OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715790613

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** In this part-question, all the objects move along the same straight line on a smooth horizontal plane. All their collisions are direct.

The masses of the objects P, Q and R and the initial velocities of P and Q (but not R) are shown in Fig. 1.1.

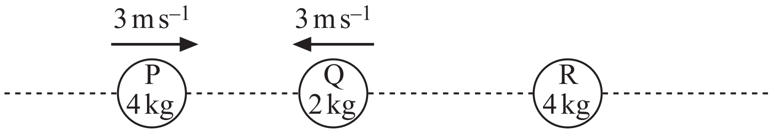


Fig. 1.1

A force of 21 N acts on P for 2 seconds in the direction PQ. P does not reach Q in this time.

(i) Calculate the speed of P after the 2 seconds. **[2]**

The force of 21 N is removed after the 2 seconds. When P collides with Q they stick together (coalesce) to form an object S of mass 6 kg.

(ii) Show that immediately after the collision S has a velocity of $8\,\mathrm{m\,s^{-1}}$ towards R. **[2]**

The collision between S and R is elastic with coefficient of restitution $\frac{1}{4}$. After the collision, S has a velocity of $5\,\mathrm{m\,s^{-1}}$ in the direction of its motion before the collision.

(iii) Find the velocities of R before and after the collision. **[6]**

(b) **In this part-question take $g = 10$.**

A particle of mass 0.2 kg is projected vertically downwards with initial speed $5\,\mathrm{m\,s^{-1}}$ and it travels 10 m before colliding with a fixed smooth plane. The plane is inclined at α to the vertical where $\tan\alpha = \frac{3}{4}$. Immediately after its collision with the plane, the particle has a speed of $13\,\mathrm{m\,s^{-1}}$. This information is shown in Fig. 1.2. Air resistance is negligible.

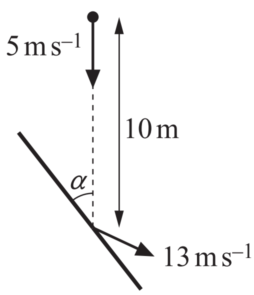


Fig. 1.2

(i) Calculate the angle between the direction of motion of the particle and the plane immediately after the collision.

Calculate also the coefficient of restitution in the collision. **[8]**

(ii) Calculate the magnitude of the impulse of the plane on the particle. **[2]**

2 A fairground ride consists of raising vertically a bench with people sitting on it, allowing the bench to drop and then bringing it to rest using brakes. Fig. 2 shows the bench and its supporting tower. The tower provides lifting and braking mechanisms. The resistances to motion are modelled as having a constant value of 400 N whenever the bench is moving up or down; the only other resistance to motion comes from the action of the brakes.

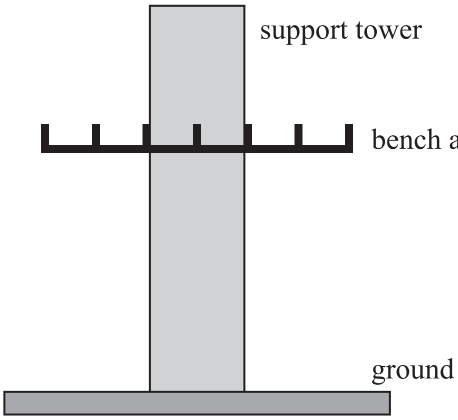


Fig. 2

On one occasion, the mass of the bench (with its riders) is 800 kg.

With the brakes not applied, the bench is lifted a distance of 6 m in 12 seconds. It starts from rest and ends at rest.

(i) Show that the work done in lifting the bench in this way is 49 440 J and calculate the average power required. **[4]**

For a short period while the bench is being lifted it has a constant speed of $0.55\,\mathrm{m\,s^{-1}}$.

(ii) Calculate the power required during this period. **[3]**

With neither the lifting mechanism nor the brakes applied, the bench is now released from rest and drops 3 m.

(iii) Using an energy method, calculate the speed of the bench when it has dropped 3 m. **[4]**

The brakes are now applied and they halve the speed of the bench while it falls a further 0.8 m.

(iv) Using an energy method, calculate the work done by the brakes. **[5]**

3 Fig. 3.1 shows a rigid, thin, **non-uniform** 20 cm by 80 cm rectangular panel ABCD of weight 60 N that is in a vertical plane. Its dimensions and the position of its centre of mass, G, are shown in centimetres. The panel is free to rotate about a fixed horizontal axis through A perpendicular to its plane; the panel rests on a small smooth fixed peg at B positioned so that AB is at 40° to the horizontal. A horizontal force in the plane of ABCD of magnitude P N acts at D away from the panel.

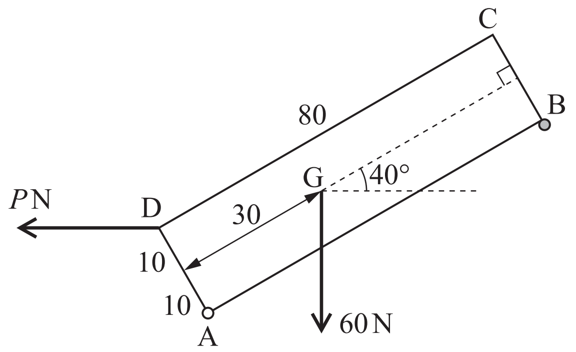


Fig. 3.1

(i) Show that the clockwise moment of the weight about A is 9.93 N m, correct to 3 significant figures. **[3]**

(ii) Calculate the value of P for which the panel is on the point of turning about the axis through A. **[2]**

(iii) In the situation where $P = 0$, calculate the vertical component of the force exerted on the panel by the axis through A. **[4]**

The panel is now placed on a line of greatest slope of a rough plane inclined at 40° to the horizontal. The panel is at all times in a vertical plane. A horizontal force in the plane ABCD of magnitude 200 N acts at D towards the panel. This situation is shown in Fig. 3.2.

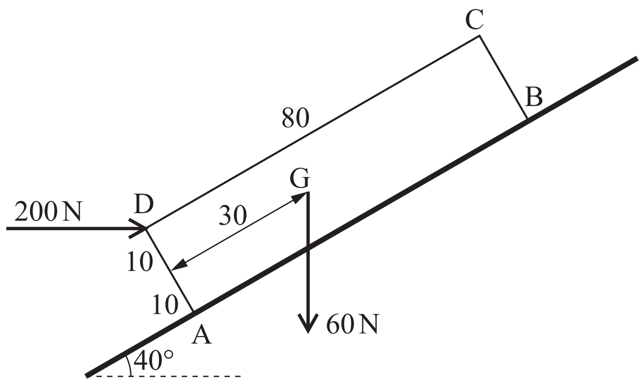


Fig. 3.2

(iv) Given that the panel is moving up the plane with acceleration up the plane of $1.75\,\text{m s}^{-2}$, calculate the coefficient of friction between the panel and the plane. **[8]**

4 **(a)** Fig. 4.1 shows a framework constructed from 4 uniform heavy rigid rods OP, OQ, PR and RS, rigidly joined at O, P, Q, R and S and with OQ perpendicular to PR. Fig. 4.1 also shows the dimensions of the rods and axes O*x* and O*y*: the units are metres.

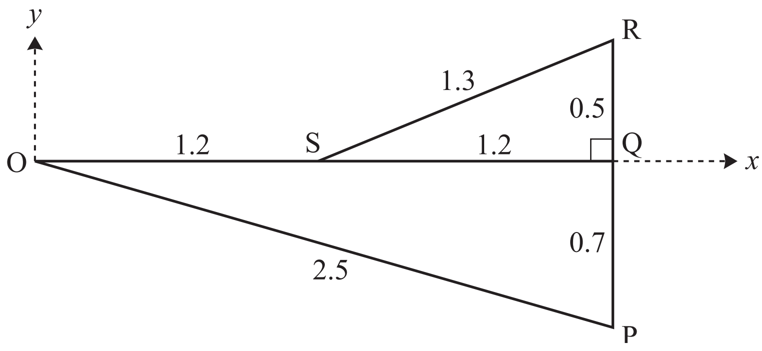


Fig. 4.1

Each rod has a mass of 0.8 kg per metre.

(i) Show that, referred to the axes in Fig. 4.1, the *x*-coordinate of the centre of mass of the framework is 1.5 and calculate the *y*-coordinate. **[5]**

The framework is freely suspended from S and a small object of mass *m* kg is attached to it at O. The framework is in equilibrium with OQ horizontal.

(ii) Calculate *m*. **[3]**

[Question 4 is continued overleaf.]

(b) Fig. 4.2 shows a framework in equilibrium in a vertical plane. The framework is made from 5 light, rigid rods OP, OQ, OR, PQ and QR. Its dimensions are indicated. PQ is horizontal and OR vertical.

The rods are freely pin-jointed to each other at O, P, Q and R. The pin-joint at O is fixed to a wall.

Fig. 4.2 also shows the external forces acting on the framework: there are vertical loads of 120 N and 60 N at Q and P respectively; a horizontal string attached to Q has tension T N; horizontal and vertical forces X N and Y N act on the framework from the pin-joint at O.

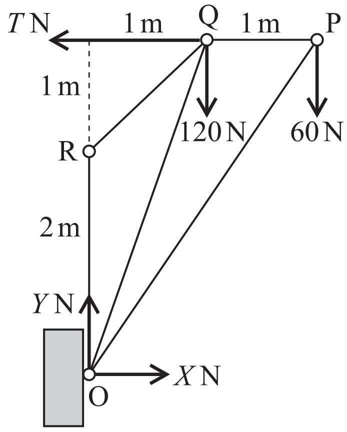


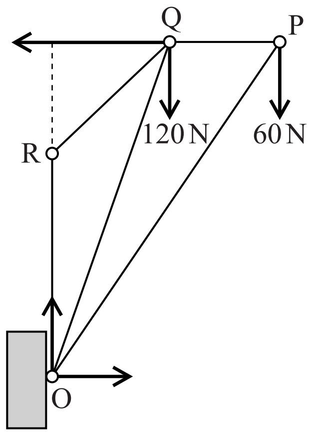
Fig. 4.2

(i) By considering only the pin-joint at R, explain why the rods OR and RQ must have zero internal force. **[2]**

(ii) Find the values of T, X and Y. **[3]**

(iii) Using the diagram in your printed answer book, show all the forces acting on the pin-joints, including those internal to the rods. **[1]**

(iv) Calculate the forces internal to the rods OP and PQ, stating whether each rod is in tension or compression (thrust). [You may leave answers in surd form. Your working in this part should correspond to your diagram in part **(iii)**.] **[5]**

4 (b) (iii)	
4 (b) (iv)	
	(answer space continued on next page)

(answer space continued on next page)

Question			Answer	Marks	Guidance
1	(a)	(i)	$3 \times 4 + 21 \times 2 = 4U$ $4U = 54$ so $U = 13.5$ and speed is 13.5 m s^{-1} **OR** $21 = 4a : a = 5.25$ and $v = 3 + 2 \times 5.25$ speed is 13.5 m s^{-1}	M1 A1 **[2]** M1 A1 **[2]**	Use of PCLM and $I = Ft$ Use of $F = ma$ and *suvat*
1	(a)	(ii)	Let V be the speed of S in direction PQ $54 - 2 \times 3 = (4 + 2)V$ $6V = 48$ so $V = 8$ and velocity is 8 m s^{-1} in direction PQ	 M1 E1 **[2]**	 PCLM for coalescence Answer given. Accept no reference to direction.
1	(a)	(iii)	Let velocities of R be u before and v after, both in the direction SR $6 \times 8 + 4u = 6 \times 5 + 4v$ $v - u = 4.5$ $\frac{v-5}{u-8} = -\frac{1}{4}$ $4v + u = 28$ Solving $u = 2$ so 2 m s^{-1} in the direction SR $v = 6.5$ so 6.5 m s^{-1} in the direction SR	 M1 A1 M1 A1 A1 A1 **[6]**	 Use of PCLM. Allow any sign convention. All masses and speeds must be correct. Any form. Use of NEL correct way up; allow sign errors Any form signs consistent with PCLM eqn cao **NOTE that a sign error in NEL leads to u =-2; this gets A0** cao. Withhold only 1 of the final A marks if the directions not clear. Directions can be inferred from a CLEAR diagram

Question			Answer	Marks	Guidance
1	(b)	(i)	Find v, the speed at which particle hits the plane		
			$\frac{1}{2} \times 0.2 \times v^2 - \frac{1}{2} \times 0.2 \times 5^2 = 0.2 \times 10 \times 10$	M1	Use of WE or *suvat* must use distance of 10 allow $g = 9.8$
			so $v^2 = 225$ and $v = 15$	A1	Answer not required ($v = 14.9$ if $g = 9.8$)
			$\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$	B1	Use of either expression or use of 36.9°
			Let velocity after be at β to the plane		
			Parallel to the plane		
			$15\cos\alpha = 13\cos\beta$	M1	Attempt to conserve velocity component parallel to plane. Allow use of 5 instead of 15
			So $\cos\beta = \frac{12}{13}$ and $\beta = 22.61..$ so 22.6° (3 s. f.)	A1	($\beta = 23.8^\circ$ if g =9.8)
			Perpendicular to the plane: $13\sin\beta = e \times 15\sin\alpha$	M1	Attempt to use NEL perpendicular to plane: Allow use of 5 instead of 15 or use $\tan\beta = e\tan\alpha$
			$\sin\beta = \frac{5}{13}$	A1	o.e. find $\tan\beta = \frac{5}{12}$
			so $13 \times \frac{5}{13} = 15 \times \frac{3}{5} \times e$ and $e = \frac{5}{9}$	A1	cao Accept 0.56 ($e = 0.589$ if g =9.8)
				[8]	
			OR: First three marks as above	M1A1B1	
			Parallel to plane, $u_x = 15\cos\alpha (= 12)$ and $v_x = u_x (= 12)$	M1	Attempt to conserve velocity component parallel to plane. Allow use of 5 instead of 15
			$\cos\beta = \frac{v_x}{v} = \frac{12}{13}$ $\beta = 22.6^\circ$	A1	
			Perpendicular to plane, $u_y = 15\sin\alpha (= 9)$ and $v_y = eu_y (= 9e)$	M1	Attempt to use NEL perpendicular to plane. Allow use of 5 instead of 15
			$v_x^2 + v_y^2 = 13^2$	A1	Use Pythagoras' theorem for velocities after collision in attempt to find e
			$12^2 + (9e)^2 = 13^2$ so $e^2 = \frac{25}{81}$ $e = \frac{5}{9}$	A1	
				[8]	

Question			Answer	Marks	Guidance
1	(b)	(ii)	Impulse is perp to plane with mod $0.2(13\sin\beta - (-15\sin\alpha))$ = 0.2(5 – (– 9)) = 2.8 N s	 M1 A1 **[2]**	 For use of $I = m(v - u)$ perp to the plane 0.2(5-9) gets M1A0 cao
2	(i)		WD is 800 × 9.8 × 6 + 400 × 6 J = 49 440 Power is 49440 ÷ 12 = 4120 W	M1 E1 M1 A1 **[4]**	WD as *Fd* Used in TWO terms Power is WD / Δt cao
2	(ii)		Power is (800 × 9.8 + 400) × 0.55 = 4532 W	M1 A1 A1 **[3]**	Power as *Fv* in one term All correct cao
2	(iii)		Let speed be *v* $\frac{1}{2}\times 800v^2 = 800\times 9.8\times 3 - 400\times 3$ $v^2 = 55.8$ so v = 7.4699… and speed is 7.47 m s^{-1} (3 s.f.)	 M1 A1 A1 A1 **[4]**	 Use of W-E equation Must include KE and at least one WD term Allow only sign errors All correct SC: Use of N2L and *suvat* : M1 Complete method A1 7.47 cao
2	(iv)		$\frac{1}{2}\times 800\times\frac{v^2}{4} - \frac{1}{2}\times 800\times v^2$ = (800 × 9.8 – 400) × 0.8 – WD WD is 22 692 so 22 700 J (3 s. f.)	M1 B1 B1 A1 A1 **[5]**	Use of W-E equation Must include 2 KE terms and a WD term Final KE term correct. FT their *v*. One correct WD term All terms present. Allow sign errors and FT their *v*. cao SC Use of N2L and *suvat*: Award maximum of B1 for 'Average force (28365) x 0.8'

Question			Answer	Marks	Guidance
					Condone using cm not m in moments in any part if consistent
3	(i)		c.w. moments about A $60\cos 40 \times 0.3 - 60\sin 40 \times 0.1$ = 9.93207… so 9.93 N m (3 s. f.)	M1 A1 E1 **[3]**	oe e.g. $60(0.3 - 0.1\tan 40)\sin 50$ or $60 \times \frac{1}{\sqrt{10}}\cos(90^\circ - \arctan 3 + 40^\circ)$ Method of dealing with moment of weight. Allow $\cos \leftrightarrow \sin$ Both weight terms correct. Allow wrong overall sign but not both terms with the same sign
3	(ii)		$P\cos 40 \times 0.2 - 9.93207... = 0$ *P* = 64.827… so 64.8 (3 s. f.)	M1 A1 **[2]**	Moments of all relevant forces attempted. No extra terms. Allow $\cos \leftrightarrow \sin$ cao (64.813… if 9.93 used)
3	(iii)		a.c. moments about A to find NR, *R*, at B $R \times 0.8 = 9.93$ or $R \times 0.8 + 60\sin 40 \times 0.1 - 60\cos 40 \times 0.3 = 0$ *R* = 12.4150… Resolve vertically *Y* – 60 + *R*cos 40 = 0 so *Y* = 50.489… so 50.5 N (3 s. f.)	M1 A1 depM1 A1 **[4]**	Attempt to use moments to find *R*. Moments of all relevant forces attempted. No extra terms. Allow $\cos \leftrightarrow \sin$ Note that mmts about B can score M1 only if mmt of horiz compt of force at A is included. If *R* is taken as vertical, M0 FT their moment of weight from (i) Not a required answer Note that the second M mark awarded in this part must be for a complete method to find *Y*: FT their calculated *R*

Question			Answer	Marks	Guidance
3	(iv)		resolve perp to plane		
			$R - 60\cos 40 - 200\sin 40 = 0$	M1	All terms present and no extra terms. Components of 60 and 200; allow $\cos \leftrightarrow \sin$
				A1	
			R = 174.52…		Not a required answer
			N2L up the plane		
			$200\cos 40 - F - 60\sin 40 = \frac{60}{9.8} \times 1.75$	M1	Use of N2L with all terms present and no extras. Components of 60 and 200; allow $\cos \leftrightarrow \sin$ Allow use of 60 for mass
				B1	Use of mass not weight
				A1	FT use of weight and/or sign errors
			F = 103.927…	A1	All correct. Not a required answer
			As friction limiting $F = \mu R$ so		
			$\mu = \frac{103.927...}{174.520...}$	M1	FT their F and their R
			= 0.59550.. so 0.596 (3 s. f.)	A1	cao
				[8]	

Question			Answer	Marks	Guidance
4	(a)	(i)	Write $d = 0.8$ $(2.5+1.2+1.3+2.4)\times d\times\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$	M1	Method for c.m (length is 7.4 m, mass is 5.92 kg)
			$=2.5d\begin{pmatrix}1.2\\ -0.35\end{pmatrix}+1.2d\begin{pmatrix}2.4\\ -0.1\end{pmatrix}+1.3d\begin{pmatrix}1.8\\ 0.25\end{pmatrix}+2.4d\begin{pmatrix}1.2\\ 0\end{pmatrix}$ OR: $(2+0.96+1.04+1.92)\times\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$ $=2\begin{pmatrix}1.2\\ -0.35\end{pmatrix}+0.96\begin{pmatrix}2.4\\ -0.1\end{pmatrix}+1.04\begin{pmatrix}1.8\\ 0.25\end{pmatrix}+1.92\begin{pmatrix}1.2\\ 0\end{pmatrix}$	B1	One rod mass and cpts correct or if done by separate *x* and *y* equations 2 rod components and masses correct. (Allow length used instead of mass)
				B1	Another rod dealt with correctly or if done by separate *x* and *y* equations, the other equation attempted with 2 rod components and masses correct. (Allow length used instead of mass)
			$7.4\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}3+2.88+2.34+2.88\\ -0.875-0.12+0.325+0\end{pmatrix}=\begin{pmatrix}11.1\\ -0.67\end{pmatrix}$ OR: $5.92\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}2.4+2.304+2.304+1.872\\ -0.7-0.096+0.26+0\end{pmatrix}=\begin{pmatrix}8.88\\ -0.536\end{pmatrix}$		
			$\bar{x}=1.5$	E1	Clearly shown, with at least one intermediate step
			$\bar{y}=-0.090540... = -0.0905$ (3 s. f.)	A1	Condone - 0.09
				[5]	

Question			Answer	Marks	Guidance
4	(a)	(ii)	EITHER: New c.m. has $\bar{x} = 1.2$ $(5.92 + m) \times 1.2 = 5.92 \times 1.5 + m \times 0$ $m = 1.48$	M1 M1 A1 **[3]**	Identifying and using a suitable condition. Complete method cao
			OR: Moment about any point is zero e.g. about S: $1.2mg = 0.3 \times 5.92g$ $m = 1.48$	M1 M1 A1 **[3]**	Identifying a suitable condition. Allow g omitted. Correct number of terms must be included cao
4	(b)	(i)	Consider the equilibrium at R Resolving horizontally gives $T_{QR} = 0$ Then resolving vertically gives $T_{OR} = 0$	 E1 E1 **[2]**	
4	(b)	(ii)	c.w. moments about O $120 \times 1 + 60 \times 2 = 3T$ so $T = 80$ Resolve to give $X = 80$ and $Y = 180$	 M1 A1 A1 **[3]**	 May also be argued by first considering internal forces FT $X = T$. Only Y =180 scores 0
4	(b)	(iii)		B1 **[1]**	All correct. Accept T, X and Y labelled but not substituted. Accept mixes of T and C. Require pairs of arrows with label on OQ, OP and PQ.
4	(b)	(iv)	Take angle OPQ as α At P ↓ $60 + T_{OP} \sin\alpha = 0$ $\sin\alpha = \frac{3}{\sqrt{13}}$: $\alpha = 56.3^\circ$ $T_{OP} = -\frac{60}{\sin\alpha} = -20\sqrt{13}$ so $20\sqrt{13}$ N (C) At P ← $T_{QP} + T_{OP}\cos\alpha = 0$ so $T_{QP} = 40$ so 40 N (T)	 M1 A1 A1 M1 A1 **[5]**	Forces internal to the rods have been taken to be tensions. Equilibrium at ANY pin-joint (not R) Correct equation(s) that leads directly to finding T_{OP} or T_{QP} o.e. Accept 72.1 N A second equilibrium equation leading to a second internal force cao T/C correct for both rods