m s^{-1}. Find the time taken for Car D to accelerate from 18 m s^{-1} to 36 m s^{-1}. **[4]**

2 **(a)** A particle P of mass m is attached to a fixed point O by a light inextensible string of length a. P is moving without resistance in a complete vertical circle with centre O and radius a. When P is at the highest point of the circle, the tension in the string is T_1. When OP makes an angle θ with the upward vertical, the tension in the string is T_2. Show that

$$T_2 = T_1 + 3mg(1 - \cos\theta).$$ **[6]**

(b) The fixed point A is 1.2 m vertically above the fixed point C. A particle Q of mass 0.9 kg is joined to A, to C, and to a particle R of mass 1.5 kg, by three light inextensible strings of lengths 1.3 m, 0.5 m and 1.8 m respectively. The particle Q moves in a horizontal circle with centre C, and R moves in a horizontal circle at the same constant angular speed as Q, in such a way that A, C, Q and R are always coplanar. The string QR makes an angle of 60° with the downward vertical. This situation is shown in Fig. 2.

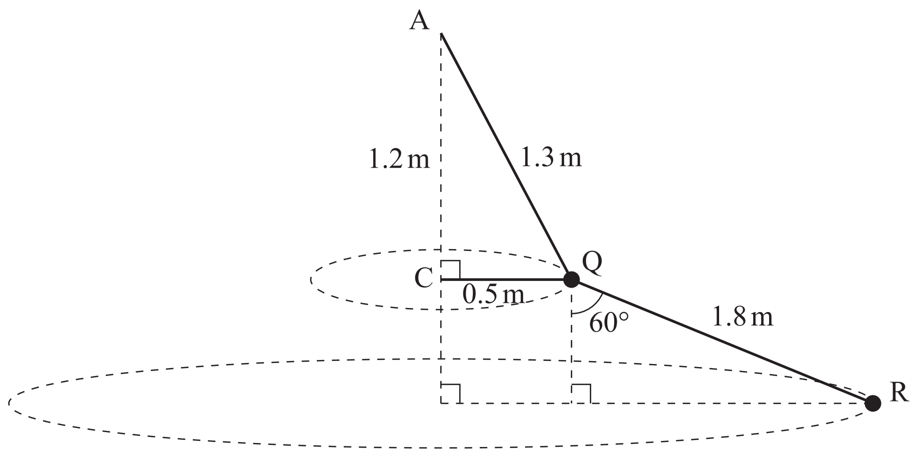

Fig. 2

(i) Find the tensions in the strings QR and AQ. **[5]**

(ii) Find the angular speed of the system. **[3]**

(iii) Find the tension in the string CQ. **[4]**

Question 3 begins on page 4.

3 Fig. 3 shows the fixed points A and F which are 9.5 m apart on a smooth horizontal surface and points B and D on the line AF such that AB = DF = 3.0 m. A small block of mass 10.5 kg is joined to A by a light elastic string of natural length 3.0 m and stiffness 12 N m^{-1}; the block is joined to F by a light elastic string of natural length 3.0 m and stiffness 30 N m^{-1}. The block is released from rest at B and then slides along part of the line AF. The block has zero acceleration when it is at a point C, and it comes to instantaneous rest at a point E.

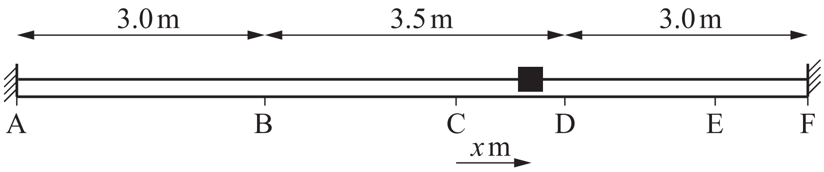

Fig. 3

(i) Find the distance BC. **[3]**

At time t s the displacement of the block from C is x m, measured in the direction AF.

(ii) Show that, when the block is between B and D, $\frac{d^2x}{dt^2} = -4x$. **[4]**

(iii) Find the maximum speed of the block. **[2]**

(iv) Find the distance of the block from C when its speed is 4.8 m s^{-1}. **[2]**

(v) Find the time taken for the block to travel from B to D. **[4]**

(vi) Find the distance DE. **[3]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis and the curve $y = \frac{x^2(a-x)}{a^2}$ for $0 \leqslant x \leqslant a$. Find the coordinates of the centre of mass of this lamina. **[9]**

(b) The region A is bounded by the x-axis, the y-axis, the curve $y = \sqrt{x^2 + 16}$ and the line $x = 3$. The region B is bounded by the y-axis, the curve $y = \sqrt{x^2 + 16}$ and the line $y = 5$. These regions are shown in Fig. 4.

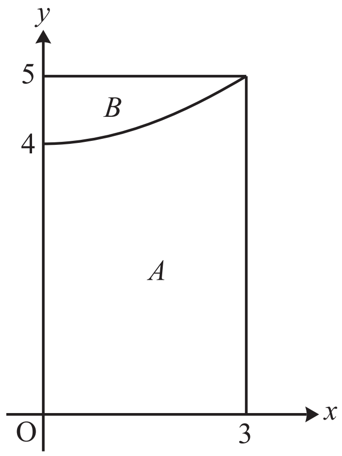

Fig. 4

(i) Find the x-coordinate of the centre of mass of the uniform solid of revolution formed when the region A is rotated through 2π radians about the x-axis. **[5]**

(ii) Using your answer to part **(i)**, or otherwise, find the x-coordinate of the centre of mass of the uniform solid of revolution formed when the region B is rotated through 2π radians about the x-axis. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$[\text{ Force }] = \text{M L T}^{-2}$	B1		
			$[\text{ Work }] = \text{M L}^2\,\text{T}^{-2}$	B1		
			$[\text{ Power }] = \text{M L}^2\,\text{T}^{-3}$	B1		
				[3]		*Deduct one mark if kg, m, s used consistently for M, L, T*
1	(ii)		$[\lambda] = \left[\dfrac{F}{v^2}\right] = \dfrac{\text{M L T}^{-2}}{(\text{L T}^{-1})^2}$	M1	Obtaining dimensions of λ	M0 if $P = \lambda U^3$ used
			$= \text{M L}^{-1}$	A1	FT $[\text{ Force }] \times \text{L}^{-2}\,\text{T}^2$	B2 (BOD) for correct answer with no working
				[2]		
1	(iii)		$[\lambda U^3] = (\text{M L}^{-1})(\text{L T}^{-1})^3 = \text{M L}^2\,\text{T}^{-3}$	M1	Obtaining dimensions of λU^3	Must be simplified
			Same as power, so dimensionally consistent	E1	Correctly shown	
				[2]		
1	(iv)		$\text{T} = \text{M}^{\alpha}(\text{M L}^2\,\text{T}^{-3})^{\beta}(\text{M L}^{-1})^{\gamma}$			
			$\beta = -\frac{1}{3}$	B1		
			$\alpha + \beta + \gamma = 0, \quad 2\beta - \gamma = 0$	M1	One equation correct (FT)	Equation from powers of M or L
			$\alpha = 1, \quad \gamma = -\frac{2}{3}$	A1A1	CAO	If A0 give SC1 for non-zero values with $\gamma = 2\beta$ OR $\alpha + \beta + \gamma = 0$
				[4]		*(SC1 will usually imply M1)*
1	(v)	(*A*)	$\lambda = \dfrac{P}{U^3} = \dfrac{35000}{45^3} = 0.384 \ (\text{kg m}^{-1})$ (3 sf)	B1		
		(*B*)	$0.3841 \times \dfrac{1}{0.454} \times 1609$	M1	For conversion factor $\dfrac{1609}{0.454}$	OR changing *both* $P\ (= 1.389 \times 10^9)$ *and* $U\ (= 100.7)$ to new units
			$= 1360 \ (\text{lb mi}^{-1})$ (3 sf)	A1	CAO	
				[3]		

Question			Answer	Marks	Guidance	
1	(vi)		$\lambda_D = \dfrac{75000}{54^3} \quad (=0.476)$	M1		
			$t_D = 9.18 \times \dfrac{1250}{800} \times \left(\dfrac{75}{35}\right)^{-\frac{1}{3}} \times \left(\dfrac{0.4763}{0.3841}\right)^{-\frac{2}{3}}$	M1 M1	Two ratios and powers correct Correct expression for new time	Or equation for k $(k=0.1983)$
		OR	$t = kmP^{-\frac{1}{3}}(PU^{-3})^{-\frac{2}{3}} = kmP^{-1}U^2$ $t_D = 9.18 \times \dfrac{1250}{800} \times \left(\dfrac{75}{35}\right)^{-1} \times \left(\dfrac{54}{45}\right)^2$		M1 Expressing t in terms of m, P, U M1 Two ratios and powers correct M1 Correct expression for new time	Or equation for k $(k=0.1983)$
			Time is 9.639 s	A1 **[4]**	CAO	
2	(a)		$T_1 + mg = m\dfrac{v_1^2}{a}$	B1		*Condone r for a*
			$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mga(1-\cos\theta)$	M1 A1	Equation involving KE and PE	
			$T_2 + mg\cos\theta = m\dfrac{v_2^2}{a}$	B1		*Condone same v for B1B1*
			$T_2 - T_1 = \dfrac{mv_2^2}{a} - \dfrac{mv_1^2}{a} + mg(1-\cos\theta)$ $T_2 - T_1 = 2mg(1-\cos\theta) + mg(1-\cos\theta)$	M1	Eliminating v_1 and v_2	*Dependent on previous M1*
			$T_2 = T_1 + 3mg(1-\cos\theta)$	E1 **[6]**		

Question			Answer	Marks	Guidance	
2	(b)	(i)	$T_{QR}\cos 60° = 1.5 \times 9.8$	M1	Resolving vertically for R	
			Tension in QR is 29.4 N	A1		
			$T_{AQ}\cos\theta = 0.9 \times 9.8 + T_{QR}\cos 60°$	M1	Resolving vertically for Q	θ is the angle CAQ
			$\frac{12}{13}T_{AQ} = 8.82 + 14.7$	A1	FT e.g.	$T\cos 22.6 = 0.9 \times 9.8 + 29.4\cos 60°$
			Tension in AQ is 25.48 N	A1	CAO	
				[5]		
2	(b)	(ii)		M1	Using $1.5r\omega^2$	Allow $1.5\frac{v^2}{r}$ provided $v = r\omega$ also seen
			$T_{QR}\sin 60° = 1.5(0.5 + 1.8\sin 60°)\omega^2$	A1		
			Angular speed is 2.87 rad s^{-1} (3 sf)	A1	FT is $\sqrt{0.2804\,T_{QR}}$	
				[3]		
2	(b)	(iii)	$T_{CQ} + T_{AQ}\sin\theta - T_{QR}\sin 60° = (0.9)(0.5)\omega^2$	B1B1	For LHS and RHS (CAO)	
			$T_{CQ} + 25.48 \times \frac{5}{13} - 29.4\sin 60° = 0.45 \times 2.871^2$	M1	Numerical equation for T_{CQ}	At most one error (FT including in B1's) and no missing terms
			Tension in CQ is 19.4 N (3 sf)	A1	CAO	
				[4]		
3	(i)			M1	Using stiffness × extension to find tension in *both* strings	Must use extension. Condone use of modulus for full marks
			If BC = y, $12y = 30(3.5 - y)$	A1	Correct equation for a distance	
			Distance BC is 2.5 m	A1		
				[3]		

Question			Answer	Marks	Guidance	
3	(ii)			B1	For $30(1.0-x)$ *or* $12(2.5+x)$	FT if BC is wrong
				M1	Equation of motion	Two forces in terms of x, and acc'n
			$30(1.0-x)-12(2.5+x)=10.5\dfrac{d^2x}{dt^2}$	A1	Allow (±) $10.5a$ on RHS	FT if BC is wrong
			$\dfrac{d^2x}{dt^2}=-4x$	E1		*When necessary, replacing a by* $-\dfrac{d^2x}{dt^2}$ *requires some explanation*
				[4]		
3	(iii)		Maximum speed is $A\omega$	M1	OR Using energy to obtain an equation for v at C	$\frac{1}{2}(30)(3.5^2)=\frac{1}{2}(30)(1.0^2)+\frac{1}{2}(12)(2.5^2)+\frac{1}{2}(10.5)v^2$
			Maximum speed is $5\ \mathrm{m\,s^{-1}}$	A1	FT is $2\times$ BC	
				[2]		
3	(iv)		$4.8^2=4(2.5^2-x^2)$	M1	Using $v^2=\omega^2(A^2-x^2)$	OR Using energy to obtain an equation for a distance
			Distance from C is 0.7 m	A1	CAO	Condone ± 0.7
				[2]		
3	(v)		$x=-2.5\cos 2t$	B1	$\pm 2.5\sin 2t$ *or* $\pm 2.5\cos 2t$	FT if BC is wrong (or A or ω)
				M1	Using $x=1.0$ to obtain a time	
			$1.0=-2.5\cos 2t$	M1	Fully correct strategy for finding the required time	e.g. $0.2058+\dfrac{1}{4}\times\dfrac{2\pi}{2}$
			Time is 0.991 s (3 sf)	A1	CAO	
				[4]		

Question			Answer	Marks	Guidance	
3	(vi)			M1	Using change of elastic energy to obtain an equation for a distance	OR Fully correct strategy using two stages of SHM
			$\frac{1}{2}(30)(3.5^2)=\frac{1}{2}(12)(\mathrm{BE}^2)$	A1		Correct (FT) equation for a distance
			BE = 5.534 Distance DE is 2.03 m (3 sf)	A1 **[3]**	CAO	
4	(a)		Area is $\int_0^a \frac{x^2(a-x)}{a^2}\,\mathrm{d}x$	M1		
			$=\left[\frac{x^3}{3a}-\frac{x^4}{4a^2}\right]_0^a \quad (=\frac{a^2}{12})$	A1		
			$\int xy\,\mathrm{d}x$	M1		
			$=\int_0^a \frac{x^3(a-x)}{a^2}\,\mathrm{d}x=\left[\frac{x^4}{4a}-\frac{x^5}{5a^2}\right]_0^a \quad (=\frac{a^3}{20})$	A1		
			$\overline{x}=\frac{\frac{1}{20}a^3}{\frac{1}{12}a^2}=\frac{3a}{5}$	A1		
			$\int\frac{1}{2}y^2\,\mathrm{d}x=\int_0^a \frac{x^4(a-x)^2}{2a^4}\,\mathrm{d}x$	M1	For $\int\ldots y^2\,\mathrm{d}x$	
			$=\left[\frac{x^5}{10a^2}-\frac{x^6}{6a^3}+\frac{x^7}{14a^4}\right]_0^a \quad (=\frac{a^3}{210})$	A2	Give A1 if just one error (e.g. omission of factor ½)	
			$\overline{y}=\frac{\frac{1}{210}a^3}{\frac{1}{12}a^2}=\frac{2a}{35} \quad (\approx 0.0571a)$	A1 **[9]**		

Question			Answer	Marks	Guidance	
4	**(b)**	**(i)**	Volume is $\int_0^3 \pi(x^2+16)\,\mathrm{d}x$	M1	π may be omitted throughout	*Condone consistent use of* $2\pi y^2$ *etc*
			$=\pi\left[\frac{x^3}{3}+16x\right]_0^3 \quad (=57\pi)$	A1		
			$\int \pi x y^2\,\mathrm{d}x$	M1		
			$=\int_0^3 \pi x(x^2+16)\,\mathrm{d}x \quad =\pi\left[\frac{x^4}{4}+8x^2\right]_0^3 \quad \left(=\frac{369}{4}\pi\right)$	A1		
			$\bar{x}=\frac{\frac{369}{4}\pi}{57\pi}=\frac{123}{76} \quad (\approx 1.62)$	A1		
				[5]		
4	**(b)**	**(ii)**	Volume of A and B combined is $\pi\times 5^2\times 3=75\pi$			
				M1	CM of composite body	
			$(18\pi)\bar{x}_B+(57\pi)\left(\frac{123}{76}\right)=(75\pi)(1.5)$	A2	Give A1 if just one error	FT values from (i)
		OR	$\int_0^3 \pi x\left(25-(x^2+16)\right)\mathrm{d}x$		M1	
			$=\frac{81}{4}\pi$		A1	
			$(18\pi)\bar{x}_B=\frac{81}{4}\pi$		A1 FT	
			$\bar{x}_B=\frac{9}{8} \quad (=1.125)$	A1	CAO	
				[4]		