OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** A particle P is executing simple harmonic motion, and the centre of the oscillations is at the point O. The maximum speed of P during the motion is $5.1\,\mathrm{m\,s^{-1}}$. When P is 6 m from O, its speed is $4.5\,\mathrm{m\,s^{-1}}$. Find the period and the amplitude of the motion. **[6]**

(b) The force F of gravitational attraction between two objects of masses m_1 and m_2 at a distance d apart is given by $F = \dfrac{Gm_1m_2}{d^2}$, where G is the universal gravitational constant.

(i) Find the dimensions of G. **[3]**

Three objects, each of mass m, are moving in deep space under mutual gravitational attraction. They move round a single circle with constant angular speed ω, and are always at the three vertices of an equilateral triangle of side R. You are given that $\omega = kG^{\alpha}m^{\beta}R^{\gamma}$, where k is a dimensionless constant.

(ii) Find α, β and γ. **[5]**

For three objects of mass 2500 kg at the vertices of an equilateral triangle of side 50 m, the angular speed is $2.0 \times 10^{-6}\,\mathrm{rad\,s^{-1}}$.

(iii) Find the angular speed for three objects of mass 4.86×10^{14} kg at the vertices of an equilateral triangle of side 30 000 m. **[4]**

2 **(a)** A fixed solid sphere with a smooth surface has centre O and radius 0.8 m. A particle P is given a horizontal velocity of $1.2\,\mathrm{m\,s^{-1}}$ at the highest point on the sphere, and it moves on the surface of the sphere in part of a vertical circle of radius 0.8 m.

(i) Find the radial and tangential components of the acceleration of P at the instant when OP makes an angle $\frac{1}{6}\pi$ radians with the upward vertical. (You may assume that P is still in contact with the sphere.) **[5]**

(ii) Find the speed of P at the instant when it leaves the surface of the sphere. **[6]**

(b) Two fixed points R and S are 2.5 m apart with S vertically below R. A particle Q of mass 0.9 kg is connected to R and to S by two light inextensible strings; Q is moving in a horizontal circle at a constant speed of $5\,\mathrm{m\,s^{-1}}$ with both strings taut. The radius of the circle is 2.4 m and the centre C of the circle is 0.7 m vertically below S, as shown in Fig. 2.

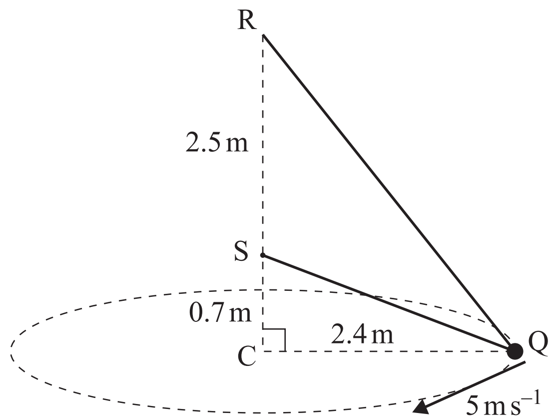

Fig. 2

Find the tension in the string RQ and the tension in the string SQ. **[7]**

3 Two fixed points X and Y are 14.4 m apart and XY is horizontal. The midpoint of XY is M. A particle P is connected to X and to Y by two light elastic strings. Each string has natural length 6.4 m and modulus of elasticity 728 N. The particle P is in equilibrium when it is 3 m vertically below M, as shown in Fig. 3.

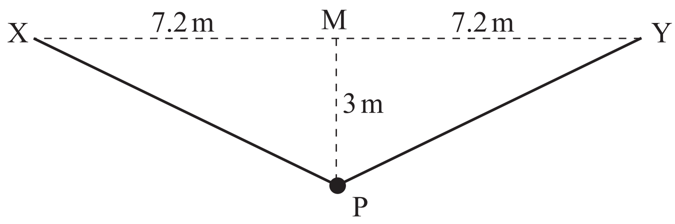

Fig. 3

(i) Find the tension in each string when P is in the equilibrium position. **[3]**

(ii) Show that the mass of P is 12.5 kg. **[3]**

The particle P is released from rest at M, and moves in a vertical line.

(iii) Find the acceleration of P when it is 2.1 m vertically below M. **[5]**

(iv) Explain why the maximum speed of P occurs at the equilibrium position. **[1]**

(v) Find the maximum speed of P. **[6]**

4 **(a)** The region enclosed between the curve $y = x^4$ and the line $y = h$ (where h is positive) is rotated about the y-axis to form a uniform solid of revolution. Find the y-coordinate of the centre of mass of this solid. **[5]**

(b) The region A is bounded by the x-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $x = 4$. The region B is bounded by the y-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $y = 6$. These regions are shown in Fig. 4.

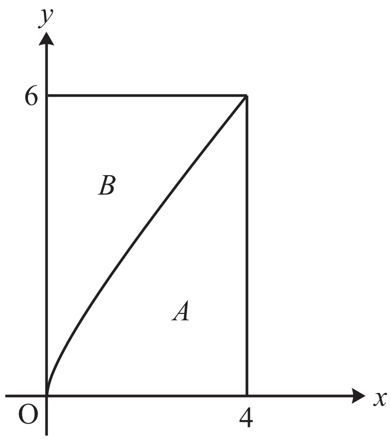

Fig. 4

(i) A uniform lamina occupies the region A. Show that the x-coordinate of the centre of mass of this lamina is 2.56, and find the y-coordinate. **[9]**

(ii) Using your answer to part **(i)**, or otherwise, find the coordinates of the centre of mass of a uniform lamina occupying the region B. **[4]**

Question			Answer	Marks	Guidance	
1	(a)		$A\omega = 5.1$	B1		
				M1	Using $v^2 = \omega^2(A^2 - x^2)$	
			$4.5^2 = \omega^2(A^2 - 6^2)$	A1		
			$4.5^2 = 5.1^2 - 36\omega^2$ $\omega = 0.4$	M1	Eliminating A or ω	
			Period ($\frac{2\pi}{\omega}$) is $5\pi = 15.7$ s (3 sf)	A1		Allow 5π
			Amplitude (A) is 12.75 m	A1 **[6]**		
1	(b)	(i)	$[F] = \mathrm{M\,L\,T^{-2}}$	B1		
			$[G] = \left[\frac{Fd^2}{m_1 m_2}\right] = \frac{\mathrm{M\,L\,T^{-2}\,L^2}}{\mathrm{M^2}}$	M1	Obtaining dimensions of G	
			$= \mathrm{M^{-1}\,L^3\,T^{-2}}$	A1 **[3]**		
1	(b)	(ii)	$\mathrm{T^{-1}} = (\mathrm{M^{-1}\,L^3\,T^{-2}})^{\alpha}\,\mathrm{M}^{\beta}\,\mathrm{L}^{\gamma}$			
			$\alpha = \frac{1}{2}$	B1		
			$-\alpha + \beta = 0$	M1	Considering powers of M	
			$\beta = \frac{1}{2}$	A1		
			$3\alpha + \gamma = 0$	M1	Considering powers of L	
			$\gamma = -\frac{3}{2}$	A1 **[5]**		*All marks FT from wrong [G] if comparable. No FT within part **(ii)**.*

Question			Answer	Marks	Guidance	
1	(b)	(iii)		M1M1	For $\left(\frac{4.86\times10^{14}}{2500}\right)^{\pm\frac{1}{2}}$ and $\left(\frac{30000}{50}\right)^{\pm\frac{3}{2}}$	*Requires* $\beta \neq 0,\ \gamma \neq 0$
			$\omega = 2.0\times10^{-6}\times\left(\frac{4.86\times10^{14}}{2500}\right)^{\frac{1}{2}}\times\left(\frac{30000}{50}\right)^{-\frac{3}{2}}$	A1	Correct equation for ω	*FT if comparable*
		OR	$2.0\times10^{-6} = k\times G^{\frac{1}{2}}\times2500^{\frac{1}{2}}\times50^{-\frac{3}{2}}$		M1 *Requires* $\beta \neq 0$ *or* $\gamma \neq 0$	
			$kG^{\frac{1}{2}} = 1.414\times10^{-5}$		M1 *Requires* $\beta \neq 0$ *and* $\gamma \neq 0$	Condone the use of any value for G (including $G = 1$)
			$\omega = 1.414\times10^{-5}\times(4.86\times10^{14})^{\frac{1}{2}}\times30000^{-\frac{3}{2}}$		A1 Correct equation for ω	*FT if comparable*
			Angular speed is $6.0\times10^{-5}\ \mathrm{rad\,s^{-1}}$	A1 **[4]**	CAO	
2	(a)	(i)		M1	Equation involving initial KE, final KE and attempt at PE	
			$\frac{1}{2}mv^2 - \frac{1}{2}m(1.2)^2 = mg(0.8 - 0.8\cos\frac{1}{6}\pi)$	A1		
			$v^2 = 3.5407$			
			Radial component $\left(\frac{v^2}{0.8}\right)$ is $4.43\ \mathrm{m\,s^{-2}}$ (3 sf)	A1		
			$(\pm)mg\sin\frac{1}{6}\pi = ma_T$	M1	Allow M1 for $\cos\frac{1}{6}\pi$ used instead of $\sin\frac{1}{6}\pi$; but M0 for $a_T = mg\sin\frac{1}{6}\pi$	
			Tangential component is $4.9\ \mathrm{m\,s^{-2}}$	A1 **[5]**	Allow $\frac{1}{2}g$	

Question			Answer	Marks	Guidance	
2	(a)	(ii)	$\frac{1}{2}mv^2 - \frac{1}{2}m(1.2)^2 = mg(0.8 - 0.8\cos\theta)$	M1	Equation involving initial KE, final KE and attempt at PE in general position	θ between OP and upward vertical Allow mgh for PE if h is linked to θ in later work
				M1	Equation involving resolved component of weight and v^2/r	
			$mg\cos\theta - R = \dfrac{mv^2}{0.8}$	A1	R may be omitted	
			Leaves surface when $R = 0$	M1	May be implied	e.g. Implied by $mg\cos\theta = \dfrac{mv^2}{0.8}$
			$v^2 - 1.44 = 2\times 9.8\times 0.8(1 - \dfrac{v^2}{7.84})$	M1	Obtaining equation in v or θ *Dependent on previous M1M1M1*	$\cos\theta = \dfrac{107}{147} = 0.728$ $\theta = 0.756$ rad or $43.3°$
			Speed is $2.39\ \mathrm{m\,s^{-1}}$	A1 **[6]**		
2	(b)		$T_R\sin\alpha + T_S\sin\beta = mg$	M1	Resolving vertically (three terms)	$\alpha = \mathrm{R\hat{Q}C} = 53.1°$, $\beta = \mathrm{S\hat{Q}C} = 16.3°$
			$0.8T_R + 0.28T_S = 0.9\times 9.8\ (= 8.82)$	A1	Allow $\sin 53.1°$, etc	
			$T_R\cos\alpha + T_S\cos\beta = m\dfrac{v^2}{r}$	M1	Horizontal equation of motion	Three terms, and v^2/r
			$0.6T_R + 0.96T_S = 0.9\times\dfrac{5^2}{2.4}\ (= 9.375)$	A1		
				M1	Obtaining T_R or T_S	*Dependent on previous M1M1*
			Tension in string RQ is 9.737 N	A1		
			Tension in string SQ is 3.68 N	A1 **[7]**		

Question			Answer	Marks	Guidance	
3	(i)		Length of each string is 7.8 m	B1		
			$T = \frac{728}{6.4}(7.8 - 6.4)$	M1	Using Hooke's law	Must use extension
			Tension is 159.25 N	A1 **[3]**		
3	(ii)		$2T\cos\theta = mg$	M1	Resolving vertically	$\theta = \text{X}\hat{\text{P}}\text{M} = 67.4°$
			$2 \times 159.25 \times \frac{5}{13} = m \times 9.8$	A1	FT	
			$m = \frac{122.5}{9.8} = 12.5\text{ kg}$	E1 **[3]**	*Working must lead to 12.5 to 3 sf*	
3	(iii)		New length of each string is 7.5 m	M1	Hooke's law with new extension	
			$T = \frac{728}{6.4}(7.5 - 6.4)\quad (= 125.125)$	A1		
			$mg - 2T\cos\theta = ma$	M1	Vertical equation of motion (3 terms)	
			$12.5 \times 9.8 - 2 \times 125.125 \times 0.28 = 12.5a$	A1	FT for incorrect T	
			Acceleration is 4.19 m s^{-2} downwards (3 sf)	A1 **[5]**	*Some indication of downwards required*	
3	(iv)		At maximum speed, acceleration is zero Acceleration is zero in equilibrium position	B1 **[1]**	Mention of zero acceleration	*Reference to $v^2 = \omega^2(A^2 - x^2)$, SHM, etc, will usually be B0*

Question			Answer	Marks	Guidance	
3	(v)		Change of PE is $12.5\times9.8\times3$ $(=367.5)$	B1		
			Initial EE is $2\times\dfrac{728\times0.8^2}{2\times6.4}$ $(=72.8)$	B1	Allow one string (36.4)	
			Final EE is $2\times\dfrac{728\times1.4^2}{2\times6.4}$ $(=222.95)$	B1	Allow one string (111.475)	
				M1	Equation involving KE, PE and EE	
			$\frac{1}{2}(12.5)v^2-367.5+222.95=72.8$	A1	FT from any B0 above All signs must be correct	*All terms must be non-zero*
			Maximum speed is $5.90\ \mathrm{m\,s^{-1}}$ (3 sf)	A1	CAO	
				[6]		
4	(a)		$V=\displaystyle\int_0^h\pi(y^{\frac{1}{4}})^2\,\mathrm{d}y$	M1	For $\int\ldots(y^{\frac{1}{4}})^2\,\mathrm{d}y$	
			$=\pi\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^h$ $\left(=\frac{2}{3}\pi h^{\frac{3}{2}}\right)$	A1	For $\frac{2}{3}y^{\frac{3}{2}}$	
			$V\bar{y}=\int\pi x^2y\,\mathrm{d}y=\displaystyle\int_0^h\pi y^{\frac{1}{2}}y\,\mathrm{d}y$	M1	For $\int x^2y\,\mathrm{d}y$	
			$=\pi\left[\frac{2}{5}y^{\frac{5}{2}}\right]_0^h$ $\left(=\frac{2}{5}\pi h^{\frac{5}{2}}\right)$	A1	For $\frac{2}{5}y^{\frac{5}{2}}$	
			$\bar{y}=\dfrac{\frac{2}{5}\pi h^{\frac{5}{2}}}{\frac{2}{3}\pi h^{\frac{3}{2}}}=\dfrac{3}{5}h$	A1		
				[5]		

Question			Answer	Marks	Guidance	
4	(b)	(i)	$A=\int_0^4 (x+\sqrt{x})\,dx$	M1	For $\int (x+\sqrt{x})\,dx$	
			$=\left[\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 \quad (=\frac{40}{3})$	A1	For $\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$	
			$A\bar{x}=\int xy\,dx=\int_0^4 x(x+\sqrt{x})\,dx$	M1	For $\int xy\,dx$	
			$=\left[\frac{1}{3}x^3+\frac{2}{5}x^{\frac{5}{2}}\right]_0^4 \quad (=\frac{512}{15})$	A1	For $\frac{1}{3}x^3+\frac{2}{5}x^{\frac{5}{2}}$	
			$\bar{x}=\frac{\frac{512}{15}}{\frac{40}{3}}=\frac{64}{25}=2.56$	E1		
			$A\bar{y}=\int \frac{1}{2}y^2\,dx=\int_0^4 \frac{1}{2}(x+\sqrt{x})^2\,dx$	M1	For $\int ...\, y^2\,dx$	
			$=\left[\frac{1}{6}x^3+\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{4}x^2\right]_0^4 \quad (=\frac{412}{15})$	A1A1	For $\frac{1}{6}x^3+\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{4}x^2$	Give A1 for two correct terms
			$\bar{y}=\frac{\frac{412}{15}}{\frac{40}{3}}=\frac{103}{50}=2.06$	A1		
				[9]		

Question			Answer	Marks	Guidance	
4	(b)	(ii)	Area of B is $24-\frac{40}{3}=\frac{32}{3}$			
			$\frac{32}{3}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+\frac{40}{3}\begin{pmatrix}2.56\\ 2.06\end{pmatrix}=24\begin{pmatrix}2\\ 3\end{pmatrix}$	M1 M1	CM of composite body Correct strategy	(One coordinate sufficient)
			$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}1.3\\ 4.175\end{pmatrix}$	A1 A1	CAO FT requires $0<\bar{y}<6$	*FT is* $6.75-1.25\bar{y}_A$ *No FT from wrong area*
		OR	$\int \frac{1}{4}\left(\sqrt{1+4y}-1\right)^2 y\,\mathrm{d}y$ *or* $\int x(6-x-\sqrt{x})\,\mathrm{d}x$ *or* $\int \frac{1}{32}\left(\sqrt{1+4y}-1\right)^4\mathrm{d}y$ *or* $\int \frac{1}{2}(6-x-\sqrt{x})(6+x+\sqrt{x})\,\mathrm{d}x$		M1 For any one of these	
			$\bar{x}=1.3,\quad \bar{y}=4.175$		M1 For one successful integration A1A1	
				[4]		

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General Comments

Candidates generally presented their answers well and demonstrated a very sound understanding of most of the topics being examined. The notable exception was Q.3 on elastic strings, where the motion was often wrongly assumed to be simple harmonic. Candidates did not appear to have any difficulty completing the paper in the time allowed.

Comments on Individual Questions

1 The simple harmonic motion problem in part (a) was well understood and was usually answered correctly. Having found the parameters of the motion, some candidates omitted the calculation of the period.

In part (b)(i) the dimensions of *G* were almost always found correctly. The method for finding the powers in part (b)(ii) was also very well understood, although a significant number started with the wrong dimensions for angular speed, usually LT^{-1}. Most candidates then used their formula correctly in part (b)(iii) to find the new angular speed.

2 In part (a)(i) the tangential acceleration was usually found correctly, although there was some sine/cosine confusion. Many attempts at the radial acceleration started with an equation of motion, but most recovered from this and then considered energy.

In part (a)(ii) most candidates produced a radial equation of motion with zero normal reaction, together with an energy equation. However, there were some difficulties with the potential energy terms and the subsequent manipulation to find the speed.

In part (b) almost all candidates obtained two equations from the vertical equilibrium and radial acceleration. There were some careless slips in the trigonometry, such as taking RC to be 2.5 m instead of 3.2 m, and several made algebraic errors when solving the simultaneous equations.

3 This was found to be by far the most difficult question, with entirely inappropriate methods often being selected for parts (iii) and (v).

Parts (i) and (ii) on the equilibrium position were usually answered correctly.

In part (iii) it was necessary to apply Newton's second law in the vertical direction. Many of those who did this were successful, although some did not give the direction of the acceleration. Many used energy to find the speed of P, but this is not the first step in any valid method for finding its acceleration. Some tried to use formulae which only apply to simple harmonic motion.

In part (iv) about half the candidates gave a satisfactory explanation, usually by stating that the acceleration is zero in the equilibrium position. There were also very many references to simple harmonic motion here.

In part (v) the expected approach was to use energy; when doing this, common errors were omission of the initial elastic energy and using the elastic energy in just one string instead of the two. However, very many attempts did not consider energy at all, usually treating the motion as if it were simple harmonic.

4 In part (a) the method for finding the centre of mass of a solid of revolution was well understood and usually carried out accurately.

In part (b)(i) the given x-coordinate of the centre of mass of the lamina was almost always found legitimately. The y-coordinate was also usually found correctly, although some omitted the factor ½ and some made errors in the expansion and integration of $(x + \sqrt{x})^2$.

In part (b)(ii) the centre of mass of a composite body was well understood, and this part was usually answered correctly.

(iC) Most candidates realised that the new mean, $\lambda = 10.5$, was to be used. Many correct answers were seen through a variety of incorrect methods for finding $P(5 \le X \le 10)$ followed; of which "$P(X \le 10) - P(X \le 5)$" and "$P(X \ge 5) - P(X \le 10)$" were typical.

(ii) Many candidates obtained full marks here though some failed to apply the required continuity correction. A few candidates lost the final accuracy mark through premature rounding of their z-value prior to using Normal tables.

(iii) Most candidates provided a suitable comment here, with remarks about "independence" being the most popular.

(iv) This proved to be one of the most challenging parts of the paper. Despite answering part (iii) correctly many candidates reverted back to the inappropriate model by combining the means rather than considering the different combinations of "pairs" and "singles". Of those attempting to consider combinations of pairs and singles only a small proportion obtained a fully correct solution; a variety of approaches was seen and those working systematically were the most successful.

3(iA) Well answered, though inappropriate "continuity corrections" were seen on occasion.

(iB) Well answered, though arithmetic errors were quite common. In several cases, –1.667 was used rather than –1.1667 often as a result of candidates misreading their own figures. A few candidates lost accuracy by prematurely rounding their z-value before using the Normal tables.

(ii) Very well answered. Most candidates scored both marks.

(iii) On the whole, this was well answered. Many candidates provided clear, accurate methods leading to correct final answers. Some candidates started out with one of the required equations containing a sign error which was not picked up, even when the error led to a negative value for σ. Most candidates identified the correct z-values. In the poorest answers, continuity corrections were attempted and z-values were changed to absurd values, such as "1 – 0.8416", before substitution into equations. Over-specification of final answers was seen, on occasion, here.

(iv) Though one of the more challenging parts, many candidates scored full marks here. A variety of correct, "non-symmetrical" solutions were seen though most opted to use z-values of $\pm$ 1.96.

4(a) This question was well answered. Many candidates scored full marks. Marks lost typically for over-assertive conclusions, typically containing words such as "not enough evidence to prove that…". The small number of candidates referring to correlation in their hypotheses often lost the first and last marks. Most candidates managed to accurately calculate the test statistic though some did not show all working as required. Most candidates stated the correct number of degrees of freedom and identified the correct critical value, though some thought that this was a 2-tailed test.

(b) Well answered. Most candidates accurately calculated the sample mean and provided hypotheses in terms of μ. Note that candidates should be discouraged from referring to the "sample population mean" when defining μ. In carrying out the test, the test statistic method proved the most popular; those who "reversed their numerator" needed to be very careful how they used their test statistic. Many appropriate, non-assertive conclusions were seen though some failed to include context in the final comments.