

Thursday 6 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

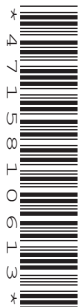
OCR supplied materials:

- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 (a) A particle P of mass 1.5 kg is connected to a fixed point by a light inextensible string of length 3.2 m. The particle P is moving as a conical pendulum in a horizontal circle at a constant angular speed of 2.5 rad s^{-1} .

(i) Find the tension in the string. [4]

(ii) Find the angle that the string makes with the vertical. [2]

- (b) A particle Q of mass m moves on a smooth horizontal surface, and is connected to a fixed point on the surface by a light elastic string of natural length d and stiffness k . With the string at its natural length, Q is set in motion with initial speed u perpendicular to the string. In the subsequent motion, the maximum length of the string is $2d$, and the string first returns to its natural length after time t .

You are given that $u = \sqrt{\frac{4kd^2}{3m}}$ and $t = Ak^\alpha d^\beta m^\gamma$, where A is a dimensionless constant.

(i) Show that the dimensions of k are MT^{-2} . [1]

(ii) Show that the equation $u = \sqrt{\frac{4kd^2}{3m}}$ is dimensionally consistent. [2]

(iii) Find α , β and γ . [4]

You are now given that Q has mass 5 kg, and the string has natural length 0.7 m and stiffness 60 N m^{-1} .

(iv) Find the initial speed u , and use conservation of energy to find the speed of Q at the instant when the length of the string is double its natural length. [5]

- 2 A particle P of mass 0.25 kg is connected to a fixed point O by a light inextensible string of length a metres, and is moving in a vertical circle with centre O and radius a metres. When P is vertically below O, its speed is 8.4 m s^{-1} . When OP makes an angle θ with the downward vertical, and the string is still taut, P has speed $v \text{ m s}^{-1}$ and the tension in the string is $T \text{ N}$, as shown in Fig. 2.

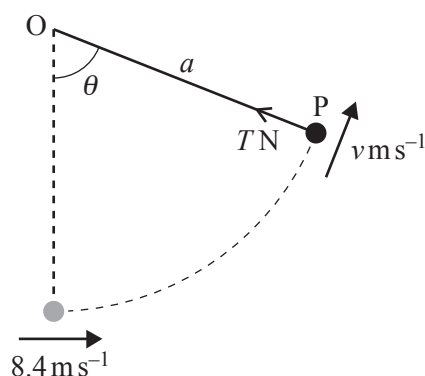


Fig. 2

- (i) Find an expression for v^2 in terms of a and θ , and show that

$$T = \frac{17.64}{a} + 7.35 \cos \theta - 4.9. \quad [7]$$

- (ii) Given that $a = 0.9$, show that P moves in a complete circle. Find the maximum and minimum magnitudes of the tension in the string. [4]
- (iii) Find the largest value of a for which P moves in a complete circle. [3]
- (iv) Given that $a = 1.6$, find the speed of P at the instant when the string first becomes slack. [4]

- 3 A light spring, with modulus of elasticity 686 N, has one end attached to a fixed point A. The other end is attached to a particle P of mass 18 kg which hangs in equilibrium when it is 2.2 m vertically below A.

(i) Find the natural length of the spring AP.

[2]

Another light spring has natural length 2.5 m and modulus of elasticity 145 N. One end of this spring is now attached to the particle P, and the other end is attached to a fixed point B which is 2.5 m vertically below P (so leaving the equilibrium position of P unchanged). While in its equilibrium position, P is set in motion with initial velocity 3.4 m s^{-1} vertically downwards, as shown in Fig. 3. It now executes simple harmonic motion along part of the vertical line AB.

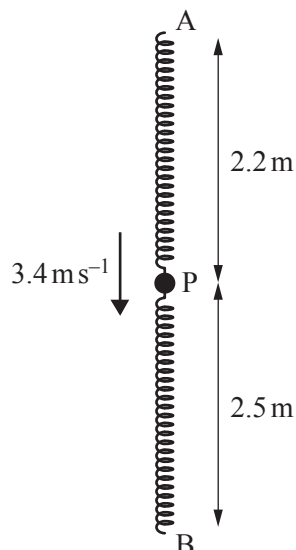


Fig. 3

At time t seconds after it is set in motion, P is x metres below its equilibrium position.

- (ii) Show that the tension in the spring AP is $(176.4 + 392x) \text{ N}$, and write down an expression for the thrust in the spring BP. [3]
- (iii) Show that $\frac{d^2x}{dt^2} = -25x$. [3]
- (iv) Find the period and the amplitude of the motion. [3]
- (v) Find the magnitude and direction of the velocity of P when $t = 2.4$. [3]
- (vi) Find the total distance travelled by P during the first 2.4 seconds of its motion. [4]

- 4 (a) A uniform solid of revolution S is formed by rotating the region enclosed between the x -axis and the curve $y = x\sqrt{4-x}$ for $0 \leq x \leq 4$ through 2π radians about the x -axis, as shown in Fig. 4.1. O is the origin and the end A of the solid is at the point $(4, 0)$.

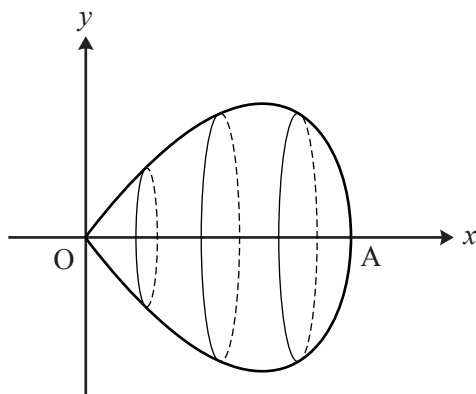


Fig. 4.1

- (i) Find the x -coordinate of the centre of mass of the solid S . [6]

The solid S has weight W , and it is freely hinged to a fixed point at O . A horizontal force, of magnitude W acting in the vertical plane containing OA , is applied at the point A , as shown in Fig. 4.2. S is in equilibrium.

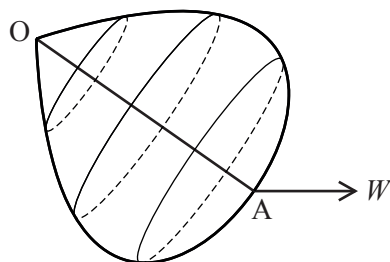


Fig. 4.2

- (ii) Find the angle that OA makes with the vertical. [3]

[Question 4(b) is printed overleaf]

- (b) Fig. 4.3 shows the region bounded by the x -axis, the y -axis, the line $y = 8$ and the curve $y = (x - 2)^3$ for $0 \leq y \leq 8$.

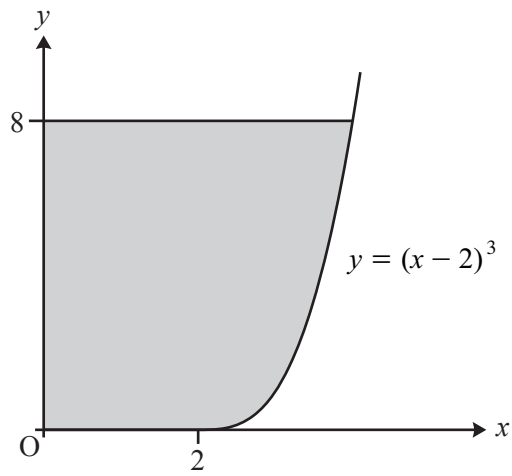


Fig. 4.3

Find the coordinates of the centre of mass of a uniform lamina occupying this region.

[9]

Question			Answer	Marks	Guidance	
1	(a)	(i)	$T \sin \theta = mr\omega^2$ $r = 3.2 \sin \theta$ $T \sin \theta = (1.5)(3.2 \sin \theta)(2.5)^2$ Tension is 30 N	M1 B1 A1 A1 [4]	Equation involving $r\omega^2$ or $l\omega^2$ $T = (1.5)(3.2)(2.5)^2$ with no wrong working earns M1B1A1	All marks in (a) can be earned anywhere in (i) or (ii)
1	(a)	(ii)	$T \cos \theta = mg$ $30 \cos \theta = 1.5 \times 9.8$ Angle is 60.7° (3 sf)	M1 A1 [2]	Resolving vertically or 1.06 rad	
1	(b)	(i)	$[k] = (\text{MLT}^{-2})\text{L}^{-1} = \text{MT}^{-2}$	E1 [1]	Can use $u = \sqrt{\frac{4kd^2}{3m}}$ or $k = \frac{\lambda}{l}$	
1	(b)	(ii)	$\left[\sqrt{\frac{4kd^2}{3m}} \right] = \left(\frac{\text{MT}^{-2}\text{L}^2}{\text{M}} \right)^{\frac{1}{2}} = \text{LT}^{-1}$ $[u] = \text{LT}^{-1}$, so eqn is dimensionally consistent	M1 E1 [2]	Obtaining dimensions of RHS Condone circular argument	
1	(b)	(iii)	$T = (\text{MT}^{-2})^\alpha \text{L}^\beta \text{M}^\gamma$ $\alpha = -\frac{1}{2}$ $\beta = 0$ $\alpha + \gamma = 0$ $\gamma = \frac{1}{2}$	B1 B1 M1 A1 [4]	Considering powers of M FT from wrong non-zero α	

Question			Answer	Marks	Guidance	
1	(b)	(iv)	$u = \sqrt{\frac{4 \times 60 \times 0.7^2}{3 \times 5}} = 2.8 \text{ ms}^{-1}$ <p>Elastic energy is $\frac{1}{2} \times 60 \times 0.7^2$ (=14.7)</p> $\frac{1}{2}(5)(2.8)^2 - \frac{1}{2}(5)v^2 = 14.7$ <p>Speed is 1.4 ms^{-1}</p>	B1 M1A1 M1 A1 [5]	M1A0 if one error Equation involving initial KE, final KE and EE <i>No FT in any part of Q1 except (iii)</i>	
2	(i)		$\frac{1}{2}m(8.4)^2 - \frac{1}{2}mv^2 = mg(a - a \cos \theta)$ $v^2 = 70.56 - 19.6a(1 - \cos \theta)$ $T - mg \cos \theta = m \frac{v^2}{a}$ $T - 2.45 \cos \theta = 0.25 \left(\frac{70.56}{a} - 19.6 + 19.6 \cos \theta \right)$ $T - 2.45 \cos \theta = \frac{17.64}{a} - 4.9 + 4.9 \cos \theta$ $T = \frac{17.64}{a} + 7.35 \cos \theta - 4.9$	M1 A1 A1 M1 A1 M1 E1 [7]	Equation involving initial KE, final KE and PE Using acceleration $\frac{v^2}{a}$ Equation relating T, a, θ	($m = 0.25$) <i>Dependent on previous M1M1</i>
2	(ii)		<p>If $a = 0.9$, $T = 14.7 + 7.35 \cos \theta$ $T > 0$ for all θ, so P moves in a complete circle</p> <p>Maximum tension is $14.7 + 7.35 = 22.05 \text{ N}$ Minimum tension is $14.7 - 7.35 = 7.35 \text{ N}$</p>	M1 E1 M1 A1 [4]	Expression for T when $a = 0.9$ Any correct explanation Using $\theta = 0$ or $\theta = \pi$ Both correct	In terms of θ or when $\theta = \pi$

Question			Answer	Marks	Guidance	
2	(iii)		<p>If P just completes the circle, $T = 0$ when $\theta = \pi$</p> $\frac{17.64}{a} - 7.35 - 4.9 = 0$ $a = 1.44$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	For 1.44	Condone $a < 1.44$ etc
2	(iv)		<p>If $a = 1.6$, $T = 6.125 + 7.35 \cos \theta$ String becomes slack when $T = 0$</p> $\cos \theta = -\frac{6.125}{7.35} = -\frac{5}{6} \quad [\theta = 2.56 \text{ rad or } 146^\circ]$ $v^2 = 70.56 - 19.6 \times 1.6(1 + \frac{5}{6})$ <p>Speed is 3.61 ms^{-1} (3 sf)</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Using expression for T when $a = 1.6$</p> <p>Obtaining an equation for v</p> <p>Or $-mg(-\frac{5}{6}) = m \frac{v^2}{1.6}$</p>	<p>Dependent on previous M1M1</p> <p>No FT in any part of Q2</p>
3	(i)		$\frac{686(2.2 - l)}{l} = 18 \times 9.8$ <p>Natural length is 1.75 m</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	Using Hooke's law	
3	(ii)		<p>Tension in AP is $\frac{686}{1.75}(0.45 + x)$ $= 176.4 + 392x$</p> <p>Thrust in BP is $\frac{145}{2.5}x$ ($= 58x$)</p>	<p>M1</p> <p>E1</p> <p>B1</p> <p>[3]</p>	Allow $-58x$	Condone thrust / tension confusion

Question			Answer	Marks	Guidance	
3	(iii)		$18 \times 9.8 - (176.4 + 392x) - 58x = 18 \frac{d^2x}{dt^2}$ $176.4 - 176.4 - 450x = 18 \frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -25x$	M1 A1 E1 [3]	Equation of motion Correct LHS equated to $\pm 18a$ Fully correct derivation	2 forces from (ii), mg and ma <i>No FT</i>
3	(iv)		Period is $\frac{2\pi}{5} = 1.26 \text{ s}$ (3 sf) $A\omega = 3.4$ Amplitude ($A = \frac{3.4}{5}$) is 0.68 m	B1 M1 A1 [3]	Allow $\frac{2\pi}{5}$	
3	(v)		$v = 3.4 \cos 5t$ When $t = 2.4$, $v = 2.87$ Magnitude of velocity is 2.87 ms^{-1} (3 sf) Since $v > 0$ the direction is downwards	M1 A1 A1 [3]	Using $\cos \omega t$ or $\sin \omega t$ <i>Dependent on M1A1</i>	$\cos \frac{2}{5}\pi t$ is M0 'Downwards' is sufficient
		OR	When $t = 2.4$, $x = -0.3649$ $v^2 = 25(0.68^2 - 0.3649^2)$ Magnitude of velocity is 2.87 ms^{-1} (3 sf) Between $1\frac{3}{4}$ and 2 periods; hence downwards		M1 Using $v^2 = \omega^2(A^2 - x^2)$ A1 A1 <i>Dependent on M1A1</i>	Earns B1M1 from (vi) <i>No FT</i> Must be justified
3	(vi)		$x = 0.68 \sin 5t$ When $t = 2.4$, $x = -0.3649$ 2.4 s is $\frac{2.4}{1.26} = 1.91$ periods (between $1\frac{3}{4}$ and 2) Distance is $8 \times 0.68 - 0.3649$ Distance is 5.08 m (3 sf)	B1 M1 M1 A1 [4]	FT (from wrong amplitude) $8A + x_{t=2.4}$ with $x_{t=2.4} < 0$ FT is $7.463A$	B1M1 can be earned in (v) Strictly, only for this

Question			Answer	Marks	Guidance	
4	(a)	(i)	$V = \int_0^4 \pi x^2 (4-x) dx$ $= \pi \left[\frac{4}{3} x^3 - \frac{1}{4} x^4 \right]_0^4 \quad (= \frac{64\pi}{3})$ $V\bar{x} = \int \pi xy^2 dx = \int_0^4 \pi x^3 (4-x) dx$ $= \pi \left[x^4 - \frac{1}{5} x^5 \right]_0^4 \quad (= 51.2\pi)$ $\bar{x} = \frac{51.2\pi}{\frac{64}{3}\pi}$ $= 2.4$	M1 A1 M1 A1 M1 A1 [6]	For $\int (x\sqrt{4-x})^2 dx$ For $\frac{4}{3}x^3 - \frac{1}{4}x^4$ For $\int xy^2 dx$ For $x^4 - \frac{1}{5}x^5$ <i>Dependent on previous M1M1</i>	π may be omitted throughout
4	(a)	(ii)	$W(2.4 \sin \theta) = W(4 \cos \theta)$ $\tan \theta = \frac{4}{2.4} = \frac{5}{3}$ $\theta = 59.0^\circ \quad (3 \text{ sf})$	M1 A1 A1 [3]	Taking moments FT Correct equation for required angle FT is $\tan^{-1} \frac{4}{\bar{x}}$	$W(2.4 \cos \phi) = W(4 \sin \phi)$ is A0 unless $\theta = 90^\circ - \phi$ also appears FT requires $\bar{x} < 4$

Question		Answer	Marks	Guidance	
4	(b)	$x = 2 + y^{\frac{1}{3}}$ $A = \int_0^8 (2 + y^{\frac{1}{3}}) dy = \left[2y + \frac{3}{4} y^{\frac{4}{3}} \right]_0^8 (= 28)$ $A\bar{x} = \int_0^8 \frac{1}{2} x^2 dy = \int_0^8 \frac{1}{2} \left(4 + 4y^{\frac{1}{3}} + y^{\frac{2}{3}} \right) dy$ $= \left[2y + \frac{3}{2} y^{\frac{4}{3}} + \frac{3}{10} y^{\frac{5}{3}} \right]_0^8 (= 49.6)$ $\bar{x} = \frac{49.6}{28} = \frac{62}{35} = 1.77 \quad (3 \text{ sf})$ $A\bar{y} = \int_0^8 xy dy = \int_0^8 \left(2y + y^{\frac{4}{3}} \right) dy$ $= \left[y^2 + \frac{3}{7} y^{\frac{7}{3}} \right]_0^8 (= \frac{832}{7})$ $\bar{y} = \frac{\frac{832}{7}}{28} = \frac{208}{49} = 4.24 \quad (3 \text{ sf})$	B1 B1 M1 B2 A1 M1 A1 A1	FT For $\int x^2 dy$ FT for $2y + \frac{3}{2} y^{\frac{4}{3}} + \frac{3}{10} y^{\frac{5}{3}}$ Give B1 for one minor slip in integration, or if $\frac{1}{2}$ omitted CAO For $\int xy dy$ FT for $y^2 + \frac{3}{7} y^{\frac{7}{3}}$ CAO	Or $32 - \left[\frac{1}{4} (x-2)^4 \right]_2^4$ Or $32 \times 2 - \int_2^4 xy dx$ Or $\frac{1}{5} (x-2)^5 + \frac{1}{2} (x-2)^4$ Or $\frac{1}{4} x(x-2)^4 - \frac{1}{20} (x-2)^5$ Or $\frac{1}{5} x^5 - \frac{3}{2} x^4 + 4x^3 - 4x^2$ <i>Must be \bar{x}</i> Or $32 \times 4 - \int_2^4 \left(\frac{1}{2} \right) y^2 dx$ Or B2 for $\frac{1}{14} (x-2)^7$ Give B1 for one minor slip in integration, or if $\frac{1}{2}$ omitted <i>Must be \bar{y}</i>
	OR	Region under curve has CM $(3.6, \frac{16}{7})$ $28\bar{x} + 4 \times 3.6 = 32 \times 2$ $\bar{x} = 1.77$ $28\bar{y} + 4 \times \frac{16}{7} = 32 \times 4$ $\bar{y} = 4.24$	[9]	B2B2 B1 (for 28) M1 A1 M1 A1	For integrals, as above