OCR

Thursday 22 May 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** The speed v of sound in a solid material is given by $v = \sqrt{\frac{E}{\rho}}$, where E is Young's modulus for the material and ρ is its density.

(i) Find the dimensions of Young's modulus. **[3]**

The density of steel is $7800\,\text{kg}\,\text{m}^{-3}$ and the speed of sound in steel is $6100\,\text{m}\,\text{s}^{-1}$.

(ii) Find Young's modulus for steel, stating the units in which your answer is measured. **[2]**

A tuning fork has cylindrical prongs of radius r and length l. The frequency f at which the tuning fork vibrates is given by $f = kc^{\alpha}E^{\beta}\rho^{\gamma}$, where $c = \frac{l^2}{r}$ and k is a dimensionless constant.

(iii) Find α, β and γ. **[4]**

(b) A particle P is performing simple harmonic motion along a straight line, and the centre of the oscillations is O. The points X and Y on the line are on the same side of O, at distances 3.9 m and 6.0 m from O respectively. The speed of P is $1.04\,\text{m}\,\text{s}^{-1}$ when it passes through X and $0.5\,\text{m}\,\text{s}^{-1}$ when it passes through Y.

(i) Find the amplitude and the period of the oscillations. **[5]**

(ii) Find the time taken for P to travel directly from X to Y. **[4]**

2 **(a)** The fixed point A is vertically above the fixed point B. A light inextensible string of length 5.4 m has one end attached to A and the other end attached to B. The string passes through a small smooth ring R of mass 0.24 kg, and R is moving at constant angular speed in a horizontal circle. The circle has radius 1.6 m, and AR = 3.4 m, RB = 2.0 m, as shown in Fig. 2.

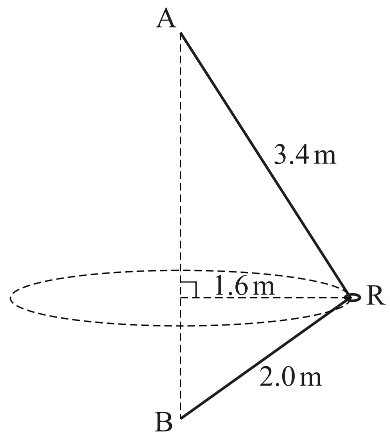


Fig. 2

(i) Find the tension in the string. **[3]**

(ii) Find the angular speed of R. **[3]**

(b) A particle P of mass 0.3 kg is joined to a fixed point O by a light inextensible string of length 1.8 m. The particle P moves without resistance in part of a vertical circle with centre O and radius 1.8 m. When OP makes an angle of 25° with the downward vertical, the tension in the string is 15 N.

(i) Find the speed of P when OP makes an angle of 25° with the downward vertical. **[3]**

(ii) Find the tension in the string when OP makes an angle of 60° with the upward vertical. **[5]**

(iii) Find the speed of P at the instant when the string becomes slack. **[5]**

3 The fixed points A and B lie on a line of greatest slope of a smooth inclined plane, with B higher than A. The horizontal distance from A to B is 2.4 m and the vertical distance is 0.7 m. The fixed point C is 2.5 m vertically above B. A light elastic string of natural length 2.2 m has one end attached to C and the other end attached to a small block of mass 9 kg which is in contact with the plane. The block is in equilibrium when it is at A, as shown in Fig. 3.

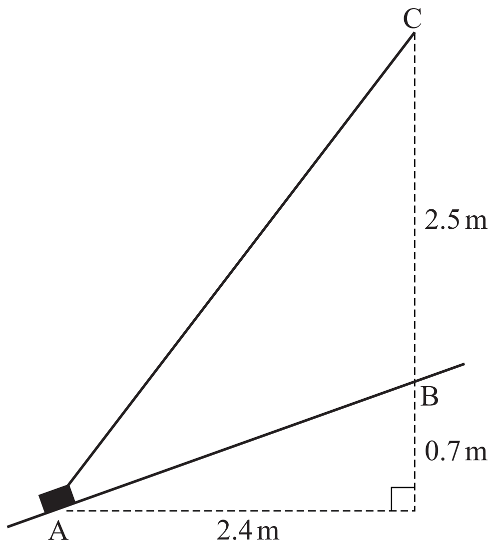


Fig. 3

(i) Show that the modulus of elasticity of the string is 37.73 N. **[5]**

The block starts at A and is at rest. A constant force of 18 N, acting in the direction AB, is then applied to the block so that it slides along the line AB.

(ii) Find the magnitude and direction of the acceleration of the block

(A) when it leaves the point A,

(B) when it reaches the point B. **[6]**

(iii) Find the speed of the block when it reaches the point B. **[6]**

4 The region R is bounded by the x-axis, the y-axis, the curve $y = e^{-x}$ and the line $x = k$, where k is a positive constant.

(i) The region R is rotated through 2π radians about the x-axis to form a uniform solid of revolution. Find the x-coordinate of the centre of mass of this solid, and show that it can be written in the form

$$\frac{1}{2} - \frac{k}{e^{2k} - 1}.$$ **[7]**

(ii) The solid in part **(i)** is placed with its larger circular face in contact with a rough plane inclined at 60° to the horizontal, as shown in Fig. 4, and you are given that no slipping occurs.

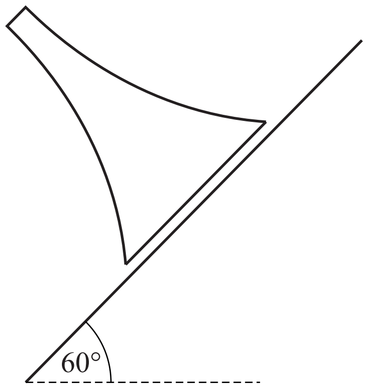


Fig. 4

Show that, whatever the value of k, the solid will not topple. **[4]**

(iii) A uniform lamina occupies the region R. Find, in terms of k, the coordinates of the centre of mass of this lamina. **[7]**

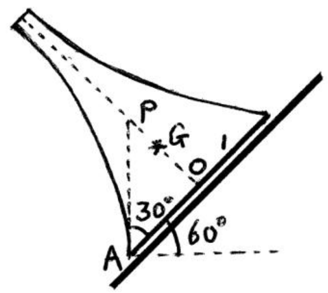
END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(a)	(i)	$[\rho] = \mathrm{ML}^{-3}$	B1		
			$[E] = [\rho v^2] = (\mathrm{ML}^{-3})(\mathrm{LT}^{-1})^2$	M1	Obtaining dimensions of *E*	
			Dimensions of Young's modulus are $\mathrm{ML}^{-1}\mathrm{T}^{-2}$	A1		
				[3]		
1	(a)	(ii)	$E = \rho v^2 = 7800 \times 6100^2 = 2.90 \times 10^{11}$ (3 sf)	B1		
			Units are $\mathrm{kg\,m^{-1}\,s^{-2}}$	B1	OR $\mathrm{N\,m^{-2}}$ OR Pa	FT provided all powers are non-zero *No FT if derived units involved*
				[2]		
1	(a)	(iii)	$\mathrm{T}^{-1} = \mathrm{L}^{\alpha}(\mathrm{ML}^{-1}\mathrm{T}^{-2})^{\beta}(\mathrm{ML}^{-3})^{\gamma}$			
			$\beta = \frac{1}{2}$	B1	CAO	
			$\gamma = -\frac{1}{2}$	B1	FT $\gamma = -\beta$	*Provided non-zero*
			$\alpha - \beta - 3\gamma = 0$	M1	Equation from powers of L	
			$\alpha = -1$	A1	CAO	
				[4]		
1	(b)	(i)	$1.04^2 = \omega^2(A^2 - 3.9^2)$	M1	Using $v^2 = \omega^2(A^2 - x^2)$	
			$0.5^2 = \omega^2(A^2 - 6.0^2)$	A1	Both equations correct	
			$\dfrac{A^2 - 15.21}{A^2 - 36} = 4.3264$	M1	Eliminating ω or *A*	$A = 6.5,\quad \omega = 0.2$
			Amplitude (*A*) is 6.5 m	A1		
			Period ($\dfrac{2\pi}{\omega}$) is $10\pi = 31.4$ s (3 sf)	A1		
				[5]		
1	(b)	(ii)	$x = 6.5\sin 0.2t$	B1	FT For $6.5\sin 0.2t$ or $6.5\cos 0.2t$	OR $(v =) 1.3\sin 0.2t$ etc
			When $x = 3.9,\ t = 3.2175$	M1	Using $x = 3.9$ or $x = 6.0$ to find a time	OR using $v = 1.04$ or $v = 0.5$
			When $x = 6.0,\ t = 5.8800$			
				M1	Fully correct strategy for required time	$4.6365 - 1.9740$ if cos is used
			Time from X to Y is 2.66 s (3 sf)	A1	CAO	
				[4]		

Question			Answer	Marks	Guidance	
2	(a)	(i)	$T\cos\alpha = T\cos\beta + 0.24\times 9.8$	M1	Resolving vertically (three terms)	$\alpha = \hat{A} = 28.1°$, $\beta = \hat{B} = 53.1°$
			$\frac{15}{17}T = \frac{3}{5}T + 2.352$	A1	Accept $\cos 28.1°$ etc	
			Tension is 8.33 N	A1 **[3]**		
2	(a)	(ii)	$T\sin\alpha + T\sin\beta = m(r\omega^2)$	M1	Eqn with resolved tension and $r\omega^2$ *One tension sufficient for M1*	Allow $\frac{v^2}{r}$ for M1
			$\frac{8}{17}T + \frac{4}{5}T = (0.24)(1.6\omega^2)$	A1		
			Angular speed is $5.25\ \text{rad s}^{-1}$	A1 **[3]**	FT is $1.819\sqrt{T}$	
2	(b)	(i)		M1	Equation with tension, resolved weight and v_1^2/r	*Accept use of mass instead of weight throughout for M marks*
			$15-(0.3)(9.8)\cos 25° = (0.3)\frac{v_1^2}{1.8}$	A1		
			Speed is $8.60\ \text{m s}^{-1}$ (3 sf)	A1 **[3]**		
2	(b)	(ii)		M1	Equation with initial KE, final KE and attempt at PE	
			$\frac{1}{2}m(v_1^2 - v_2^2) = mg(1.8\cos 25° + 1.8\cos 60°)$	A1		
			$v_2^2 = 24.40$			
				M1	Equation with tension, resolved weight (using 60°) and v_2^2/r	
			$T + (0.3)(9.8)\cos 60° = (0.3)\frac{v_2^2}{1.8}$	A1		
			Tension is 2.60 N (3 sf)	A1 **[5]**	FT is $\frac{v_1^2}{6} - 9.739$	*SC* For 60° with *downward* vertical give A1 for 11.4 N (after M1A0M1A0), i.e. 3/5

Question			Answer	Marks	Guidance	
2	(b)	(iii)		M1	Equation with resolved weight in general position, and v_3^2/r	May also include T
			$(m)g\cos\theta = (m)\dfrac{v_3^2}{1.8}$	A1		θ is angle between OP and upward vertical
				M1	Equation with KE and attempt at PE in general position	
			$\frac{1}{2}m(v_2^2 - v_3^2) = mg(1.8)(\cos\theta - \cos 60°)$	A1	OR	$\frac{1}{2}m(v_1^2 - v_3^2) = mg(1.8)(\cos\theta + \cos 25°)$
			$24.40 - v_3^2 = 2v_3^2 - 9.8\times 1.8$			$\cos\theta = 0.794$, $\theta = 0.653\text{ rad} = 37.4°$
			Speed is 3.74 m s^{-1} (3 sf)	A1	CAO	
				[5]		
3	(i)		$T\cos\beta = mg\sin\alpha$	M1	Resolving parallel to slope	α is angle of slope, $\beta = \text{C}\hat{\text{A}}\text{B}$ $\alpha = 16.26°$, $\beta = 53.13 - \alpha = 36.87°$
			$0.8T = (9)(9.8)(0.28)$	A1	Accept $\cos 36.9°$ etc	
		OR	$T\sin\gamma + R\cos\alpha = mg$ $T\cos\gamma = R\sin\alpha$ $0.8T + 0.96R = 9\times 9.8$ $0.6T = 0.28R$		M1 Resolving vertically *and* horizontally A1 Both equations correct	γ is between string and horizontal $\gamma = 53.13°$ (R is normal reaction)
			$T = 30.87$	A1	Accept anything rounding to 31	*Dep on M1A1 (May be implied)*
			$T = \dfrac{\lambda(4.0 - 2.2)}{2.2}$	B1	Correct equation linking T and λ	
			Modulus of elasticity is 37.73 N	E1	*Working must lead to 37.73 to 4 sf*	
				[5]		
3	(ii)	(*A*)	Resultant force is 18 N (up the slope)	M1	Or $18 + T\cos\beta - mg\sin\alpha$	
			Acceleration is 2 m s^{-2} in direction AB	A1	*Accept positive direction indicated clearly on diagram*	*Just '2' implies M1A0*
				[2]		

Question			Answer	Marks	Guidance	
3	(ii)	(B)	At B, tension is $\frac{37.73\times(2.5-2.2)}{2.2}$ $(=5.145)$	B1		
			$18+T_B\sin\alpha-mg\sin\alpha=ma$	M1	Equation of motion	At least two forces required for M1
			$18+5.145\times0.28-9\times9.8\times0.28=9a$	A1	FT for wrong tension	
			Acceleration is 0.584 m s^{-2} in direction BA (3 sf)	A1	CAO	
				[4]		
3	(iii)		WD by force is 18×2.5 $(=45)$	B1		
			EE at A is $\frac{37.73\times1.8^2}{2\times2.2}$ $(=27.783)$ EE at B is $\frac{37.73\times0.3^2}{2\times2.2}$ $(=0.77175)$	B1	For either of these	
			Change in PE is $9\times9.8\times0.7$ $(=61.74)$	B1		
				M1	Equation involving KE and at least two of WD, EE, PE	
			$45+27.783=0.77175+61.74+\frac{1}{2}(9)v^2$	A1	FT from any B0 above, but all 5 terms must be non-zero and all signs correct	
			Speed is 1.51 m s^{-1} (3 sf)	A1	CAO	*Dependent on previous 5 marks*
				[6]		

Question			Answer	Marks	Guidance	
4	(i)		Volume is $\int_0^k \pi\left(e^{-x}\right)^2 dx$	M1	*π may be omitted throughout*	
			$= \pi\left[-\frac{1}{2}e^{-2x}\right]_0^k \quad \{=\frac{1}{2}\pi(1-e^{-2k})\}$	A1	For $-\frac{1}{2}e^{-2x}$	
			$\int \pi x y^2 \, dx$	M1		
			$= \int_0^k \pi x e^{-2x} dx = \pi\left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^k$	A1A1	For $-\frac{1}{2}xe^{-2x}$ and $-\frac{1}{4}e^{-2x}$	
			$= \frac{1}{4}\pi(1-2ke^{-2k}-e^{-2k})$			
			$\bar{x} = \dfrac{1-2ke^{-2k}-e^{-2k}}{2(1-e^{-2k})}$	A1	Any correct form	
			$= \dfrac{1-e^{-2k}}{2(1-e^{-2k})} - \dfrac{2ke^{-2k}}{2(1-e^{-2k})} = \dfrac{1}{2} - \dfrac{k}{e^{2k}-1}$	E1		
				[7]		
4	(ii)		OG $< \frac{1}{2}$ for all values of k	B1	OR $\dfrac{k}{e^{2k}-1} > 0$ o.e. stated or implied *Allow* $\bar{x} \to \frac{1}{2}$ *as* $k \to \infty$ *for B1*	
				M1	Trigonometry in OAP or OAG	
			$OP = (1)\tan 30° = \dfrac{1}{\sqrt{3}} \quad (=0.577)$	A1	Or $O\hat{A}G < \tan^{-1}\frac{1}{2} \quad (=26.6°)$	
			$OG < OP$ (or $O\hat{A}G < 30°$) so G is to the right of AP and solid will not topple	E1	Fully correct explanation	
				[4]		

Question			Answer	Marks	Guidance	
4	(iii)		Area is $\int_0^k \mathrm{e}^{-x}\,\mathrm{d}x \;=\left[\,-\mathrm{e}^{-x}\,\right]_0^k \quad (=1-\mathrm{e}^{-k})$	B1		
			$\int xy\,\mathrm{d}x$	M1		
			$=\int_0^k x\mathrm{e}^{-x}\,\mathrm{d}x \;=\left[\,-x\mathrm{e}^{-x}-\mathrm{e}^{-x}\,\right]_0^k$	A1		
			$\bar{x}=\dfrac{1-k\mathrm{e}^{-k}-\mathrm{e}^{-k}}{1-\mathrm{e}^{-k}}$	A1	Any correct form	e.g. $1-\dfrac{k}{\mathrm{e}^k-1}$
			$\int \frac{1}{2}y^2\,\mathrm{d}x$	M1	For $\int \ldots y^2\,\mathrm{d}x$	
			$=\int_0^k \frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x \;=\left[\,-\frac{1}{4}\mathrm{e}^{-2x}\,\right]_0^k$	A1		
			$\bar{y}=\dfrac{1-\mathrm{e}^{-2k}}{4(1-\mathrm{e}^{-k})}$	A1	Any correct form	e.g. $\frac{1}{4}(1+\mathrm{e}^{-k})$
				[7]		