Oxford Cambridge and RSA

Wednesday 13 May 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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1 A thin uniform rigid rod JK of length 1.2 m and weight 30 N is resting on a rough circular cylinder which is fixed to a floor. The axis of symmetry of the cylinder is horizontal and at all times the rod is perpendicular to this axis.

Initially, the rod is horizontal and its point of contact with the cylinder is 0.4 m from K. It is held in equilibrium by resting on a small peg at J. This situation is shown in Fig. 1.1.

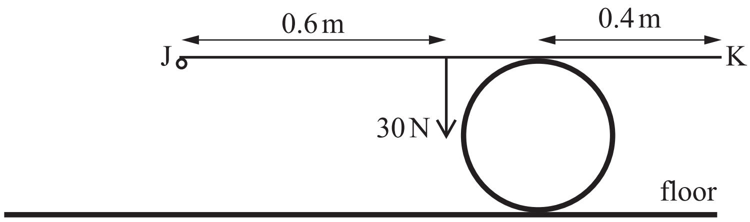


Fig. 1.1

(i) Calculate the force exerted by the peg on the rod and also the force exerted by the cylinder on the rod. **[3]**

A small object of weight W N is attached to the rod at K.

(ii) Find the greatest value of W for which the rod maintains its contact at J. **[2]**

The object at K is removed. Fig. 1.2 shows the rod resting on the cylinder with its end J on the floor, which is smooth and horizontal. The point of contact of the rod with the cylinder is 0.3 m from K. Fig. 1.2 also shows the normal reaction, S N, of the floor on the rod, the normal reaction, R N, of the cylinder on the rod and the frictional force F N between the cylinder and the rod.

Suppose the rod is in equilibrium at an angle of $\theta°$ to the horizontal, where $\theta < 90$.

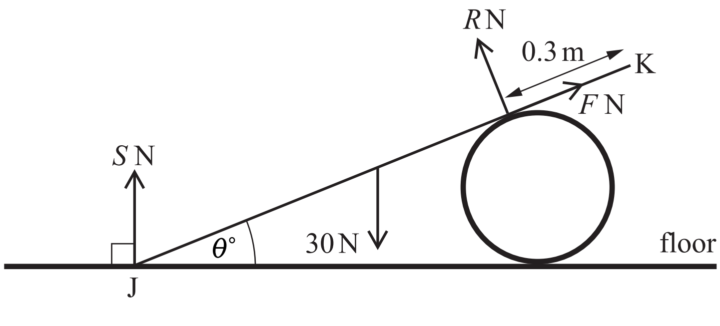


Fig. 1.2

(iii) Find S. Find also expressions in terms of θ for R and F. **[8]**

The coefficient of friction between the cylinder and the rod is μ.

(iv) Determine a relationship between μ and θ. **[3]**

2 Fig. 2 shows a wedge of angle 30° fixed to a horizontal floor. Small objects P, of mass 8 kg, and Q, of mass 10 kg, are connected by a light inextensible string that passes over a smooth pulley at the top of the wedge. The part of the string between P and the pulley is parallel to a line of greatest slope of the wedge. Q hangs freely and at no time does either P or Q reach the pulley or P reach the floor.

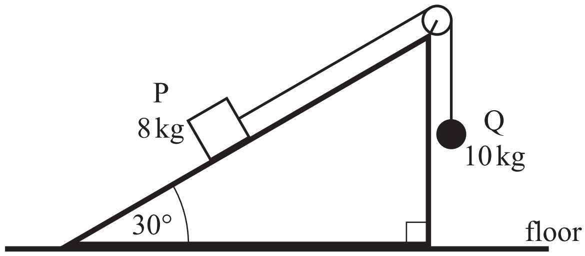


Fig. 2

(i) Assuming the string remains taut, find the change in the gravitational potential energy of the system when Q descends h m, stating whether it is a loss or a gain. **[3]**

Object P makes smooth contact with the wedge. The system is set in motion with the string taut.

(ii) Find the speed at which Q hits the floor if

(A) the system is released from rest with Q a distance of 1.2 m above the floor, **[3]**

(B) instead, the system is set in motion with Q a distance of 0.3 m above the floor and P travelling **down** the slope at $1.05\,\mathrm{m\,s^{-1}}$. **[3]**

The sloping face is roughened so that the coefficient of friction between object P and the wedge is 0.9. The system is set in motion with the string taut and P travelling down the slope at $2\,\mathrm{m\,s^{-1}}$.

(iii) How far does P move before it reaches its lowest point? **[5]**

(iv) Determine what happens to the system after P reaches its lowest point. **[2]**

(v) Calculate the power of the frictional force acting on P in part **(iii)** at the moment the system is set in motion. **[2]**

Question 3 begins on page 4.

3 A uniform heavy lamina occupies the region shaded in Fig. 3. This region is formed by removing a square of side 1 unit from a square of side a units (where $a > 1$).

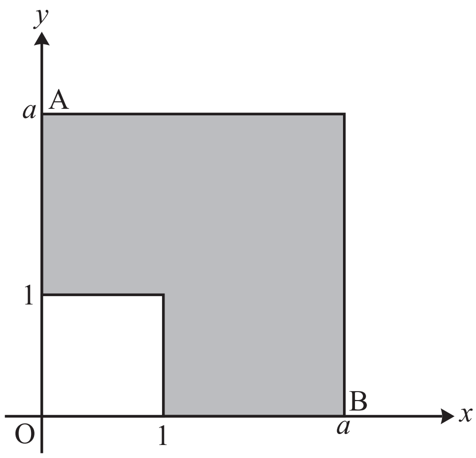


Fig. 3

Relative to the axes shown in Fig. 3, the centre of mass of the lamina is at $(\bar{x}, \bar{y})$.

(i) Show that $\bar{x} = \bar{y} = \dfrac{a^2 + a + 1}{2(a+1)}$.

[You may need to use the result $\dfrac{a^3 - 1}{2(a^2 - 1)} = \dfrac{a^2 + a + 1}{2(a+1)}$.] **[5]**

(ii) Show that the centre of mass of the lamina lies on its perimeter if $a = \frac{1}{2}(1 + \sqrt{5})$. **[3]**

In another situation, $a = 4$.

A particle of mass one third that of the lamina is attached to the lamina at vertex B; the lamina with the particle is freely suspended from vertex A and hangs in equilibrium. The positions of A and B are shown in Fig. 3.

(iii) Calculate the angle that AB makes with the vertical. **[10]**

4 **(a)** Two discs, P of mass 4 kg and Q of mass 5 kg, are sliding along the same line on a smooth horizontal plane when they collide. The velocity of P before the collision and the velocity of Q after the collision are shown in Fig. 4. P loses $\frac{5}{9}$ of its kinetic energy in the collision.

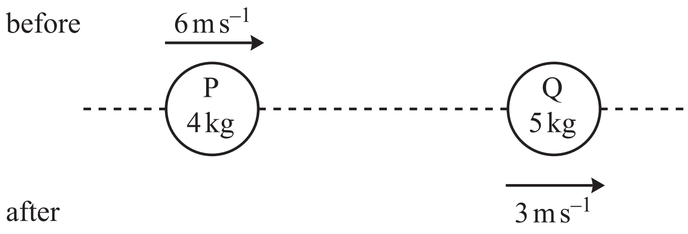


Fig. 4

(i) Show that after the collision P has a velocity of $4\,\mathrm{m\,s^{-1}}$ in the opposite direction to its original motion. **[4]**

While colliding, the discs are in contact for $\frac{1}{5}$ s.

(ii) Find the impulse on P in the collision and the average force acting on the discs. **[4]**

(iii) Find the velocity of Q before the collision and the coefficient of restitution between the two discs. **[4]**

(b) A particle is projected from a point 2.5 m above a smooth horizontal plane. Its initial velocity is $5.95\,\mathrm{m\,s^{-1}}$ at an angle θ below the horizontal, where $\sin\theta = \frac{15}{17}$. The coefficient of restitution between the particle and the plane is $\frac{4}{5}$.

(i) Show that, after bouncing off the plane, the greatest height reached by the particle is 2.5 m. **[5]**

(ii) Calculate the horizontal distance between the two points at which the particle is 2.5 m above the plane. **[3]**

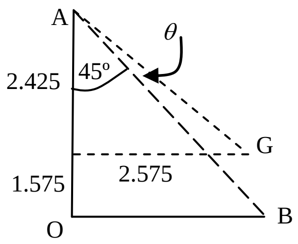
END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		Suppose $\uparrow$ U N at J and V N at point of contact with cylinder Taking moments about point of contact with the cylinder or J $0.8U - 0.2 \times 30 = 0$ or $0.8V - 0.6 \times 30 = 0$ so $U = 7.5$ or $V = 22.5$ Resolve $\uparrow$ $U + V - 30 = 0$ so $V = 22.5$ or $U = 7.5$	M1 A1 B1	A correct moments equation FT use of 1st answer. (Or use moments again)	3
	(ii)		Taking moments about point of contact with the cylinder $0.4W - 0.2 \times 30 = 0$ so $W = 15$	M1 A1	A correct moments equation and reaction at J = 0 Award SC2 for 15 seen WWW	2
	(iii)		Taking moments about point of contact $S \times 0.9\cos\theta - 30 \times 0.3\cos\theta = 0$ so $S = 10$ cw moments about J $30 \times 0.6\cos\theta - R \times 0.9 = 0$ so $R = 20\cos\theta$ resolve up the rod $S\sin\theta + F - 30\sin\theta = 0$ so $F = 20\sin\theta$ **SC** Resolving vert and horiz leads to simultaneous eqns. $R = 20\cos\theta$ $F = 20\sin\theta$	M1 A1 M1 A1 A1 M1 A1 A1 M1 A1 M1 A1 A1 A1	A moments equation about point of contact: allow sin instead of cos: Must be trig fn in both terms. Allow slip in distances if clearly taking moments about point of contact Award SC2 for 10 seen WWW Attempt at moments or resolution, involving F or R but not both, with all appropriate terms present and no extras. No need to substitute for S, if it is present. Or $R = (30 - S)\cos\theta$ o.e. FT their S Attempt at another moments or resolution, that will enable F or R to be found, with all appropriate terms present and no extras No need to substitute for S, if it is present. Or $F = (30 - S)\sin\theta$ o.e. FT their S One eqn in F and R Second eqn in F and R AND attempt to solve simult eqns for F or R Or $R = (30 - S)\cos\theta$ o.e. FT their S Or $F = (30 - S)\sin\theta$ o.e.FT their S	8

Question			Answer	Marks	Guidance	
	(iv)		Need $F \le \mu R$ $20\sin\theta \le \mu \times 20\cos\theta$ so $\mu \ge \tan\theta$	B1 M1 A1	Use of $F \le \mu R$ or $F = \mu R$ or $F < \mu R$ Needs an inequality, using their F and R from **(iii)** FT incorrect S (Strict inequality gets 2/3)	3
				16		
2	**(i)**		The system *loses* $10gh - 8g\frac{h}{2} = 6gh$ or $58.8h$ *J*	M1 B1 A1	Difference of GPEs Use of $\frac{h}{2}$ or $h\sin 30^\circ$ (sight of $39.2h$) Clearly shown: *Loss* must be stated	3
	(ii)	**(*A*)**	$\frac{1}{2}\times 18\times v^2 = 6g\times 1.2$ (= 70.56) $v = 2.8$ so 2.8 m s^{-1} OR $10g - 8g\sin 30^\circ = 18a$ $a = g/3$ or 3.26: $v^2 = 0.8g = 7.84$: $v = 2.8$	M1 B1 A1 M1 B1 A1	Use of WE equation (a KE = a GPE) Use of '18'. cao Use N2L AND use *suvat* Use of '18'. cao	3 3
	(ii)	**(*B*)**	Q passes through point 0.3 m above floor going down at 1.05 m s^{-1} $\frac{1}{2}\times 18\times V^2 - \frac{1}{2}\times 18\times 1.05^2 = 6g\times 0.3$ ($V^2 = 3.0625$) and $V = 1.75$ so 1.75 m s^{-1} OR $10g - 8g\sin 30^\circ = 18a$ and *suvat* eqn $a = g/3$ (= 3.26) ($V^2 = 3.0625$) and $V = 1.75$ so 1.75 m s^{-1}	M1 A1 A1 M1 A1 A1	WE with 2 KE terms and a difference in GPE terms: must be using 18; may consider motion of P and motion of Q separately Allow FT on their $6g$ cao Use N2L (both particles considered) AND use *suvat:* must be using 18 cao cao	3

Question			Answer	Marks	Guidance	
	(iii)		$F_{\max} = 0.9 \times 8g\cos 30$	M1	Attempt to use $F = \mu R$ with weight of $8g$	
				A1	May not be evaluated (61.10675…or $35.28\sqrt{3}$)	
			$\frac{1}{2} \times 18 \times 2^2 = 6gh + F_{\max}h$	M1	WE. Correct KE term (with 18), a GPE term and Fh. Allow sign errors	
				A1	All correct	
			h = 0.30023… so 0.300 m (3 s. f.)	A1	cao	5
			OR			
			$F_{\max} = 0.9 \times 8g\cos 30$	M1	Attempt to use $F = \mu R$ with mass of 8	
				A1	May not be evaluated (61.10675…)	
			$F + 10g - 8g\sin 30^\circ = 18a$	M1	Use N2L for both particles, or combined system, must be 18*a*; all terms present and use *suvat*	
			$a = -6.66$	A1	cao	
			h = 0.30023… so 0.300 m (3 s. f.)	A1	cao	5
	(iv)		Could P stay at rest in equilibrium? Force up the plane on P is 10g = 98 N. Force down plane on P due to its weight is 8$g$sin30 = 39.2 N	M1	Attempt to compare forces up and down the plane including friction OR: Force needed for equilibrium = $10g - 8g\sin 30^\circ = 58.8$ and compare with max frictional force 61.1	
			39.2 + $F_{\max} \approx 100.3 > 98$ so yes, stays at rest	E1	A clear argument 58.8 is less than 61.1 so equilibrium possible	2
	(v)		2× 61.10675… = 122.21… so 122 W (3 s. f.)	M1 F1	Attempt to use $P = Fv$ with v = 2 and their frictional force	2
				18		

Question			Answer	Marks	Guidance	
3		(i)	either $\bar{x} = \bar{y}$ by symmetry For $\bar{x}$ $a^2\sigma\frac{a}{2} = (a^2-1)\sigma\bar{x} + 1\sigma\frac{1}{2}$ Hence $\frac{a^3}{2} - \frac{1}{2} = (a^2-1)\bar{x}$ and $\bar{x} = \frac{(a^3-1)}{2(a^2-1)}$: $\bar{x} = \frac{a^2+a+1}{2(a+1)}$	B1 M1 A1 A1 E1	Consider a square side 1 removed from a square side a Or by calculation Allow σ omitted Any one term correct All correct At least one intermediate step shown	5
			or $\bar{x} = \bar{y}$ by symmetry $((a-1)a + 1(a-1))\sigma\bar{x} = (a-1)a \times \frac{a}{2}\sigma + (a-1)\frac{(a+1)}{2}\sigma$ …. hence $(a+1)\bar{x} = \frac{a^2}{2} + \frac{(a+1)}{2}$ and $\bar{x} = \frac{a^2+a+1}{2(a+1)}$	B1 M1 A1 A1 E1	Splitting horizontally into 2 rectangles Or by calculation Allow σ omitted. o.e. Any one term correct All correct Dividing by (a – 1) o.e. Some simplification shown [*Or splitting vertically into 2 rectangles or…*]	
		(ii)	Need $\bar{x} = \frac{a^2+a+1}{2(a+1)} = 1$ so $a^2+a+1 = 2a+2$ and $a^2-a-1=0$ Solving for the +ve root gives $a = \frac{1+\sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$	B1 M1 A1	 oe	3

Question			Answer	Marks	Guidance	
			OR Substitute expression for a into expression for $\bar{x}$ and attempt to simplify Obtain $\bar{x} = 1$ This is on perimeter of shape	M1 A1 B1	Includes an attempt at squaring out.. Decimals used M0	3
		(iii)	a = 4 gives $\bar{x} = \bar{y} = 2.1$ Take the mass of the particle to be m $4m\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m\begin{pmatrix}4\\0\end{pmatrix} + 3m\begin{pmatrix}2.1\\2.1\end{pmatrix}$ $\bar{x} = 2.575$ $\bar{y} = 1.575$ G is vertically below A  $\theta = \arctan\frac{2.575}{2.425} - 45$ $\theta = 45 - \arctan\frac{2.425}{2.575}$ = 1.71835… so 1.72° (3 s. f.)	B1 M1 A1 A1 A1 B1 M1 M1 A1 A1	 Allow if a not substituted as 4 At least 1 correct term Allow if a not substituted as 4 FT for 1 of the final answer marks if there is a single error May be implied Note: Can use cosine rule in triangle AGB: AG = 3.537, AB = 5.657, GB = 2.124: M1 for attempt to find all three lengths A1 all correct M1 use cosine rule A1 1.72 cao Correct angle attempted (may be scored below) Use of arctan oe to find angle OAG must be using their 2.575 and 4 - their 1.575 Use of appropriate lengths. FT their values. cao	10
				18		

Question			Answer	Marks	Guidance	
4	(a)	(i)	Let speed of P after collision be u m s^{-1}			
			$\frac{1}{2}\times 4\times u^2 = \frac{1}{2}\times 4\times 36\times \frac{4}{9}$	M1	Use of KE; allow use of $\frac{5}{9}$ in an equation involving only u	
				B1	Correct use of $\frac{4}{9}$ of a KE term	
			$u = 4$	E1	Condone ± 4	
			Cannot be in the direction of Q as 4 > 3	E1	Can use PCLM and NEL to show 'in direction of Q' gives negative e	4
		(ii)	Taking $\leftarrow$ +ve			
			Impulse is $4(4-(-6))$	M1	Allow sign error. Must be 4(final vel - initial vel); condone 4(6 +4)	
			= 40 N s	A1	Note: impulse on Q = 5(3 - - 5) = 40 needs to be related to impulse on P to score any marks	
			Let force be F N			
			$40 = F\times \frac{1}{5}$	M1	Use of Impulse = Ft	
			so $F = 200$	A1	Sign consistent with sign of impulse	4
		(iii)	Taking $\rightarrow$ +ve and vel of Q as U m s^{-1}			
			PCLM $24+5U=-4\times 4+5\times 3$	M1	o.e. e.g. using impulse	
			so $U=-5$ i.e. to the left	A1	Must state direction but could be implied by diagram or equivalent	
			$e=\dfrac{4+3}{6+5}=\dfrac{7}{11}$ (0.636 to 3 sig figs)	M1		
				F1	FT their U	4

Question			Answer	Marks	Guidance	
	(b)	(i)	Vert cpt of vel in m s^{-1} is $5.95\times\frac{15}{17}=5.25$	B1	May be implied	
			Particle hits the plane with vert speed			
			$\sqrt{5.25^2+2\times 9.8\times 2.5}$	M1	Must be vertical component	
			= 8.75	A1	May be implied	
			Particle leaves plane with vert speed			
			8.75×0.8 = 7	B1	Award for the ×0.8 on vert cpt (dependent on M1)	
			Height reached is given by			
			$0=7^2-2\times 9.8\times h$ so h = 2.5	E1	Clearly shown cwo	
						5
		(ii)	Time taken in seconds is			
			down $\frac{8.75-5.25}{9.8}=\frac{5}{14}=0.357$:up $\frac{7}{9.8}=\frac{5}{7}=0.714$:	M1	Finding both times and adding	
			so $\frac{15}{14}$ s	F1	FT only their 8.75 from (i)	
			Horiz distance is $\frac{15}{14}\times 5.95\times\frac{8}{17}$ = 3 m	A1	cao	
						3
				20		