ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

4763

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

2R–9J05

1 **(a)** Two light elastic strings, each having natural length 2.15 m and stiffness 70 N m^{-1}, are attached to a particle P of mass 4.8 kg. The other ends of the strings are attached to fixed points A and B, which are 1.4 m apart at the same horizontal level. The particle P is placed 2.4 m vertically below the midpoint of AB, as shown in Fig. 1.

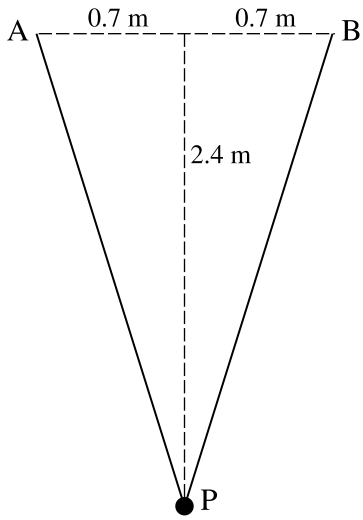

Fig. 1

(i) Show that P is in equilibrium in this position. **[6]**

(ii) Find the energy stored in the string AP. **[2]**

Starting in this equilibrium position, P is set in motion with initial velocity 3.5 m s^{-1} vertically upwards. You are given that P first comes to instantaneous rest at a point C where the strings are slack.

(iii) Find the vertical height of C above the initial position of P. **[4]**

(b) **(i)** Write down the dimensions of force and stiffness (of a spring). **[2]**

A particle of mass m is performing oscillations with amplitude a on the end of a spring with stiffness k. The maximum speed v of the particle is given by $v = cm^{\alpha}k^{\beta}a^{\gamma}$, where c is a dimensionless constant.

(ii) Use dimensional analysis to find α, β and γ. **[4]**

2 A hollow hemisphere has internal radius 2.5 m and is fixed with its rim horizontal and uppermost. The centre of the hemisphere is O. A small ball B of mass 0.4 kg moves in contact with the smooth inside surface of the hemisphere.

At first, B is moving at constant speed in a horizontal circle with radius 1.5 m, as shown in Fig. 2.1.

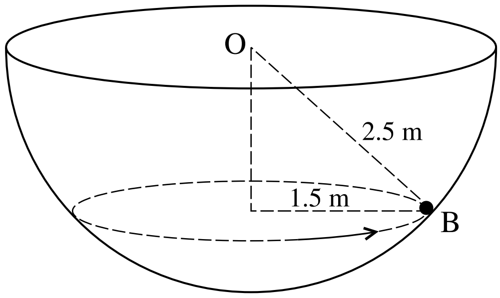

Fig. 2.1

(i) Find the normal reaction of the hemisphere on B. **[3]**

(ii) Find the speed of B. **[3]**

The ball B is now released from rest on the inside surface at a point on the same horizontal level as O. It then moves in part of a vertical circle with centre O and radius 2.5 m, as shown in Fig. 2.2.

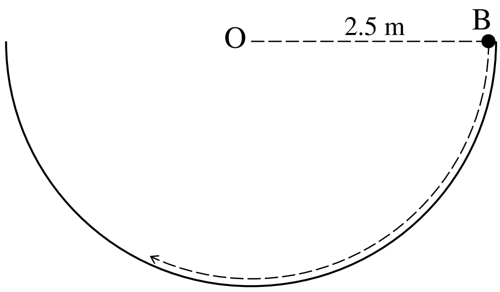

Fig. 2.2

(iii) Show that, when B is at its lowest point, the normal reaction is three times the weight of B. **[4]**

For an instant when the normal reaction is twice the weight of B, find

(iv) the speed of B, **[5]**

(v) the tangential component of the acceleration of B. **[3]**

3 In this question, give your answers in an exact form.

The region R_1 (shown in Fig. 3) is bounded by the x-axis, the lines $x = 1$ and $x = 5$, and the curve $y = \frac{1}{x}$ for $1 \leqslant x \leqslant 5$.

(i) A uniform solid of revolution is formed by rotating the region R_1 through 2π radians about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[5]**

(ii) Find the coordinates of the centre of mass of a uniform lamina occupying the region R_1. **[7]**

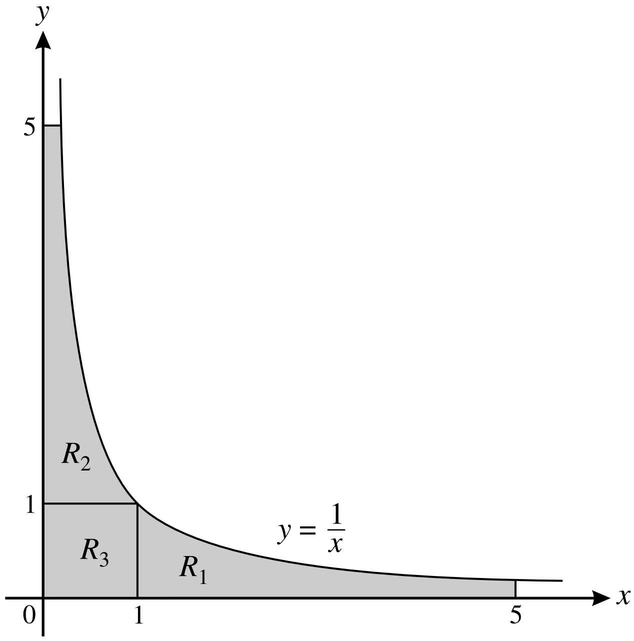

Fig. 3

The region R_2 is bounded by the y-axis, the lines $y = 1$ and $y = 5$, and the curve $y = \frac{1}{x}$ for $\frac{1}{5} \leqslant x \leqslant 1$. The region R_3 is the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1).

(iii) Write down the coordinates of the centre of mass of a uniform lamina occupying the region R_2. **[2]**

(iv) Find the coordinates of the centre of mass of a uniform lamina occupying the region consisting of R_1, R_2 and R_3 (shown shaded in Fig. 3). **[4]**

4 A particle P is performing simple harmonic motion in a vertical line. At time t s, its displacement x m above a fixed point O is given by

$$x = A \sin \omega t + B \cos \omega t$$

where A, B and ω are constants.

(i) Show that the acceleration of P, in m s^{-2}, is $-\omega^2 x$. **[3]**

When $t = 0$, P is 16 m *below* O, moving with velocity 7.5 m s^{-1} *upwards*, and has acceleration 1 m s^{-2} *upwards*.

(ii) Find the values of A, B and ω. **[4]**

(iii) Find the maximum displacement, the maximum speed, and the maximum acceleration of P. **[5]**

(iv) Find the speed and the direction of motion of P when $t = 15$. **[2]**

(v) Find the distance travelled by P between $t = 0$ and $t = 15$. **[4]**

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GCE

Mathematics (MEI)

Advanced GCE **4763**

Mechanics 3

Mark Scheme for June 2010

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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1(a)(i)	$\text{AP} = \sqrt{2.4^2 + 0.7^2} = 2.5$ Tension $T = 70 \times 0.35 \quad (= 24.5)$ Resultant vertical force on P is $2T\cos\theta - mg$ $= 2 \times 24.5 \times \frac{2.4}{2.5} - 4.8 \times 9.8$ $= 47.04 - 47.04 = 0$ Hence P is in equilibrium	M1 A1 M1 B1 B1 E1	**6**	 Attempting to resolve vertically For $T \times \frac{2.4}{2.5}$ (or $T\cos 16.3°$ *etc*) For 4.8×9.8 Correctly shown
(ii)	$\text{EE} = \frac{1}{2} \times 70 \times 0.35^2$ Elastic energy is 4.2875 J	M1 A1	**2**	(M0 for $\frac{1}{2} \times 70 \times 0.35$) *Note* If 70 is used as modulus instead of stiffness: (i) M1A0M1B1B1E0 (ii) M1 A1 for 1.99
(iii)	Initial $\text{KE} = \frac{1}{2} \times 4.8 \times 3.5^2$ By conservation of energy $4.8 \times 9.8h = 2 \times 4.2875 + \frac{1}{2} \times 4.8 \times 3.5^2$ $47.04h = 8.575 + 29.4$ Height is 0.807 m (3 sf)	B1 M1 F1 A1	**4**	 Equation involving EE, KE and PE (A0 for 0.8) ft is $\frac{2 \times \text{(ii)} + 29.4}{47.04}$
(b)(i)	$[\text{ Force }] = \text{M L T}^{-2}$ $[\text{ Stiffness }] = \text{M T}^{-2}$	B1 B1	**2**	*Deduct 1 mark if units are used*
(ii)	$\text{L T}^{-1} = \text{M}^{\alpha}(\text{M T}^{-2})^{\beta}\text{L}^{\gamma}$ $\gamma = 1$ $\beta = \frac{1}{2}$ $0 = \alpha + \beta$ $\alpha = -\frac{1}{2}$	 B1 B1 M1 A1	**4**	 Considering powers of M *When [Stiffness] is wrong in (i), allow all marks ft provided the work is comparable and answers are non-zero*

2 (i)	$R\cos\theta = mg$ [θ is angle between OB and vertical] $R\times 0.8 = 0.4\times 9.8$ Normal reaction is 4.9 N	M1 A1 A1 **3**	Resolving vertically
(ii)	$R\sin\theta = m\dfrac{v^2}{r}$ $4.9\times 0.6 = 0.4\times\dfrac{v^2}{1.5}$ $v^2 = 11.025$ Speed is $3.32\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 A1 **3**	For acceleration $\dfrac{v^2}{r}$ or $r\omega^2$ or $4.9\times 0.6 = 0.4\times 1.5\omega^2$ ft is $1.5\sqrt{R}$
(iii)	By conservation of energy $\frac{1}{2}mu^2 = mg\times 2.5$ $u^2 = 5g$ ($u = 7$) $R - mg = m\times\dfrac{u^2}{2.5}$ $R - mg = 2mg$ $R = 3mg$	M1 A1 M1 E1 **4**	Equation involving KE and PE Vertical equation of motion (must have three terms) Correctly shown or $R = 11.76$ and $3\times 0.4\times 9.8 = 11.76$
(iv) **(v)**	$\frac{1}{2}mv^2 = mg\times 2.5\cos\theta$ $v^2 = 5g\cos\theta$ $R - mg\cos\theta = m\times\dfrac{v^2}{2.5}$ When $R = 2mg$ ($= 7.84$), $2mg - mg\cos\theta = \dfrac{mv^2}{2.5}$ $2mg - \dfrac{mv^2}{5} = \dfrac{mv^2}{2.5}$ $7.84 - 0.08v^2 = 0.16v^2$ $v^2 = \dfrac{98}{3}$ Speed is $5.72\ \mathrm{m\,s^{-1}}$ (3 sf) $\cos\theta = \dfrac{v^2}{5g} = \dfrac{2}{3}$ ($\theta = 48.2°$ or 0.841 rad) Tangential acceleration is $g\sin\theta$ Tangential acceleration is $7.30\ \mathrm{m\,s^{-2}}$ (3 sf)	M1 A1 M1 M1 M1 A1 M1 A1 **8**	*Mark (iv) and (v) as one part* Equation involving KE, PE and an angle (θ is angle with vertical) [$\frac{1}{2}mv^2 = mgh$ can earn M1A1, but only if $\cos\theta = h/2.5$ appears somewhere] Equation of motion towards O (must have three terms, and the weight must be resolved) Obtaining an equation for v Obtaining an equation for θ *These two marks are each dependent on M1M1 above* [$g\sin\theta$ in isolation only earns M1 if the angle θ is clearly indicated]

3 (i)	Volume is $\int_1^5 \pi\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	π may be omitted throughout *Limits not required*
	$= \pi\left[-\frac{1}{x} \right]_1^5 \quad (= \frac{4}{5}\pi)$	A1	For $-\frac{1}{x}$
	$\int \pi x y^2 \,\mathrm{d}x = \int_1^5 \pi x\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	*Limits not required*
	$= \pi\left[\ln x \right]_1^5 \quad (= \pi \ln 5)$	A1	For $\ln x$
	$\bar{x} = \frac{\pi \ln 5}{\frac{4}{5}\pi} = \frac{5\ln 5}{4}$ *(2.012)*	A1 **5**	*SR* If exact answers are not seen, deduct only the first A1 affected
(ii)	Area is $\int_1^5 \frac{1}{x}\mathrm{d}x$	M1	*Limits not required*
	$= \left[\ln x \right]_1^5 \quad (= \ln 5)$	A1	For $\ln x$
	$\int x y \,\mathrm{d}x = \int_1^5 x\left(\frac{1}{x}\right)\mathrm{d}x \quad \left(= \left[x \right]_1^5 = 4 \right)$	M1	*Limits not required*
	$\bar{x} = \frac{4}{\ln 5}$ *(2.485)*	A1	
	$\int \frac{1}{2} y^2 \,\mathrm{d}x = \int_1^5 \frac{1}{2}\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	For $\int \left(\frac{1}{x}\right)^2 \mathrm{d}x$
	$= \left[-\frac{1}{2x} \right]_1^5 \quad (= \frac{2}{5})$	A1	For $-\frac{1}{2x}$
	$\bar{y} = \frac{2/5}{\ln 5} = \frac{2}{5\ln 5}$ *(0.2485)*	A1 **7**	
(iii)	CM of R_2 is $\left(\frac{2}{5\ln 5}, \frac{4}{\ln 5} \right)$	B1B1 ft **2**	*Do not penalise inexact answers in this part*
(iv)		B1	For CM of R_3 is $(\frac{1}{2}, \frac{1}{2})$ (one coordinate is sufficient)
	$\bar{x} = \frac{(\ln 5)\left(\frac{4}{\ln 5}\right) + (\ln 5)\left(\frac{2}{5\ln 5}\right) + (1)\left(\frac{1}{2}\right)}{\ln 5 + \ln 5 + 1}$	M1	Using $\sum mx$ with three terms
		M1	Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum
	CM is $\left(\frac{4.9}{2\ln 5 + 1}, \frac{4.9}{2\ln 5 + 1} \right)$ *(1.161 , 1.161)*	A1 cao **4**	

4 (i)	$v = \frac{\mathrm{d}x}{\mathrm{d}t} = A\omega\cos\omega t - B\omega\sin\omega t$ $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 M1 E1 **3**	 Finding the second derivative Correctly shown
(ii)	$B = -16$ $\omega = 0.25$ $A = 30$	B1 B1 B2 **4**	When A is wrong, give B1 for a correct equation involving A [e.g. $A\omega = 7.5$ or $7.5^2 = \omega^2(A^2 + B^2 - 16^2)$] or for $A = -30$
(iii)	Maximum displacement is $(\pm)\sqrt{A^2 + B^2}$ Maximum displacement is 34 m Maximum speed is $(\pm)\ 34\omega$ Maximum acceleration is $(\pm)\ 34\omega^2$ Maximum speed is $8.5\ \mathrm{m\,s^{-1}}$ Maximum acceleration is $2.125\ \mathrm{m\,s^{-2}}$	M1 A1 M1 F1 F1 **5**	Or $7.5^2 = \omega^2(\mathrm{amp}^2 - 16^2)$ Or finding t when $v = 0$ and substituting to find x For either (any valid method) Only ft from $\omega \times$ amp Only ft from $\omega^2 \times$ amp
(iv)	$v = 7.5\cos 0.25t + 4\sin 0.25t$ When $t = 15$, $v = 7.5\cos 3.75 + 4\sin 3.75$ $= -8.44$ Speed is $8.44\ \mathrm{m\,s^{-1}}$ (3 sf); downwards	M1 A1 **2**	
(v)	Period $\frac{2\pi}{\omega} \approx 25$ s, so $t = 0$ to $t = 15$ is less than one period When $t = 15$, $x = 30\sin 3.75 - 16\cos 3.75$ $= -4.02$ Distance travelled is $16 + 34 + 34 + 4.02$ Distance travelled is 88.0 m (3 sf)	M1 M1 M1 A1 cao **4**	 Take account of change of direction Fully correct strategy for distance

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4763 Mechanics 3

General comments

This paper was found to be relatively straightforward; there was a lot of very competent work and the marks were generally high. About half the candidates scored 60 marks or more (out of 72), and very few scored less than 30. Nevertheless, the questions on circular motion and simple harmonic motion caused problems for a substantial proportion of the candidates.

Comments on individual questions

1) *(Elasticity and dimensional analysis)*
This question was very well answered and the average mark was about 16 (out of 18). Approximately 40% of the candidates scored full marks.

(a)(i) This was well understood, and most candidates were able to show that the system is in equilibrium.

(a)(ii) The elastic energy stored in the string was almost always calculated correctly.

(a)(iii) The use of conservation of energy was well understood and only a few failed to include all of gravitational, elastic and kinetic energy in their equation. Sign errors were also quite rare, although a substantial number (perhaps 40% of the candidates) forgot to double the elastic energy found in part (ii) to account for the two strings. There were some who simply found the point at which the strings just became slack.

(b)(i) The dimensions were usually given correctly, although there were some errors with stiffness.

(b)(ii) The method for finding the powers was very well known, and it was almost always carried out accurately.

2) *(Circular motion)*
This was found to be the most difficult question, but even so about 20% of the candidates scored full marks. The average mark was about 12.
Many candidates used an angle θ in their work without indicating (on their answer script) which angle this was and this often led to confusion and errors. In these comments, θ is the angle between OB and the vertical.

(i) Most candidates found the normal reaction correctly, but a fairly common error was to resolve in the direction of OB ($R = mg \cos \theta$) instead of vertically ($R \cos \theta = mg$).

(ii) The use of v^2 / r for the acceleration towards the centre was well understood, although a few used $r = 2.5$ instead of $r = 1.5$.

(iii) This was answered quite well using conservation of energy and considering the vertical forces at the lowest point. Most candidates worked numerically, but a few produced elegant algebraic solutions.

(iv) This was the most challenging item on the question paper. Candidates needed to obtain two equations involving *v* and θ, and solve them simultaneously. The first equation, from forces towards the centre ($2mg - mg\cos\theta = mv^2 / r$), was usually correct, although failure to resolve the weight properly was a fairly common error. However, the second equation, from conservation of energy ($\frac{1}{2}mv^2 = mgr\cos\theta$), was very often omitted or incorrect. Many candidates thought they could just resolve vertically ($2mg\cos\theta = mg$) and obtained $\theta = 60°$.

(v) A good proportion of the candidates knew that the tangential acceleration was $g\sin\theta$ (although quite a few gave it as $mg\sin\theta$), but most had not been able to find the value of θ.

3) *(Centres of mass)*
This was the best answered question, with an average mark of about 16. Approximately 45% of the candidates scored full marks, and most of the errors were minor slips such as failing to give answers in an exact form.

(i) The method for finding the centre of mass of a solid of revolution was well understood and most candidates carried it out accurately.

(ii) Most candidates found the centre of mass of the lamina correctly. Only a few made errors in evaluating the definite integrals, but a factor of ½ was sometimes omitted from the *y*-coordinate.

(iii) The great majority of candidates realised that they could simply interchange the coordinates of the centre of mass of R_1. A few did not use the symmetry, and worked it all out from scratch.

(iv) Finding the centre of mass of the combined lamina was generally done well, although some just added up the coordinates of the three parts and divided by three.

4) *(Simple harmonic motion)*
The average mark on this question was about 14, and about 25% of the candidates scored full marks.

(i) Almost every candidate proved the given result satisfactorily.

(ii) Most candidates understood how to find *B* and ω, although $B = 16$ (instead of $B = -16$) and $\omega = 4$ (instead of $\omega = 0.25$) were fairly common errors. Many used complicated methods to find *A*, such as $v^2 = \omega^2(A^2 + B^2 - x^2)$, rather than simply differentiating to find *v* and substituting $t = 0$.

(iii) Many answered this efficiently, calculating $a = \sqrt{A^2 + B^2}$, ωa and $\omega^2 a$ to find the maximum displacement, speed and acceleration. More complicated (but correct) methods were very often used, such as finding the value of *t* when $v = 0$ to calculate the maximum displacement.

(iv) Almost all candidates differentiated and substituted $t = 15$, although there were some slips in the differentiation, and a large number had their calculators in degree mode.

(v) A fair proportion answered this correctly, often supporting their argument with a clear displacement-time graph. However, the majority did not consider carefully enough how P had moved and changed direction. Again the use of degree mode prevented many from obtaining the correct answer.