



Wednesday 18 May 2016 - Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \, \text{m} \, \text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail
 of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

 Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document. 1 (a) Two model railway trucks are moving freely on a straight horizontal track when they are in a direct collision.

The trucks are P of mass $0.5 \, \text{kg}$ and Q of mass $0.75 \, \text{kg}$. They are initially travelling in the same direction. Just before they collide P has a speed of $4 \, \text{m s}^{-1}$ and Q has a speed of $1 \, \text{m s}^{-1}$, as shown in Fig. 1.1.

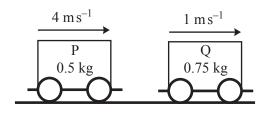


Fig. 1.1

- (i) Suppose that the speed of P is halved in the collision and that its direction of motion is not changed. Find the speed of Q immediately after the collision and find the coefficient of restitution. [5]
- (ii) Show that it is not possible for both the speed of P to be halved in the collision and its direction of motion to be reversed. [3]

Both of the model trucks have flat horizontal tops. They are each travelling at the speeds they had immediately after the collision.

Part of the mass of Q is a small object of mass 0.1 kg at rest at the edge of the top of Q nearest P. The object falls off, initially with negligible velocity relative to Q.

(iii) Determine the speed of Q immediately after the object falls off it, making your reasoning clear.

Part of the mass of P is an object of mass $0.05\,\mathrm{kg}$ that is fired horizontally from the top of P, parallel to and in the opposite direction to the motion of P. Immediately after the object is fired, it has a speed of $10\,\mathrm{m\,s^{-1}}$ relative to P.

- (iv) Determine the speed of P immediately after the object has been fired from it. [4]
- **(b)** The velocities of a small object immediately before and after an elastic collision with a horizontal plane are shown in Fig. 1.2.

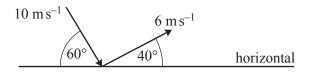


Fig. 1.2

Show that the plane cannot be smooth.

[3]

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2 (a) A bullet of mass 0.04 kg is fired into a fixed uniform rectangular block along a line through the centres of opposite parallel faces, as shown in Fig. 2.1.

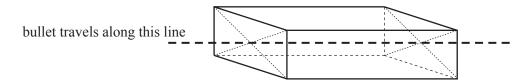


Fig. 2.1

The bullet enters the block at 50 m s⁻¹ and comes to rest after travelling 0.2 m into the block.

(i) Calculate the resistive force on the bullet, assuming that this force is constant. [3]

Another bullet of the same mass is fired, as before, with the same speed into a similar block of mass 3.96 kg. The block is initially at rest and is free to slide on a smooth horizontal plane.

- (ii) By considering linear momentum, find the speed of the block with the bullet embedded in it and at rest relative to the block. [2]
- (iii) By considering mechanical energy, find the distance the bullet penetrates the block, given the resistance of the block to the motion of the bullet is the same as in part (i). [4]
- (b) Fig. 2.2 shows a block of mass 6 kg on a uniformly rough plane that is inclined at 30° to the horizontal.

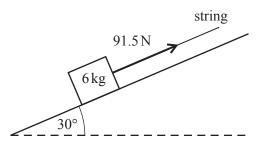


Fig. 2.2

A string with a constant tension of $91.5\,\mathrm{N}$ parallel to the plane pulls the block up a line of greatest slope. The speed of the block increases from $1\,\mathrm{m\,s^{-1}}$ to $7\,\mathrm{m\,s^{-1}}$ over a distance of $8\,\mathrm{m}$.

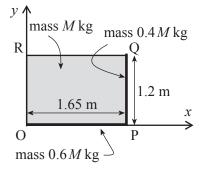
(i) Use an energy method to find the magnitude of the frictional force acting on the block.

Calculate the coefficient of friction between the block and the plane.

[8]

(ii) Calculate the power of the tension in the string when the block has a speed of $7 \,\mathrm{m \, s^{-1}}$. [2]

Fig. 3.1 shows a thin planar uniform rigid rectangular sheet of metal, OPQR, of width 1.65 m and height 1.2 m. The mass of the sheet is *M* kg. The sides OP and PQ have thin rigid uniform reinforcements attached with masses 0.6*M* kg and 0.4*M* kg, respectively. Fig. 3.1 also shows coordinate axes with origin at O.



The sheet with its reinforcements is to be used as an inn sign.

(i) Calculate the coordinates of the centre of mass of the inn sign. [4]

Fig. 3.1

The inn sign has a weight of 300 N. It hangs in equilibrium with QR horizontal when vertical forces Y_Q N and Y_R N act at Q and R respectively.

(ii) Calculate the value of
$$Y_0$$
 and show that $Y_R = 120$. [3]

The inn sign is hung from a framework, ABCD, by means of two light vertical inextensible wires attached to the sign at Q and R and the framework at B and C, as shown in Fig. 3.2. QR and BC are horizontal. The framework is made from light rigid rods AB, BC, CA and CD freely pin-jointed together at A, B and C and to a vertical wall at A and D. Fig. 3.3 shows the dimensions of the framework in metres as well as the external forces X_A N, Y_A N acting at A and X_D N, Y_D N acting at D.

You are given that $\sin \alpha = \frac{5}{13}$, $\cos \alpha = \frac{12}{13}$, $\sin \beta = \frac{4}{5}$ and $\cos \beta = \frac{3}{5}$.

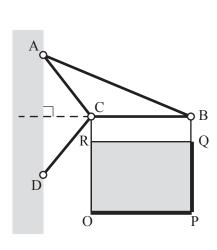


Fig. 3.2

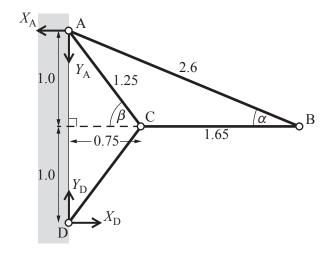


Fig. 3.3

(iii) Mark on the diagram in your Printed Answer Book all the forces acting on the pin-joints at A, B, C and D, including those internal to the rods, when the inn sign is hanging from the framework.

(iv) Show that
$$X_D = 261$$
. [2]

(v) Calculate the forces internal to the rods AB, BC and CD, stating whether each rod is in tension or thrust (compression). Calculate also the values of Y_D and Y_A. [Your working in this part should correspond to your diagram in part (iii).]
 [8]

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4 Fig. 4.1 shows a hollow circular cylinder open at one end and closed at the other. The radius of the cylinder is 0.1 m and its height is h m. O and C are points on the axis of symmetry at the centres of the open and closed ends, respectively. The thin material used for the closed end has four times the density of the thin material used for the curved surface.

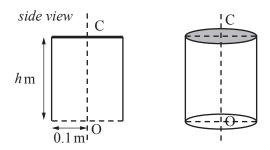


Fig. 4.1

Cylinders of this type are made with different values of h.

(ii) Show that $50h^2 + 5h - 3 = 0$ and hence that h = 0.2.

(i) Show that the centres of mass of these cylinders are on the line OC at a distance $\frac{5h^2 + 2h}{2 + 10h}$ m from O.

[7]

Fig. 4.2 shows one of the cylinders placed with its open end on a slope inclined at an angle α to the horizontal, where $\tan \alpha = \frac{2}{3}$. The cylinder does not slip but is on the point of tipping.

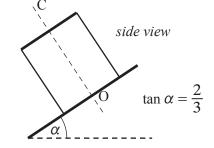


Fig. 4.2

Fig. 4.3 shows another of the cylinders that has weight 42 N and h = 0.5. This cylinder has its open end on a rough horizontal plane. A force of magnitude T N is applied to a point P on the circumference of the closed end. This force is at an angle β with the horizontal such that $\tan \beta = \frac{3}{4}$ and the force is in the vertical plane containing O, C and P. The cylinder does not slip but is on the point of tipping.

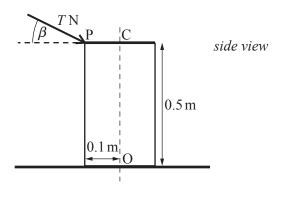


Fig. 4.3

(iii) Calculate T. [5]

END OF QUESTION PAPER

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3 (iii)	A spare copy of this diagram can be found on page 16
	X_{A} 1.0 Y_{A} 1.25 0.0 1.0 Y_{D} X_{D} X_{D} X_{D}
3 (iv)	

Q	Question		Answer	Marks	Guidance		
1	(a)	(i)	Take + ve → PCLM				
			$0.5 \times 4 + 0.75 \times 1 = 0.5 \times 2 + 0.75 \times v_{Q}$	M1 A1	Application of PCLM. Allow sign errors Correct. Any form		
			so $v_{Q} = \frac{7}{3}$	A1	Exact or anything that rounds to 2.33 or better		
			NEL $\frac{\frac{7}{3} - 2}{1 - 4} = -e$	M1	NEL. Accept sign errors but not approach/separation		
			so $e = \frac{1}{9}$ (or 0.11)	A1ft [5]	ft their v_Q		
		(ii)	Suppose direction reversed. Given LM conserved $0.5 \times 4 + 0.75 \times 1 = -0.5 \times 2 + 0.75 \times v_Q$ so $v_Q = 5$	B1	Award for the correct LM equation		
			NEL gives $\frac{5+2}{1-4} = -e \text{ so } e = \frac{7}{3}$	M1	Using their re-calculated v_Q (not equal to 7/3 from (i))		
			e > 1 so not an elastic collision	E1 [3]	www		
			OR for last two marks: KE after collision = 10.375, before collision = 4.375:	M1			
			Increase in energy not possible, because no work put into system	E1			

Q	uestic	n	Answer	Marks	Guidance	
		(iii)	No (external horizontal) force acts on the truck	B1	Force or momentum considered or correct momentum equation	
			so no change in momentum of truck (less object)			
			(So no change in velocity. Still) $\frac{7}{3}$ m s ⁻¹ .	B1	FT their value from (i): seen	
				[2]		
		(iv)	Before 0.5 kg at 2 m s ⁻¹ \rightarrow After 0.05 kg at U m s ⁻¹ \leftarrow and 0.45 kg at V m s ⁻¹ \rightarrow PCLM	M1	Allow if $(10-2) = 8$ used instead of U	
			$0.5 \times 2 = -0.05 \times U + 0.45 \times V$	M1	Allow only sign errors	
			U+V=10	B1	oe: relative velocity used correctly	
			Solving, $V = 3 \text{ so } 3 \text{ m s}^{-1}$	A1 [4]	cao	
					SC1 Using $U = 10$, giving $V = 10/3$	
	(b)		Consider the LM parallel to the plane	M1	Accept considering the horizontal components of velocity	
					before and after o.e. and arguing/stating they should be the same	
			Before: $m \times 10\cos 60 = 5m$ After: $m \times 6\cos 40 \approx 4.6$ m	A1	Need not include m . Using sine gets $0/3$	
			Not the same. (LM not conserved.) Plane cannot be smooth.	E1 [3]	Accept arguments from velocity	

C	Questic	on	Answer	Marks	Guidance
2	(a)	(i)	WD against resistance = KE lost	M1	
			$\frac{1}{2} \times 0.04 \times 50^2 = 0.2F$	A1	
			so $F = 250$ and resistive force is 250 N	A1	Accept -250
			OR:		
			Use suvat: $v^2 = u^2 + 2as$: $a = -\frac{2500}{0.4}a = -\frac{2500}{0.4}$ Use N2L: $F = 0.04 \times a = -250 F = 0.04 \times a = -250$ Resistive force = 250 N	M1 A1 A1	Complete method: suvat and N2L Correct a Correct F
		(**)	DCLM	[3]	
		(ii)	PCLM $0.04 \times 50 = (3.96 + 0.04)V$	M1	
			so $V = 0.5$ so 0.5 m s^{-1}	A1 [2]	cao
		(iii)	Energy lost is		
			$\frac{1}{2} \times 0.04 \times 50^2 - \frac{1}{2} \times 4 \times 0.5^2$	M1	Correct masses
			= 49.5 J equating WD against resistance to energy lost	A1ft	ft their 0.5 from (ii). May be implied
			250x = 49.5	M1	ft their 250 $x = a$ difference in non-zero KEs
			so $x = 0.198$ and distance is 0.198 m	A1 [4]	cao

Quest	ion	Answer	Marks	Guidance
(b)	(i)	Using W-E equation. Friction is F N		
		$\frac{1}{2} \times 6 \times (7^2 - 1^2) = (91.5 - F - 6g \sin 30) \times 8$	M1	o.e. All 5 terms present, no extras. Allow sign errors
		so $F = 44.1$	B1 B1 M1 A1	KE terms (both) Resolved weight or GPE term WD is Force × distance cao
		(As slipping) $F = \mu R$	M1	Used
		$R = 6g\cos 30^\circ = 6g \times \frac{\sqrt{3}}{2}$	B1	Does not need to be evaluated
		$R = 6g\cos 30^{\circ} = 6g \times \frac{\sqrt{3}}{2}$ so $\mu = \frac{44.1}{3g \times \sqrt{3}} = \frac{1.5}{\sqrt{3}} = \frac{\sqrt{3}}{2}$ (0.8660)	A1	cao Any form. 0.87 or better
			[8]	Using suvat and N2L: Max possible is B1 for resolved weight, then last 3 marks. Award SC(4) if N2L and suvat used, and μ correct, www
	(ii)	Power is $T \times v$ so $91.5 \times 7 = 640.5 \text{ W}$	M1 A1 [2]	cao accept 640 or 641. Must be their final answer

	Question		Answer	Marks	Guidance	
3		(i)	$2M\left(\frac{\overline{x}}{\overline{y}}\right) = M\left(\frac{0.825}{0.6}\right) + 0.6M\left(\frac{0.825}{0}\right) + 0.4M\left(\frac{1.65}{0.6}\right)$	M1	Complete method	
				A1	At least 2 RHS vector terms or 3 component terms correct	
			$\overline{x} = 0.99$ $\overline{y} = 0.42$	A1 A1		
			9 0.12	[4]		
		(ii)	c.w moments about R $300 \times 0.99 - 1.65 \times Y_0 = 0$	M1	Or moments about Q to find $Y_{\mathbb{R}}$	
			so $Y_Q = 180$	A1	Must be established not using given Y_R	
			$Y_{\rm Q} + Y_{\rm R} = 300 \text{ so } Y_{\rm R} = 120$	E1	AG	
				[3]		
		(iii)	Mark in 120 N and 180 N and all internal forces	B1 [1]	Accept labelled internal forces marked as T or C	
		(iv)	c.w moments about A $120 \times 0.75 + 180 \times 2.4 - 2 \times X_D = 0$ Or $300 \times (0.99 + 0.75) - 2 \times X_D = 0$	M1	Appropriate moments considered	
			so $X_{\rm D} = 261$	E1 [2]	Convincingly shown	

Question	Answer	Marks	Guidance
(v)	At B $\uparrow T_{AB} \sin \alpha - 180 = 0$	M1	This solution takes all internal forces +ve when T(ensions) Equilibrium at a pin-joint to find one of required forces (all relevant forces)
	so $T_{AB} = 180 \times \frac{13}{5} = 468$ so force in AB is 468 N (T)	A1	Do not need T/C here
	$\leftarrow T_{\rm BC} + T_{\rm AB} \cos \alpha = 0$	M1	2 nd equilibrium at the same or another pin-joint to find another required force (all relevant forces)
	so $T_{BC} = -468 \times \frac{12}{13} = -432$ so force in BC is 432 N (C)	F1	FT their values. Do not need T/C here
	At D		
	$\rightarrow T_{\rm DC}\cos\beta + X_{\rm D} = 0$	M1	3 rd equilibrium at a pin-joint (complete method to find third required force)
	so $T_{DC} = -261 \times \frac{5}{3} = -435$ so force in DC is 435 N (C)	F1	FT their values and all T/C correct
	$\uparrow Y_{\rm D} + T_{\rm DC} \sin \beta = 0$		
	so $Y_D = 435 \times \frac{4}{5} = 348$ so 348 N	В1	cao for the first of Y_D or Y_A found
	$Y_{\rm D} - Y_{\rm A} = 300$ so $Y_{\rm A} = 48$ so 48 N	B1	ft for the second of Y_D or Y_A : difference = 300
		[8]	

Qu	estion	Answer	Marks	Guidance
4	(i)	Let centre of mass be at G G is on CO by symmetry Let $CG = Y$ and curved surface density be σ	B1	Accept σ taken to be 1 without comment. o. e.
		$\left[(\pi(0.1)^2 \times 4\sigma + 2\pi \times 0.1 \times h \times \sigma) Y = \pi(0.1)^2 \times 4\sigma \times h + 2\pi \times 0.1 \times h \times \sigma \times \frac{h}{2} \right]$	M1	Complete method
			B1	'masses' in correct ratios: 0.04: 0.2h
			B1 A1	Correct use of 'h' and 'h/2' All correct
		so $Y = \frac{(5h^2 + 2h)}{2 + 10h}$	E1	Convincingly shown
	(44)		[6]	
	(ii)	Let the lowest point of contact of the cylinder with the plane be A On point of tipping G is vertically above A	B1	May be shown on a diagram
		Angle AGO is α	B1	May be shown on a diagram can be implied by subsequent work
		$\tan \alpha = \frac{0.1}{Y} = \frac{2}{3}$	M1	Allow reciprocal of RHS
		so $0.3 = 2 \times \frac{(2h+5h^2)}{2+10h}$	A1	
		so $50h^2 + 5h - 3 = 0$ AG Either	A1	cwo and convincingly shown
		(5h-1)(10h+3)=0	M1	Clear evidence of 2 roots
		(only positive root is) $h = 0.2$.	E1	No need to comment on the negative root
		or		and the second of the second o
		$50 \times (0.2)^2 + 5 \times 0.2 - 3 = 2 + 1 - 3 = 0$	B1	
		And this is the only positive root	E1 [7]	Need statement but no need to show this.

Question	Answer	Marks	Guidance	
(iii)	$(\sin \beta = 0.6; \cos \beta = 0.8)$ c.w. moments about 'furthest' point of the base (through which acts the NR)	M1	Both forces present in a moments equation	
	$T\cos\beta \times 0.5 - T\sin\beta \times 0.2 - 42 \times 0.1 = 0$	M1 A1	Attempt to find moment of force of magnitude <i>T</i> in horizontal and vertical components oe Correct distances oe	
	(so $T(0.4-0.12)=4.2$)	A1	Correct equation, numerical values of cos/sin do not need to be substituted	
	so $T = 15$	A1 [5]	cao	

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General Comments:

The standard of the solutions presented by candidates was pleasing. Most candidates were able to make a reasonable attempt at most parts of the paper. There was some evidence that candidates felt rushed towards the end of the paper.

As always, candidates should draw clear and labelled diagrams and these are always appropriate when dealing with forces or velocities. A lot of potentially very good work was marred by sign errors that, perhaps, could have been avoided by having a clear diagram.

Comments on Individual Questions:

Question No. 1

Momentum and Impulse

Candidates showed that they were able to write down equations using the principle of conservation of linear momentum and Newton's experimental law, but these equations were often inaccurate in the detail. This appeared to be due to a lack of understanding of some of the situations described in the question.

- (a)(i) Most candidates scored full marks.
- (ii) The best approach was to find the resulting speed of Q in this new situation and show that it led to an invalid value for the coefficient of restitution. A significant number of candidates did not realise that this part of the question had to refer to a different situation to that in part (i). As a consequence, they carried forward the values of the coefficient of restitution and the speed of Q found in part (i).
- (iii) There were some good, well-reasoned answers to this part, leading to the conclusion that the speed of Q was unchanged and equal to the value found in part (i). A common error was to attempt to find a new value for the speed of Q. Other candidates realised that the speed of Q was unchanged but were not able to support their realisation with a convincing argument involving the absence of an external force or conservation of momentum for the truck.
- (iv) Only a minority of candidates earned more than a single mark. Most candidates did not appreciate the significance of the fact that the speed of the object was given *relative* to P. This relative speed was often used as the actual speed.
- (b) The majority of candidates considered the horizontal components of the velocity before and after impact and concluded that since these components were not equal, the plane could not be smooth. A common error was to interpret an elastic collision as one in which the coefficient of restitution must be equal to one. This led them to compare the vertical components of the velocity before and after impact.

Question No. 2

Work, Energy and Power

As is often the case in problems on the topic of work and energy, an alternative method of solution using Newton's second law and *suvat* equations is available. In parts of the question where no method is specified, either approach is acceptable, but when there is an instruction to use a method involving a particular part of the specification such as "use an energy method" candidates should follow the instruction.

- (a)(i) Almost all candidates scored full marks.
- (ii) Most candidates used linear momentum to find the speed of the block. Some candidates did not include the mass of the bullet, using 3.96 kg instead of 4 kg as the final mass.
- (iii) The majority of candidates followed the instruction to consider mechanical energy, and equated a loss in kinetic energy to a work done against the resistance. A common error was to use incorrect masses in the kinetic energy terms, neglecting to include the mass of the bullet embedded in the block.
- (b)(i) Most candidates used an energy method, as instructed, but a significant minority of candidates used Newton's second law and *suvat* equations. The energy equation required 5 terms: two kinetic energy terms; a gravitational potential energy term; two work done terms, one involving the tension and the other involving the frictional force. The common errors were
- omitting the work done by the tension
- including the weight component term twice, once as a work done term and once as a gravitational potential energy term
- omitting one of the kinetic energy terms
- using forces instead of work done terms in an energy equation.
- (ii) Most candidates multiplied their frictional force from part (b)(i) by the given speed and gained both marks.

Question No. 3

Forces and Equilibrium

Candidates appeared to be confident with the content of this question and there were many very good, well-presented solutions.

- (i) Almost all candidates produced clear and concise moments equations followed by accurate working to give the coordinates of the centre of mass of the inn sign.
- (ii) Again, almost all candidates scored full marks.
- (iii) The single mark available in this part required the labelling of the internal forces to all of the rods and the external forces at B and C. The internal force in each rod should be marked with a pair of arrows and a label such as T_{AB} . This would help candidates to produce accurate equations when evaluating the magnitudes of the forces.
- (iv) Most candidates took moments about A and used the results from part (ii) to find X_D .
- (v) There were many fully correct solutions to this part. The clear and logical presentation of most of these solutions was very pleasing and demonstrated an ability to work out a strategy before embarking on writing down equations. A minority of candidates resolved horizontally and vertically at A, B, C and D and then faltered in trying to evaluate the forces that were required. Apart from numerical and sign errors, the most common errors were to assume that, because triangle ACD is isosceles, $Y_A = Y_D$ and the internal force in AC = the internal force in CD.

Question No. 4

Centre of mass

There were many pleasing responses to this question, with good diagrams and logically presented work. When a given answer has to be shown, it is important to remember that clear explanations are required. The majority of candidates produced moments equations that were dimensionally correct, but a minority of candidates omitted the distance part of at least one of the terms in their equations.

- (i) Most candidates knew the method of approach to use, but there were two common errors:
- the ratio of the densities of the two materials was reversed or ignored
- the formula used for the surface area of the cylinder was incorrect.

Many candidates did not address the fact that they needed to show that the centre of mass of the cylinder was on OC. A simple statement that this was true by symmetry was sufficient.

- (ii) Candidates who drew a clear diagram usually gained full marks in this part. The significant number of candidates who did not draw a diagram often had the expression for tan α upside down.
- (iii) Almost all candidates attempted to take moments about an appropriate point. Most candidates attempted to resolve T into two components. A very common error was to consider only one of the components of the force T. Some candidates worked with T and attempted to find the perpendicular distance from the tipping point to the line of action of T. This was rarely accomplished successfully.