A 56
B 48
C 41
D 34
E 27

OCR
Oxford Cambridge and RSA

Wednesday 17 May 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Fig. 1.1 shows the masses and speeds of two small uniform circular discs, A and B, sliding towards one another on a smooth horizontal surface. Each of the discs is moving with its centre on the line shown in the figure; there is a barrier which is perpendicular to this line. The discs collide and separate.

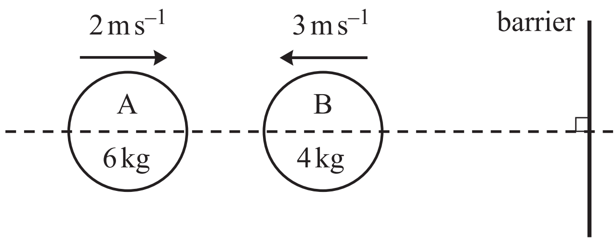


Fig. 1.1

(i) By considering only linear momentum, explain how you know that the direction of motion of each disc must be reversed in the collision. **[2]**

You are now given that the coefficient of restitution is e.

(ii) Show that after the collision A has speed $2e$. Find an expression in terms of e for the speed of B after the collision. **[5]**

(iii) Find an expression in terms of e for the magnitude of the impulse in the collision. **[1]**

Three seconds after its collision with A, disc B has a perfectly elastic direct collision with the barrier shown in Fig. 1.1 and later collides again with A.

(iv) What time will elapse between the two collisions of the discs? **[3]**

In a different situation, B has an oblique impact with a smooth barrier. This barrier is inclined at an angle α to the direction of motion of B. The coefficient of restitution in this collision is $\frac{1}{3}$ and the direction of motion of B is turned through 90° as a result of the collision, as shown in Fig. 1.2.

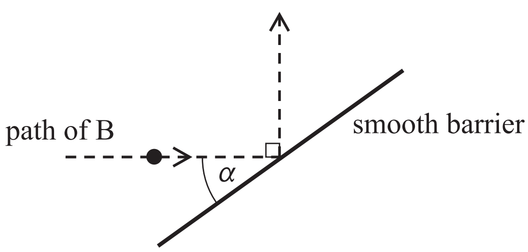


Fig. 1.2

(v) Calculate α, giving your answer in degrees. **[5]**

2 **In this question take $g = 10$.**

Fig. 2 shows a small object Q, of mass 5 kg, which slides in a straight line up a rough ramp. The point D is at the top of the ramp, where the ramp is joined to a horizontal platform. The point C on the ramp is a vertical distance of 2 m below D. CD is inclined at an angle α to the horizontal.

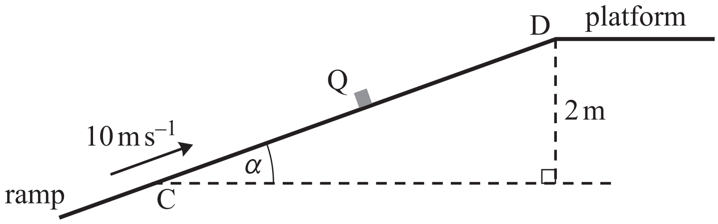


Fig. 2

Q is struck so that it moves up a line of greatest slope of the ramp. Q passes through C with a speed of $10\,\mathrm{m\,s^{-1}}$ and comes to rest at D.

The motion of Q is first modelled by assuming that the only resistance to its motion is friction with the plane.

(i) Without assuming that the ramp is uniformly rough, calculate the work done by the frictional force as Q travels from C to D. **[3]**

Now assume that the coefficient of friction between the object and the plane has the constant value $\frac{5}{8}$.

(ii) Show that the work done by the frictional force acting on Q as it travels from C to D may be expressed as $\frac{125}{2\tan\alpha}$.

Calculate $\tan\alpha$. **[7]**

A new ramp is built that is inclined at an angle greater than α to the horizontal. This also ends at D. Q is now struck so that it moves up a line of greatest slope of the new ramp. As in the previous situation, it passes through a point at a vertical level 2 m below D at a speed of $10\,\mathrm{m\,s^{-1}}$. The coefficient of friction between Q and the new ramp is the same as that between Q and the old ramp.

(iii) Does Q now come to rest below D or still come to rest at D or is Q still moving at D?
You should explain your answer but you need not produce detailed calculations. **[2]**

Subsequently Q is moving on the horizontal platform and is made to travel in a straight line by a force which has a constant power of 50 W. The resistance to the motion of Q is F N, where F is constant. The velocity and acceleration of Q at time t s are $v\,\mathrm{m\,s^{-1}}$ and $a\,\mathrm{m\,s^{-2}}$ in the direction of its motion.

(iv) Write down the equation of motion of Q in terms of F, v and a. **[3]**

(v) Given that Q is travelling at a constant speed of $4\,\mathrm{m\,s^{-1}}$, calculate F. **[1]**

(vi) Show that a can only be constant if it is zero.

Q changes speed from $U\,\mathrm{m\,s^{-1}}$ to $V\,\mathrm{m\,s^{-1}}$ in T s. Explain why it would not be appropriate to calculate the distance travelled in this time as $\frac{1}{2}T(U+V)$. **[3]**

3 **(a)** Fig. 3.1 shows a framework JKL in a vertical plane. The framework is made from three light rigid rods JK, KL and LJ which are freely pin-jointed to each other at J, K and L. The pin-joint at J is attached to a fixed vertical wall; the pin-joint at L is in contact with a fixed smooth horizontal beam. JK is 5 m, KL is 3 m and LJ is 4 m. Angle KLJ is 90°.

The framework is held in equilibrium with JK horizontal by means of a single applied force of 80 N acting at K parallel to JL. Fig. 3.1 shows this force and the angle α between JL and JK. Fig. 3.1 also shows the horizontal component, X N, of the force on the framework due to J being attached to the wall. Note that the diagram does not show the vertical component of the force acting at J nor the force on the framework at L due to contact with the beam.

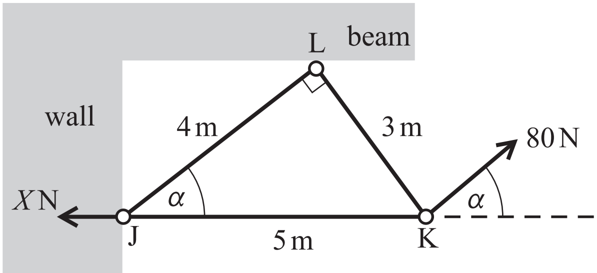


Fig. 3.1

Show that $X = 64$.

By first considering the equilibrium of the pin-joint at K, or otherwise, calculate the forces internal to the rods JK, KL and LJ, stating whether each rod is in tension or thrust (compression). **[8]**

(b) Fig. 3.2 shows a uniform heavy ladder AB, of weight W N, standing on rough horizontal ground and resting on a smooth peg at C. The ladder has length 5 m. C is 3 m above the ground and is a horizontal distance of 1.25 m from A. The ladder is in equilibrium.

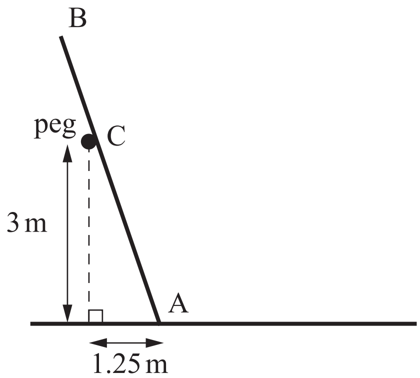


Fig. 3.2

Show that the force exerted on the ladder by the peg at C is $\dfrac{50W}{169}$.

Calculate the range of possible values of the coefficient of friction between the ladder and the ground. **[12]**

4 **(a)** In this part question, all coordinates refer to the axes shown in Fig. 4.1.

Fig. 4.1 shows a uniform rectangular lamina OABC with OA and OC on the x- and y-coordinate axes. The units of the axes are metres.

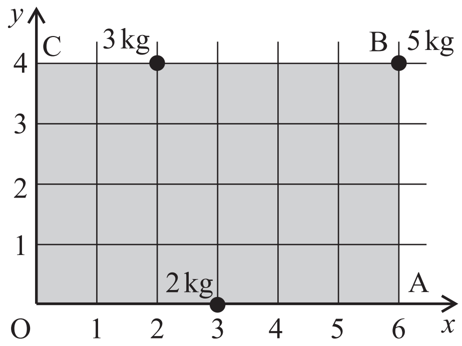


Fig. 4.1

The mass of the lamina is 2 kg and 3 heavy particles are fixed to it. The masses of these particles are 2 kg, 3 kg and 5 kg and they are at points with coordinates (3, 0), (2, 4) and (6, 4), respectively.

(i) Calculate the coordinates of the combined centre of mass of the lamina and the 3 particles. **[3]**

A particle of mass m kg is now fixed to the lamina at a point $(X, 0)$, with $0 < X \leqslant 6$, so that the combined centre of mass of the original 3 particles, the lamina and the mass m kg lies on OB (part of the line with equation $y = \frac{2}{3}x$).

(ii) Establish that $mX = 6$. **[4]**

(b) Fig. 4.2 shows a thin heavy uniform wire ABC bent into the shape of a semi-circle with centre O and radius r. More of the same wire, DE, has its centre at O and lies on the straight line AOC. The distance DE is kr, where $k \geqslant 2$ is a constant. The two wires are joined at A and C to form object P.

You may use without proof the information that the centre of mass of the curved semi-circular section of the wire, ABC, lies on the line OB at a distance $\frac{2r}{\pi}$ from O.

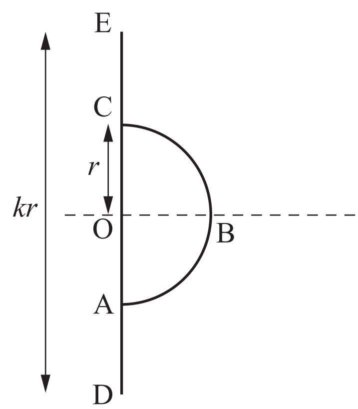


Fig. 4.2

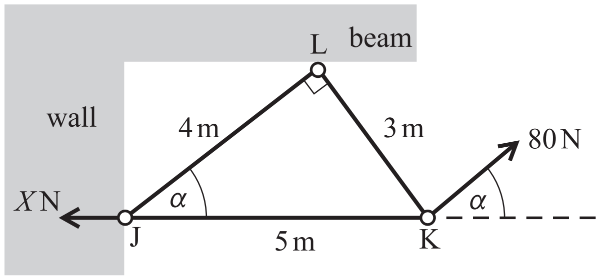
(i) Show that the centre of mass of P lies on OB at a distance $\frac{2r}{\pi + k}$ from O. **[4]**

Two light inelastic strings are attached to P, one at E and the other at B. Both strings are vertical and P is in equilibrium with DE vertical. The tension in the string attached at E is T_E and the tension in the string attached at B is T_B.

(ii) Find the value of k for which $T_E = 2T_B$. **[6]**

END OF QUESTION PAPER

A spare copy of Fig. 3.1 can be found on page 16



3 (a)

(answer space continued on next page)

Question			Answer	Marks	Guidance
1		(i)	The total LM of 0 is conserved Cannot both be in same direction (nor in original direction)	M1 A1 **[2]**	Both 'LM = 0' and 'conserved' stated or implied in the answer, e.g. $6 \times 2 - 4 \times 3 = 0$ and 'conserved' Accept "They must be going in opposite directions"
		(ii)	$\rightarrow$+ve and 'after' velocities $\leftarrow v_A$ and $\rightarrow v_B$ PCLM $6 \times 2 + 4 \times -3 = -6v_A + 4v_B$ so $3v_A = 2v_B$ NEL $\dfrac{v_B - (-v_A)}{-3-2} = -e$ so $v_B + v_A = 5e$ Solving, $v_A = 2e$ and $v_B = 3e$	 M1 A1 M1 A1 A1 **[5]**	 Use of PCLM. Accept LHS as 0 without comment oe Use of NEL. Must be attempt at separation/approach (right way up) oe (consistent with PCLM signs) cao both, speeds not velocities required (v_A is AG)
		(iii)	**either** Using A: $\leftarrow$ $6(2e + 2) = 12(e + 1)$	 B1	 cao oe
			or Using B: $\rightarrow$ $4(3e + 3) = 12(e + 1)$	 B1	 cao oe
				[1]	
		(iv)	**either** ▪Speed of B unchanged by impacting barrier; ▪B returns to its original position after 6 s; ▪in this time A has moved 12*e*; ▪relative speed after 6 s is *e*. Time taken is 6 s + time to catch up (12 s) so 18 s	 B1ft B1ft B1	 2 of these 4 statements made or implied, ft speed of B from (ii) all of these 4 statements made or implied, ft speed of B from (ii) cao
			or ▪Speed of B unchanged by impacting barrier; ▪B travels distance 9e to barrier; ▪A is now 15*e* from barrier ▪relative speed after 6 s is *e*. Time taken is 3 s + time to catch up (15 s) so 18 s	 B1ft B1ft B1	 2 of these 4 statements made or implied, ft speed of B from (ii) all of these 4 statements made or implied, ft speed of B from (ii) cao

Question			Answer	Marks	Guidance
				[3]	
		(v)	Vel of B before impact is v at α to barrier; vel of B after impact is v^1 at β to barrier $\beta = 90 - \alpha$	B1	Allow any specified value of v
			either Use $\tan\beta = e\tan\alpha$	M1	May be quoted without proof
			$\tan\beta = \frac{1}{3}\tan\alpha$	A1	
			or $v\cos\alpha = v^1\cos\beta$ and $v^1\sin\beta = \frac{1}{3}v\sin\alpha$	M1	Award for either statement seen
			$\tan\beta = \frac{1}{3}\tan\alpha$	A1	
			or $v_x = u_x$ and $v_y = \frac{1}{3}u_y$	M1	Both statements needed or seen on diagram
			$\tan\beta = \frac{v_y}{v_x} = \frac{1}{3}\frac{u_y}{u_x} = \frac{1}{3}\tan\alpha$	A1	
			then so $\frac{1}{\tan\alpha} = \frac{1}{3}\tan\alpha$	A1	Any form, in terms of $\tan\alpha$ only
			so $\tan^2\alpha = 3$ and $\tan\alpha = \sqrt{3}$ (+ve root) and $\alpha = 60°$	A1 **[5]**	cao

Question			Answer	Marks	Guidance	
2		(i)	$0-\frac{1}{2}\times5\times10^2=-5\times10\times2-\text{WD}$	M1	M1 use of W-E equation with WD, KE and GPE. Allow sign errors	
				A1	Any form. Allow sign error in WD term only	
			So WD = 150. Work done is 150 J	A1 **[3]**	−150 gets 2/3 (Allow 152 from g = 9.8)	
		(ii)	Say Q moves d up ramp and friction is F WD = Fd	M1	used	
			$d=\dfrac{2}{\sin\alpha}$ so WD = $\dfrac{2F}{\sin\alpha}$	A1		
			(Since sliding) $F=\mu R$	M1	used	
			Resolving perp to ramp $R=5\times10\cos\alpha$	B1	$5g\cos\alpha$	
			so $F=50\times\frac{5}{8}\cos\alpha=\frac{125}{4}\cos\alpha$			
			Hence WD is $\dfrac{2}{\sin\alpha}\times\dfrac{125}{4}\cos\alpha=\dfrac{125}{2\tan\alpha}$	A1	AG Properly shown	
			so 150 = $\dfrac{125}{2\tan\alpha}$	M1	Equate WD to 150. FT(i)	
			and $\tan\alpha=\dfrac{125}{2\times150}=\dfrac{5}{12}$ (0.417 to 3 s. f.)	A1 **[7]**	cao aef NOT implied by 22.6	
		(iii)	**either** Suppose α is greater, $\cos\alpha$ is less so $F=50\times\frac{5}{8}\cos\alpha$ is less			
			also $\sin\alpha$ is greater so $d=\dfrac{2}{\sin\alpha}$ is less	B1	Award for either F or d stated to be less OR still moving at D	
			Hence Fd = WD is less. GPE the same so Q			

Question			Answer	Marks	Guidance	
			is still moving	B1	Award for both F and d stated to be less AND still moving at D	
			or From (ii) use WD = $\frac{125}{2\tan\alpha}$ If α is greater, $\tan\alpha$ is greater Hence WD is less. GPE the same so Q is still moving	B1 B1	Award for WD stated to be less OR still moving at D Award for WD stated to be less,with reference to established result, AND still moving at D	
				[2]		
		(iv)	Using N2L $\frac{50}{v} - F = 5a$	M1 B1 A1 **[3]**	Use of N2L All terms present ($D - F = 5a$ is sufficient) Use of *Power* = *Driving force* × *speed* Any form	
		(v)	Put $a = 0$ and $v = 4$, $F = 12.5$.	B1 **[1]**		
		(vi)	a and v are the only things in the equation that can change so they are both constant or both vary If $a \neq 0$, v must vary so a must vary. Hence if a is constant it must be zero (giving constant v) *suvat* equations only valid for constant accn	B1 B1 B1 **[3]**	F is constant OR a and v are the only variables Must be convincing	

Question			Answer	Marks	Guidance
3	(a)				In this part, award the best possible mark for marks 3 to 8. M1A1 for a correct resolution and force (even if not the first one given) M1F1 for second resolution and force M1 for third resolution A1 All forces correct and all T/C correct
			$\sin\alpha = 0.6;\ \cos\alpha = 0.8$	B1	Either of these
			Horizontal equilibrium of whole framework: $80\cos\alpha - X = 0$ so $X = 64$	B1	
			Take the internal forces to be +ve in tension		Any convention may be used
			At K↑ $80\sin\alpha + T_{KL}\cos\alpha = 0$	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix
			so $T_{KL} = -60$ a force in KL of 60 N (C)	A1	
			either At K resolve parallel to JL: $80 - T_{JK}\cos\alpha = 0$	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix
			or At K→ $80\cos\alpha - T_{KL}\sin\alpha - T_{JK} = 0$	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix
			so $T_{JK} = 100$ a force in JK of 100 N (T)	F1	FT their value for KL
			either At J→ $T_{JK} + T_{LJ}\cos\alpha - X = 0$	M1	
			so $T_{LJ} = -45$ a force in LJ of 45 N (C)	A1	All values correct and all T/C correct
			or At L→ $T_{KL}\sin\alpha - T_{LJ}\cos\alpha = 0$	M1	
			so $T_{LJ} = -45$ a force in LJ of 45 N (C)	A1	All values correct and all T/C correct
				[8]	

Question			Answer	Marks	Guidance
	(b)		Take: weight of ladder as W N; angle of ladder with horiz as α. Forces: at A, R N ↑, F N ←; at C, S N perp to AB		
			$\tan\alpha = \frac{3}{1.25} = \frac{12}{5}$; $\sin\alpha = \frac{12}{13}$; $\cos\alpha = \frac{5}{13}$	B1	Award for any of these seen. oe for different angle chosen (Award for 67.4)
			ac moments about A $2.5W\cos\alpha - 3.25S = 0$ (3) $S = \frac{50W}{169}$	M1 B1 A1	Moments with all terms present The distance AC is 3.25 (may be implied) Shown: A0 if 67.4 stated with no evidence of 12/5 o.e.
			In equilib; consider vert and horiz cpts ↑ $R + S\cos\alpha - W = 0$ (1) ← $F - S\sin\alpha = 0$ (2)	M1 A1 M1 A1	Award for either. Allow cos/sin mix, allow sign errors oe (R = 0.886W or 1947/2197 W) Second resolution. Allow cos/sin mix, allow sign errors oe (F = 0.273W or 600/2197 W)
			We have $\mu \geq \frac{F}{R}$ Substitute for sinα and cosα and for F and R from (1) and (2). Eliminate S.	M1	Used and attempt to get all in terms of W (Accept '='at this stage)
			so $\mu \geq \dfrac{\frac{50W}{169} \times \frac{12}{13}}{W - \frac{50W}{169} \times \frac{5}{13}}$	F1	Correct ft, dependent on all of previous 3 M marks, all in terms of W
			so $\mu \geq \frac{600W}{(169 \times 13 - 250)W} = \frac{200}{649}$	A1	This value is 0.3081664…: 600/1947

Question			Answer	Marks	Guidance	
			Hence $\mu \geq 0.308$ (3 s. f.)	A1 **[12]**	cao as final answer. Must have > as well as =.	

Question			Answer	Marks	Guidance
4	(a)	(i)	$(2+2+3+5)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=2\begin{pmatrix}3\\2\end{pmatrix}+2\begin{pmatrix}3\\0\end{pmatrix}+3\begin{pmatrix}2\\4\end{pmatrix}+5\begin{pmatrix}6\\4\end{pmatrix}$	M1	Complete method for CoM Using 24 instead of 2 for mass of lamina is NOT a MR: award M1 max. Omitting mass of lamina, M1max. Allow one error
			so $12\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}48\\36\end{pmatrix}$		
			and the CoM is at (4, 3)	A1 A1 **[3]**	One coordinate cao Other coordinate cao
		(ii)	Require		
			$\begin{pmatrix}48\\36\end{pmatrix}+m\begin{pmatrix}X\\0\end{pmatrix}=(12+m)\begin{pmatrix}p\\ \frac{2p}{3}\end{pmatrix}$ for some p	M1	General method for attempt to get new CoM on OB
				B1	Dealing with finding a point on the line OB: using $y=\frac{2}{3}x$ in some way
			Hence $48+mX=(12+m)p$ and $36=(12+m)\frac{2p}{3}$	M1	Consider the components and attempt to eliminate, or equivalent
			Eliminate $(12+m)p$ to get $36=\frac{2}{3}(48+mX)$ so $mX=6$	A1 **[4]**	Convincingly done (must follow from correct (48, 36))
	(b)	(i)	Let the line density of the wire be λ.		Do not penalise for stating $\lambda = 1$ or assuming this without comment
			$\bar{y}=0$, by considering symmetry	B1	Need a statement including symmetry (or calculation for this coordinate)
			DE has mass $kr\lambda$, CoM at (0, 0). Semi-circle has mass $\pi r\lambda$, CoM at $\left(\frac{2r}{\pi},\ 0\right)$.	B1	Both masses correct (but see above, accept kr and πr)

Question			Answer	Marks	Guidance
			$(kr\lambda+\pi r\lambda)\bar{x}=kr\lambda\times 0+\pi r\lambda\times\frac{2r}{\pi}$	M1	Method for CoM. Accept 1st term on RHS not present
			so $(k+\pi)\bar{x}=2r$ and $\bar{x}=\frac{2r}{k+\pi}$	A1	Must be convincing
				[4]	
		(ii)	Suppose the weight of P is W and CoM at G **Method A** Take c.w moments about G		Condone use of length of wire or of mass instead of weight in correct ratios
			$T_E\times\frac{2r}{\pi+k}-T_B\times\left(r-\frac{2r}{\pi+k}\right)=0$	M2	Use of moments with all appropriate forces
				A1	A correct moments equation
			Substitute $T_E=2T_B$ giving	M1	Substitute and attempt elimination
			$\frac{4r}{\pi+k}=r-\frac{2r}{\pi+k}$	A1	o.e.
			so $4r=r(\pi+k)-2r$		
			so $k=6-\pi$ (2.86 to 3 s. f.)	A1	cao
			Method B		
			a.c moments about O: $W\times\frac{2r}{\pi+k}-T_B\times r=0$	M1 A1	Use of moments with all appropriate forces A correct moments equation. (Allow $\pi r+kr$ as W)
			so $T_B=\frac{2W}{\pi+k}$		
			Using vertical components: $T_B+T_E=W$	M1	Or take moments about a second point, for example B
			so $T_E=W-\frac{2W}{\pi+k}$		

Question			Answer	Marks	Guidance	
			$= \dfrac{W(\pi + k - 2)}{\pi + k}$			
			$\dfrac{T_E}{T_B} = 2 = \dfrac{\frac{W(\pi + k - 2)}{\pi + k}}{\frac{2W}{\pi + k}}$	M1	Using their expressions for tensions	
			so $4 = \pi + k - 2$	A1	Some simplification seen	
			so $k = 6 - \pi$ (2.86 to 3 s. f.)	A1	cao	
			Method C			
			T_E:T_B = GB:OG	M2		
			GB = $r - \dfrac{2r}{\pi + k} = \left(\dfrac{r(\pi + k - 2)}{\pi + k}\right)$	A1		
			$\dfrac{T_E}{T_B} = 2 = \dfrac{\frac{W(\pi + k - 2)}{\pi + k}}{\frac{2W}{\pi + k}}$	M1	Using their expressions for lengths	
			so $4 = \pi + k - 2$	A1	Some simplification seen	
			so $k = 6 - \pi$ (2.86 to 3 s. f.)	A1	cao	
				[6]		