

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4763

Mechanics 3

Monday

22 MAY 2006

Morning

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

- 1 (a) (i) Find the dimensions of power. [3]

In a particle accelerator operating at power P , a charged sphere of radius r and density ρ has its speed increased from u to $2u$ over a distance x . A student derives the formula

$$x = \frac{28\pi r^3 u^2 \rho}{9P}.$$

- (ii) Show that this formula is not dimensionally consistent. [5]
- (iii) Given that there is only one error in this formula for x , obtain the correct formula. [3]
- (b) A light elastic string, with natural length 1.6 m and stiffness 150 N m^{-1} , is stretched between fixed points A and B which are 2.4 m apart on a smooth horizontal surface.
- (i) Find the energy stored in the string. [2]

A particle is attached to the mid-point of the string. The particle is given a horizontal velocity of 10 m s^{-1} perpendicular to AB (see Fig. 1.1), and it comes instantaneously to rest after travelling a distance of 0.9 m (see Fig. 1.2).

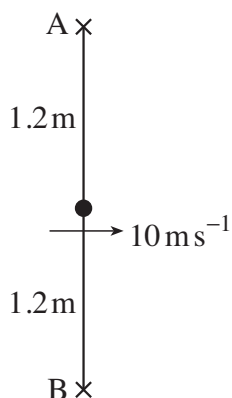


Fig. 1.1

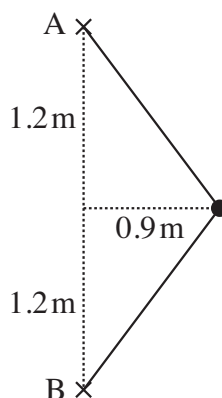


Fig. 1.2

- (ii) Find the mass of the particle. [5]

- 2 (a) A particle P of mass 0.6 kg is connected to a fixed point by a light inextensible string of length 2.8 m. The particle P moves in a horizontal circle as a conical pendulum, with the string making a constant angle of 55° with the vertical.
- (i) Find the tension in the string. [2]
- (ii) Find the speed of P. [4]
- (b) A turntable has a rough horizontal surface, and it can rotate about a vertical axis through its centre O. While the turntable is stationary, a small object Q of mass 0.5 kg is placed on the turntable at a distance of 1.4 m from O. The turntable then begins to rotate, with a constant angular acceleration of 1.12 rad s^{-2} . Let $\omega \text{ rad s}^{-1}$ be the angular speed of the turntable.

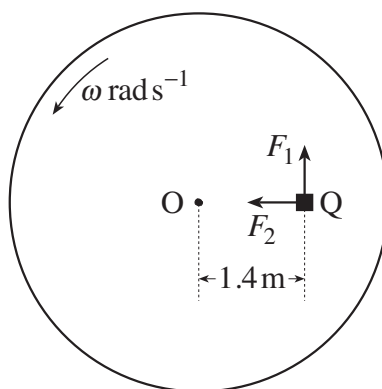


Fig. 2

- (i) Given that Q does not slip, find the components F_1 and F_2 of the frictional force acting on Q perpendicular and parallel to QO (see Fig. 2). Give your answers in terms of ω where appropriate. [4]

The coefficient of friction between Q and the turntable is 0.65.

- (ii) Find the value of ω when Q is about to slip. [5]
- (iii) Find the angle which the frictional force makes with QO when Q is about to slip. [3]

- 3 A fixed point A is 12m vertically above a fixed point B. A light elastic string, with natural length 3 m and modulus of elasticity 1323 N, has one end attached to A and the other end attached to a particle P of mass 15 kg. Another light elastic string, with natural length 4.5 m and modulus of elasticity 1323N, has one end attached to B and the other end attached to P.

(i) Verify that, in the equilibrium position, $AP = 5$ m. [3]

The particle P now moves vertically, with both strings AP and BP remaining taut throughout the motion. The displacement of P above the equilibrium position is denoted by x m (see Fig. 3).

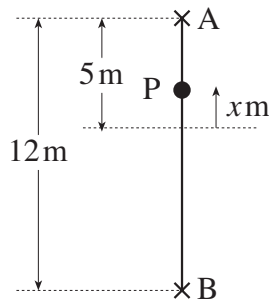


Fig. 3

(ii) Show that the tension in the string AP is $441(2-x)$ N and find the tension in the string BP. [3]

(iii) Show that the motion of P is simple harmonic, and state the period. [4]

The minimum length of AP during the motion is 3.5 m.

(iv) Find the maximum length of AP. [1]

(v) Find the speed of P when $AP = 4.1$ m. [3]

(vi) Find the time taken for AP to increase from 3.5 m to 4.5 m. [4]

- 4 The region bounded by the curve $y = \sqrt{x}$, the x -axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x -axis to form a uniform solid of revolution.

(i) Find the x -coordinate of the centre of mass of this solid. [6]

From this solid, the cylinder with radius 1 and length 3 with its axis along the x -axis (from $x = 1$ to $x = 4$) is removed.

(ii) Show that the centre of mass of the remaining object, Q, has x -coordinate 3. [5]

This object Q has weight 96 N and it is supported, with its axis of symmetry horizontal, by a string passing through the cylindrical hole and attached to fixed points A and B (see Fig. 4). AB is horizontal and the sections of the string attached to A and B are vertical. There is sufficient friction to prevent slipping.

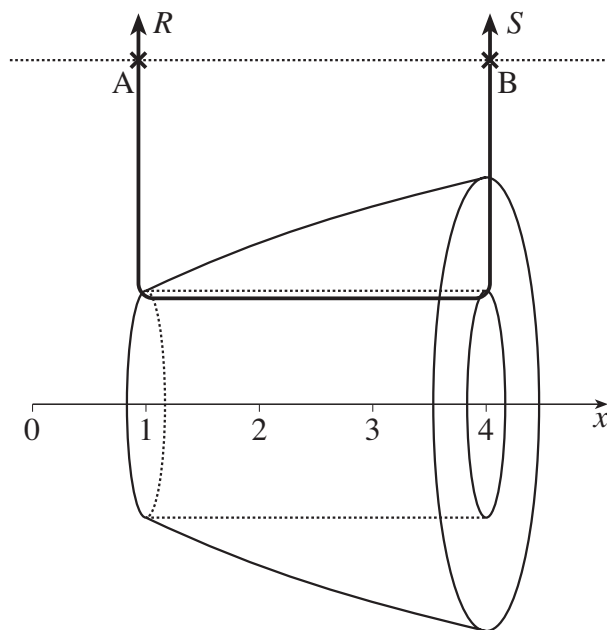


Fig. 4

(iii) Find the support forces, R and S , acting on the string at A and B

(A) when the string is light, [4]

(B) when the string is heavy and uniform with a total weight of 6 N. [3]

1(a)(i)	$[\text{Force}] = \text{MLT}^{-2}$ $[\text{Power}] = [\text{Force}] \times [\text{Distance}] \div [\text{Time}]$ $= [\text{Force}] \times \text{LT}^{-1}$ $= \text{ML}^2 \text{T}^{-3}$	B1 M1 A1 3	or $[\text{Energy}] = \text{ML}^2 \text{T}^{-2}$ or $[\text{Energy}] \times \text{T}^{-1}$
(ii)	$[\text{RHS}] = \frac{(\text{L})^3 (\text{LT}^{-1})^2 (\text{ML}^{-3})}{\text{ML}^2 \text{T}^{-3}}$ $= \text{T}$ $[\text{LHS}] = \text{L}$ so equation is not consistent	B1B1 M1 A1 E1 5	For $(\text{LT}^{-1})^2$ and (ML^{-3}) Simplifying dimensions of RHS With all working correct (cao) SR '... $\text{L} = \frac{28}{9} \pi \text{T}$, so inconsistent' can earn B1B1M1A1E0
(iii)	$[\text{RHS}]$ needs to be multiplied by LT^{-1} which are the dimensions of u Correct formula is $x = \frac{28 \pi r^3 u^3 \rho}{9P}$	M1 A1 A1 cao 3	RHS must appear correctly
	OR $x = k r^\alpha u^\beta \rho^\gamma P^\delta$ $\beta = 3$ $x = \frac{28 \pi r^3 u^3 \rho}{9P}$	M1 A1 A1	Equating powers of one dimension
(b)(i)	Elastic energy is $\frac{1}{2} \times 150 \times 0.8^2$ $= 48 \text{ J}$	M1 A1 2	<i>Treat use of modulus</i> $\lambda = 150 \text{ N as MR}$
(ii)	In extreme position, length of string is $2\sqrt{1.2^2 + 0.9^2}$ ($= 3$) elastic energy is $\frac{1}{2} \times 150 \times 1.4^2$ ($= 147$) By conservation of energy, $147 - 48 = \frac{1}{2} \times m \times 10^2$ Mass is 1.98 kg	B1 M1 M1 A1 A1 5	for $\sqrt{1.2^2 + 0.9^2}$ or 1.5 or 3 allow M1 for $(2 \times) \frac{1}{2} \times 150 \times 0.7^2$ Equation involving EE and KE

2 (a)(i)	Vertically, $T \cos 55^\circ = 0.6 \times 9.8$ Tension is 10.25 N	M1 A1 2	
(ii)	Radius of circle is $r = 2.8 \sin 55^\circ$ ($= 2.294$)	B1	
	Towards centre, $T \sin 55^\circ = 0.6 \times \frac{v^2}{2.8 \sin 55^\circ}$	M2	Give M1 for one error
	OR $T \sin 55^\circ = 0.6 \times (2.8 \sin 55^\circ) \times \omega^2$ $\omega = 2.47$ $v = (2.8 \sin 55^\circ) \omega$	M1 M1	or $T = 0.6 \times 2.8 \times \omega^2$ <i>Dependent on previous M1</i>
	Speed is 5.67 ms^{-1}	A1 4	
(b)(i)	Tangential acceleration is $r \alpha = 1.4 \times 1.12$ $F_1 = 0.5 \times 1.4 \times 1.12$ $= 0.784 \text{ N}$ Radial acceleration is $r \omega^2 = 1.4 \omega^2$ $F_2 = 0.5 \times 1.4 \omega^2$ $= 0.7 \omega^2 \text{ N}$	M1 A1 M1 A1 4	SR $F_1 = -0.784$, $F_2 = -0.7 \omega^2$ <i>penalise once only</i>
(ii)	Friction $F = \sqrt{F_1^2 + F_2^2}$ Normal reaction $R = 0.5 \times 9.8$ About to slip when $F = \mu \times 0.5 \times 9.8$ $\sqrt{0.784^2 + 0.49 \omega^4} = 0.65 \times 0.5 \times 9.8$ $\omega = 2.1$	M1 M1 A1 A1 A1 cao 5	For LHS and RHS <i>Both dependent on M1M1</i>
(iii)	$\tan \theta = \frac{F_1}{F_2}$ $= \frac{0.784}{0.7 \times 2.1^2}$ Angle is 14.25°	M1 A1 A1 3	Allow M1 for $\tan \theta = \frac{F_2}{F_1}$ etc Accept 0.249 rad

3 (i)	$T_{AP} = \frac{1323}{3} \times 2 \quad (= 882)$ $T_{BP} = \frac{1323}{4.5} \times 2.5 \quad (= 735)$ $T_{AP} - mg - T_{BP} = 882 - 15 \times 9.8 - 735 = 0$ <p>so P is in equilibrium</p>	B1 B1 E1 3	
	<p>OR $\frac{1323}{3}(AP - 3) = \frac{1323}{4.5}(BP - 4.5) + 15 \times 9.8$</p> $AP + BP = 12 \text{ and solving, } AP = 5$	B2 E1	<p>Give B1 for one tension correct</p>
(ii)	<p>Extension of AP is $5 - x - 3 = 2 - x$</p> $T_{AP} = \frac{1323}{3}(2 - x) = 441(2 - x)$ <p>Extension of BP is $7 + x - 4.5 = 2.5 + x$</p> $T_{BP} = \frac{1323}{4.5}(2.5 + x) = 294(2.5 + x)$	E1 B1 B1 3	
(iii)	$441(2 - x) - 15 \times 9.8 - 294(2.5 + x) = 15 \frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -49x$ <p>Motion is SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.898 \text{ s}$</p>	M1 A1 M1 A1 4	<p>Equation of motion involving 3 forces</p> <p>Obtaining $\frac{d^2x}{dt^2} = -\omega^2 x \text{ (+c)}$</p> <p>Accept $\frac{2}{7}\pi$</p>
(iv)	<p>Centre of motion is $AP = 5$</p> <p>If minimum value of AP is 3.5, amplitude is 1.5</p> <p>Maximum value of AP is 6.5 m</p>	B1 1	
(v)	<p>When $AP = 4.1$, $x = 0.9$</p> <p>Using $v^2 = \omega^2(A^2 - x^2)$</p> $v^2 = 49(1.5^2 - 0.9^2)$ <p>Speed is 8.4 ms^{-1}</p> <p>OR $x = 1.5 \sin 7t$</p> <p>When $x = 0.9$, $7t = 0.6435$ ($t = 0.0919$)</p> $v = 7 \times 1.5 \cos 7t$ $= 10.5 \cos(0.6435)$ $= 8.4$	M1 A1 A1 3 M1 A1 A1	<p>Accept ± 8.4 or -8.4</p> <p>or $x = 1.5 \cos 7t$</p> <p>or $7t = 0.9273$ ($t = 0.1325$)</p> <p>or $v = -7 \times 1.5 \sin 7t$</p> $= (-) 10.5 \sin(0.9273)$

(vi)	$x = 1.5 \cos 7t$ When $1.5 \cos 7t = 0.5$ Time taken is 0.176 s	M1	For $\cos(\sqrt{49}t)$ or $\sin(\sqrt{49}t)$
		A1	or $x = 1.5 \sin 7t$ <i>M1A1 above can be awarded in (v) if not earned in (vi)</i>
		M1	or other fully correct method to find the required time e.g. $0.400 - 0.224$ or $0.224 - 0.049$
		A1	4 Accept 0.17 or 0.18

4 (i)	$\int \pi y^2 dx = \int_1^4 \pi x dx$ $= \left[\frac{1}{2} \pi x^2 \right]_1^4 = 7.5\pi$ $\int \pi x y^2 dx$ $= \int_1^4 \pi x^2 dx = \left[\frac{1}{3} \pi x^3 \right]_1^4 (= 21\pi)$ $\bar{x} = \frac{21\pi}{7.5\pi}$ $= 2.8$	M1 A1 M1 A1 M1 A1 6	π may be omitted throughout
(ii)	Cylinder has mass $3\pi\rho$ Cylinder has CM at $x = 2.5$ $(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$ $\bar{x} = 3$	B1 B1 M1 A1 E1 5	Or volume 3π Relating three CMs (ρ and / or π may be omitted) or equivalent, e.g. $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$ Correctly obtained
(iii)(A)	Moments about A, $S \times 3 - 96 \times 2 = 0$ $S = 64 \text{ N}$ Vertically, $R + S = 96$ $R = 32 \text{ N}$	M1 A1 M1 A1 4	Moments equation or another moments equation <i>Dependent on previous M1</i>
(B)	Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$ Vertically, $R + S = 96 + 6$ $R = 35 \text{ N}, S = 67 \text{ N}$	M1 A1 A1 3	Moments equation Both correct
	OR Add 3 N to each of R and S $R = 35 \text{ N}, S = 67 \text{ N}$	M1 A2	<i>Provided</i> $R \neq S$ Both correct