

# Thursday 31 May 2012 – Morning

## A2 GCE MATHEMATICS (MEI)

4763 Mechanics 3

### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION FOR CANDIDATES

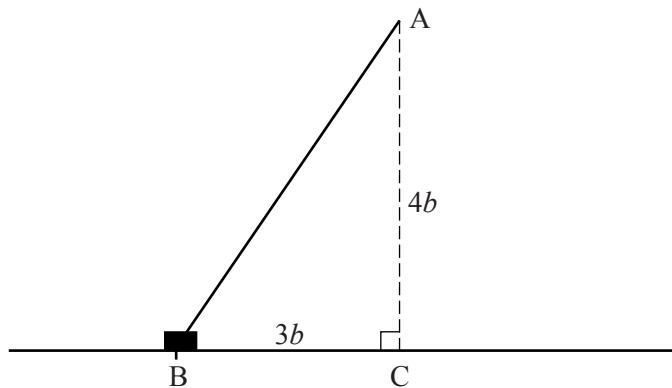
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The fixed point A is at a height  $4b$  above a smooth horizontal surface, and C is the point on the surface which is vertically below A. A light elastic string, of natural length  $3b$  and modulus of elasticity  $\lambda$ , has one end attached to A and the other end attached to a block of mass  $m$ . The block is released from rest at a point B on the surface where  $BC = 3b$ , as shown in Fig. 1. You are given that the block remains on the surface and moves along the line BC.



**Fig. 1**

- (i) Show that immediately after release the acceleration of the block is  $\frac{2\lambda}{5m}$ . [4]
- (ii) Show that, when the block reaches C, its speed  $v$  is given by  $v^2 = \frac{\lambda b}{m}$ . [4]
- (iii) Show that the equation  $v^2 = \frac{\lambda b}{m}$  is dimensionally consistent. [3]

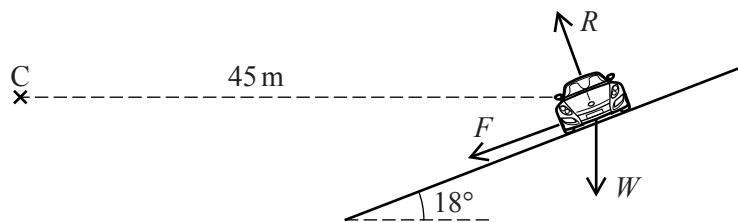
The time taken for the block to move from B to C is given by  $km^\alpha b^\beta \lambda^\gamma$ , where  $k$  is a dimensionless constant.

- (iv) Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ . [4]

When the string has natural length 1.2 m and modulus of elasticity 125 N, and the block has mass 8 kg, the time taken for the block to move from B to C is 0.718 s.

- (v) Find the time taken for the block to move from B to C when the string has natural length 9 m and modulus of elasticity 20 N, and the block has mass 75 kg. [3]

- 2 (a) Fig. 2 shows a car of mass 800 kg moving at constant speed in a horizontal circle with centre C and radius 45 m, on a road which is banked at an angle of  $18^\circ$  to the horizontal. The forces shown are the weight  $W$  of the car, the normal reaction,  $R$ , of the road on the car and the frictional force  $F$ .



**Fig. 2**

- (i) Given that the frictional force is zero, find the speed of the car. [4]
- (ii) Given instead that the speed of the car is  $15 \text{ m s}^{-1}$ , find the frictional force and the normal reaction. [7]
- (b) One end of a light inextensible string is attached to a fixed point O, and the other end is attached to a particle P of mass  $m$  kg. Starting with the string taut and P vertically below O, P is set in motion with a horizontal velocity of  $7 \text{ m s}^{-1}$ . It then moves in part of a vertical circle with centre O. The string becomes slack when the speed of P is  $2.8 \text{ m s}^{-1}$ .

Find the length of the string. Find also the angle that OP makes with the upward vertical at the instant when the string becomes slack. [7]

- 3 A particle Q is performing simple harmonic motion in a vertical line. Its height,  $x$  metres, above a fixed level at time  $t$  seconds is given by

$$x = c + A\cos(\omega t - \phi)$$

where  $c$ ,  $A$ ,  $\omega$  and  $\phi$  are constants.

- (i) Show that  $\ddot{x} = -\omega^2(x - c)$ .

[3]

Fig. 3 shows the displacement-time graph of Q for  $0 \leq t \leq 14$ .

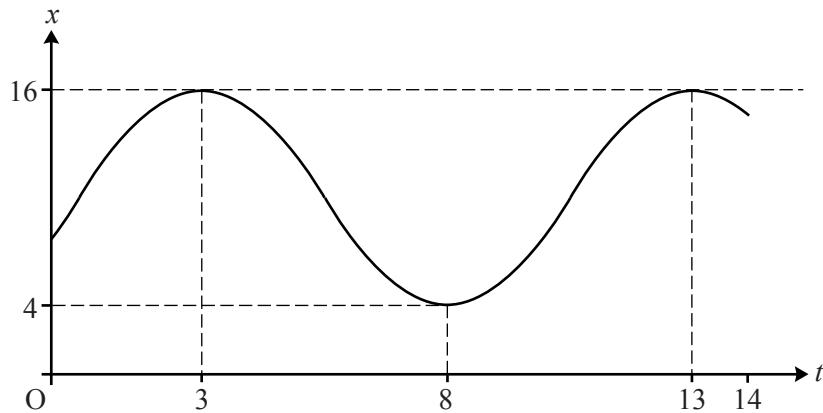


Fig. 3

- (ii) Find exact values for  $c$ ,  $A$ ,  $\omega$  and  $\phi$ .

[6]

- (iii) Find the maximum speed of Q.

[2]

- (iv) Find the height and the velocity of Q when  $t = 0$ .

[3]

- (v) Find the distance travelled by Q between  $t = 0$  and  $t = 14$ .

[4]

- 4 (a) A uniform lamina occupies the region bounded by the  $x$ -axis, the  $y$ -axis and the curve  $y = 3 - \sqrt{x}$  for  $0 \leq x \leq 9$ . Find the coordinates of the centre of mass of this lamina. [9]
- (b) Fig. 4.1 shows the region bounded by the line  $x = 2$  and the part of the circle  $y^2 = 25 - x^2$  for which  $2 \leq x \leq 5$ . This region is rotated about the  $x$ -axis to form a uniform solid of revolution  $S$ .

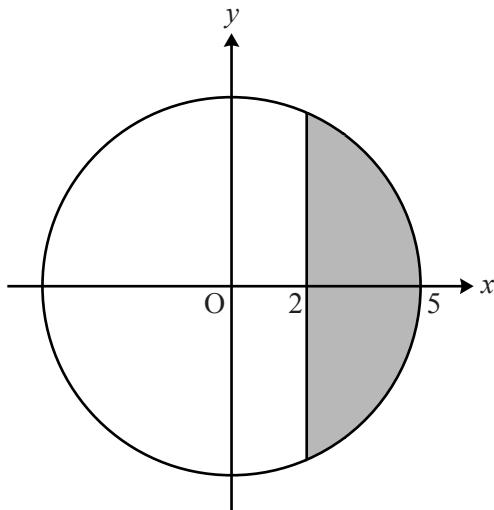


Fig. 4.1

- (i) Find the  $x$ -coordinate of the centre of mass of  $S$ .

[5]

The solid  $S$  rests in equilibrium with its curved surface in contact with a rough plane inclined at  $25^\circ$  to the horizontal. Fig. 4.2 shows a vertical section containing AB, which is a diameter and also a line of greatest slope of the flat surface of  $S$ . This section also contains XY, which is a line of greatest slope of the plane.

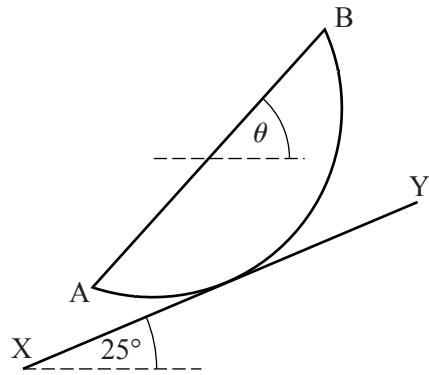


Fig. 4.2

- (ii) Find the angle  $\theta$  that AB makes with the horizontal.

[4]

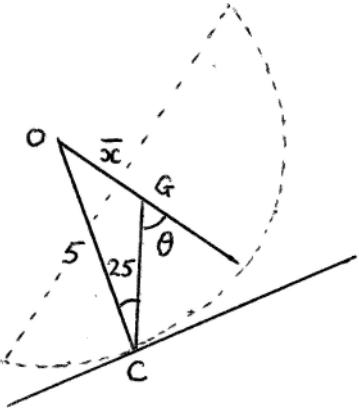
Question		Answer	Marks	Guidance
1	(i)	$AB = 5b, \text{ so } T = \frac{\lambda(2b)}{3b} \quad (= \frac{2}{3}\lambda)$ <p>Horizontal component is  <math>T \cos \theta = \frac{3}{5}T</math> where <math>\theta = \angle ABC</math></p> <p>Acceleration is <math>\frac{\frac{3}{5}(\frac{2}{3}\lambda)}{m} = \frac{2\lambda}{5m}</math></p>	M1 A1 B1 E1 [4]	For use of $\frac{\lambda x}{l}$ For $\frac{3}{5}T$ or $T \cos 53.1^\circ$ Allow use of $\cos 53.1^\circ$
1	(ii)	$\frac{1}{2}mv^2 = \frac{\lambda(2b)^2}{2(3b)} - \frac{\lambda b^2}{2(3b)}$ $\frac{1}{2}mv^2 = \frac{\lambda b}{2}$ $v^2 = \frac{\lambda b}{m}$	B1B1 M1 E1 [4]	For $\frac{\lambda(2b)^2}{2(3b)}$ and $\frac{\lambda b^2}{2(3b)}$ Equation involving KE and EE
1	(iii)	$[\lambda] = MLT^{-2}$ $[\text{RHS}] = \frac{(MLT^{-2})(L)}{M} = L^2 T^{-2}$ $[\text{LHS}] = (LT^{-1})^2$ , so it is dimensionally consistent	B1 M1 E1 [3]	Can be implied from part (iv) Obtaining dimensions of RHS Correctly shown Must see dimensions of $\lambda, b, m$ and simplification

Question		Answer	Marks	Guidance
1	(iv)	$T = M^\alpha L^\beta (MLT^{-2})^\gamma$ $\gamma = -\frac{1}{2}$ $\alpha + \gamma = 0, \beta + \gamma = 0$ $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$	B1 M1 A1A1 [4]	CAO Considering powers of M or L FT $\alpha = -\gamma, \beta = -\gamma$ (provided non-zero)
1	(v)	$0.718 = k(8)^{\frac{1}{2}}(0.4)^{\frac{1}{2}}(125)^{-\frac{1}{2}}$ $k = 4.4875$  $t = (4.4875)(75)^{\frac{1}{2}}(3)^{\frac{1}{2}}(20)^{-\frac{1}{2}}$  New time is 15.1 s (3 sf)	M1 M1 A1 [3]	Obtaining equation for $k$  Obtaining expression for new time  CAO <i>No penalty for using b=1.2 and b=9</i>
2	(a)	$R \cos 18^\circ = 800 \times 9.8 \quad (R = 8243)$  $R \sin 18^\circ = 800 \times \frac{v^2}{45}$  $\tan 18^\circ = \frac{v^2}{45 \times 9.8}$  Speed is $12.0 \text{ ms}^{-1}$ (3 sf)	M1 M1 A1 A1 [4]	Resolving vertically Horizontal equation of motion  Might also include $F$ Might also include $F$
2	(a)	$R \cos 18^\circ = F \sin 18^\circ + 800 \times 9.8$  $R \sin 18^\circ + F \cos 18^\circ = 800 \times \frac{15^2}{45}$  Frictional force is 1380 N (3 sf) Normal reaction is 8690 N (3 sf)	M1 A1 M1 A1 M1 A1 A1 [7]	Resolving vertically (three terms)  Horizontal equation (three terms)  Obtaining a value for $F$ or $R$  <i>Dependent on previous M1M1</i>

Question		Answer	Marks	Guidance
2	(b)	$\frac{1}{2}m(7^2 - 2.8^2) = mg(a + a \cos \theta)$ $a(1 + \cos \theta) = 2.1$ $mg \cos \theta = m \times \frac{2.8^2}{a}$ $a \cos \theta = 0.8$ Length of string is 1.3 m Angle with upward vertical is $52.0^\circ$ (3 sf)	M1 A1 M1 A1 M1 A1 A1 [7]	Equation involving KE and PE Correct equation involving $a$ and $\theta$ Radial equation of motion Correct equation involving $a$ and $\theta$ Eliminating $\theta$ or $a$ Dependent on previous M1M1 A0 for $128^\circ$ or $38^\circ$
3	(i)	$\dot{x} = -A\omega \sin(\omega t - \phi)$ $\ddot{x} = -A\omega^2 \cos(\omega t - \phi)$ $\ddot{x} = -\omega^2(x - c)$	B1 M1 E1 [3]	Allow one error
3	(ii)	$c = 10$ $A = 6$ $\frac{2\pi}{\omega} = 10$ $\omega = \frac{\pi}{5}$ $x = 16$ when $t = 3 \Rightarrow 3\omega - \phi = 0$ $\phi = \frac{3\pi}{5}$	B1 B1 M1 A1 M1 A1 [6]	Accept $A = -6$ Using $\frac{2\pi}{\omega}$ Accept $\omega = -\frac{\pi}{5}$ Obtaining simple relationship between $\phi$ and $\omega$ . NB $\phi = 3$ is M0 NB other values possible If exact values not seen, give A0A1 for both $\omega = 0.63$ and $\phi = 1.9$ Max 5/6 if values are not consistent

Question		Answer	Marks	Guidance	
3	(iii)	Maximum speed is $A\omega$ Maximum speed is $\frac{6\pi}{5}$ or 3.77 ms <sup>-1</sup> (3 sf)	M1  A1  [2]	Or e.g. evaluating $\dot{x}$ when $t = 5.5$ FT is $  A\omega  $ (must be positive)	
3	(iv)	When $t = 0$ , height is 8.15 m (3 sf) $v = -\frac{6\pi}{5} \sin(\frac{\pi t}{5} - \frac{3\pi}{5})$ When $t = 0$ , velocity is 3.59 ms <sup>-1</sup> (3 sf)	B1  M1  A1  [3]	FT is $c + A \cos \phi$ (provided $4 < x < 16$ ) Or $v^2 = \left(\frac{\pi}{5}\right)^2 (6^2 - 1.854^2)$ FT is $A\omega \sin \phi$ (must be positive)	Must use radians <i>Allow one error in differentiation</i> ( $\phi = 3$ gives $x = 4.06$ , $v = 0.532$ )
3	(v)	When $t = 0$ , $x = 8.146$ When $t = 14$ , $x = 14.854$ $(16 - 8.146) + 12 + 12 + (16 - 14.854)$ Distance is 33 m	M1  M1  M1  A1  [4]	Finding $x$ when $t = 14$ $(16 - 14.854)$ used Fully correct strategy CAO	Correct (FT) value, or evidence of substitution, required ( $\phi = 3$ gives $x = 15.3$ ) Requires $4 < x(14) < 16$ Also requires $4 < x(0) < 16$

Question		Answer	Marks	Guidance
4	(a)	$A = \int_0^9 (3 - \sqrt{x}) dx$ $= \left[ 3x - \frac{2}{3}x^{\frac{3}{2}} \right]_0^9 (= 9)$ $A\bar{x} = \int xy dx = \int_0^9 x(3 - \sqrt{x}) dx$ $= \left[ \frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} \right]_0^9 (= 24.3)$ $\bar{x} = \frac{24.3}{9} = 2.7$ $A\bar{y} = \int \frac{1}{2}y^2 dx = \int_0^9 \frac{1}{2}(3 - \sqrt{x})^2 dx$ $= \left[ \frac{9}{2}x - 2x^{\frac{3}{2}} + \frac{1}{4}x^2 \right]_0^9 (= 6.75)$ $\bar{y} = \frac{6.75}{9} = 0.75$	M1 A1 M1 A1 A1 M1 M1 A1 A1 [9]	For $3x - \frac{2}{3}x^{\frac{3}{2}}$ For $\int xy dx$ For $\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}}$ For $\int \dots y^2 dx$ Expanding (three terms) and integrating <i>(allow one error)</i> For $\frac{9}{2}x - 2x^{\frac{3}{2}} + \frac{1}{4}x^2$ Or $\int_{(0)}^{(3)} (3-y)^2 y dy$ Or $\frac{9}{2}y^2 - 2y^3 + \frac{1}{4}y^4$

Question			Answer	Marks	Guidance
4	(b)	(i)	$V = \int_2^5 \pi(25-x^2)dx$ $= \pi \left[ 25x - \frac{1}{3}x^3 \right]_2^5 (= 36\pi )$ $V \bar{x} = \int \pi xy^2 dx = \int_2^5 \pi x(25-x^2)dx$ $= \pi \left[ \frac{25}{2}x^2 - \frac{1}{4}x^4 \right]_2^5 (= \frac{441\pi}{4} )$ $\bar{x} = \frac{\frac{441\pi}{4}}{36\pi} = \frac{49}{16} (= 3.0625 )$	M1 A1 M1 A1 A1 [5]	For $\int \dots (25-x^2)dx$ For $25x - \frac{1}{3}x^3$ For $\int xy^2 dx$ For $\frac{25}{2}x^2 - \frac{1}{4}x^4$ Accept 3.1 from correct working
4	(b)	(ii)	 $\frac{\sin \theta}{5} = \frac{\sin 25^\circ}{\bar{x}}$ $\theta = 43.6^\circ$	M1 M1 M1 A1 [4]	CG is vertical (may be implied) Using triangle OGC or equivalent Accept art $43^\circ$ or $44^\circ$ from correct work FT is $\sin^{-1}\left(\frac{2.113}{\bar{x}}\right)$ Provided $2.113 < \bar{x} < 5$