OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** Fig. 1.1 shows the velocities of a tanker of mass 120 000 tonnes before and after it changed speed and direction.

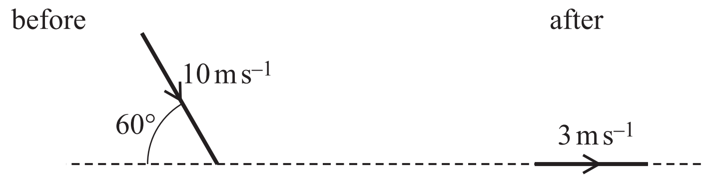


Fig. 1.1

Calculate the magnitude of the impulse that acted on the tanker. **[4]**

(b) An object of negligible size is at rest on a horizontal surface. It explodes into two parts, P and Q, which then slide along the surface.

Part P has mass 0.4 kg and speed $6\,\mathrm{m\,s^{-1}}$. Part Q has mass 0.5 kg.

(i) Calculate the speed of Q immediately after the explosion. State how the directions of motion of P and Q are related. **[2]**

The explosion takes place at a distance of 0.75 m from a raised vertical edge, as shown in Fig. 1.2. P travels along a line perpendicular to this edge.

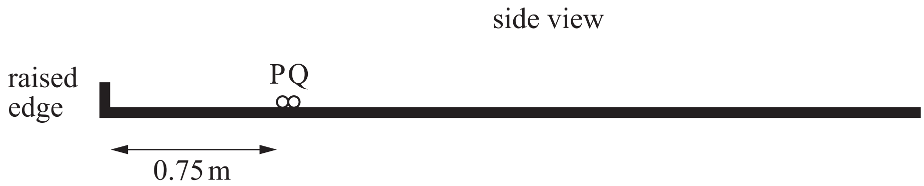


Fig. 1.2

After the explosion, P has a perfectly elastic direct collision with the raised edge and then collides again directly with Q. The collision between P and Q occurs $\frac{2}{3}$ s after the explosion. Both collisions are instantaneous.

The contact between P and the surface is smooth but there is a constant frictional force between Q and the surface.

(ii) Show that Q has speed $2.7\,\mathrm{m\,s^{-1}}$ just before P collides with it. **[4]**

(iii) Calculate the coefficient of friction between Q and the surface. **[4]**

(iv) Given that the coefficient of restitution between P and Q is $\frac{1}{8}$, calculate the speed of Q immediately after its collision with P. **[5]**

2 This question is about 'kart gravity racing' in which, after an initial push, unpowered home-made karts race down a sloping track.

The moving karts have only the following resistive forces and these both act in the direction opposite to the motion.

- A force R, called rolling friction, with magnitude $0.01Mg\cos\theta$ N where M kg is the mass of the kart and driver and θ is the angle of the track with the horizontal
- A force F of varying magnitude, due to air resistance

A kart with its driver has a mass of 80 kg.

One stretch of track slopes uniformly downwards at 4° to the horizontal. The kart travels 12 m down this stretch of track. The total work done by the kart against both rolling friction and air resistance is 455 J.

(i) Calculate the work done against air resistance. **[4]**

(ii) During this motion, the kart's speed increases from $2\,\text{m}\,\text{s}^{-1}$ to $v\,\text{m}\,\text{s}^{-1}$. Use an energy method to calculate v. **[5]**

To reach the starting line, the kart (with the driver seated) is pushed *up* a slope against rolling friction and air resistance.

At one point the slope is at 5° to the horizontal, the air resistance is 15 N, the acceleration of the kart is $1.5\,\text{m}\,\text{s}^{-2}$ up the slope and the power of the pushing force is 405 W.

(iii) Calculate the speed of the kart at this point. **[7]**

3 The object shown shaded in Fig. 3.1 is cut from a flat sheet of thin rigid uniform material; LMJK, OAIJ, AEFH and CDEB are rectangles. The grid-lines in Fig. 3.1 are 1 cm apart.

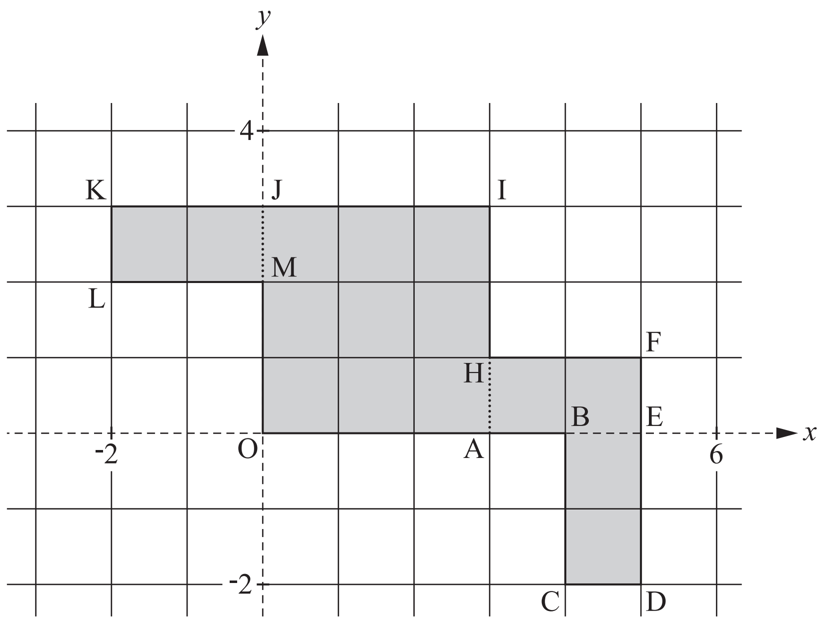


Fig. 3.1

(i) Calculate the coordinates of the centre of mass of the object referred to the axes shown in Fig. 3.1. **[5]**

The object is freely suspended from the point K and hangs in equilibrium.

(ii) Calculate the angle that KI makes with the vertical. **[4]**

The mass of the object is 0.3 kg.

A particle of mass m kg is attached to the object at a point on the line OJ so that the new centre of mass is at the centre of the square OAIJ.

(iii) Calculate the value of m and the position of the particle referred to the axes shown in Fig. 3.1. **[6]**

The extra particle is now removed and the object shown in Fig. 3.1 is folded: LMJK is folded along JM so that it is perpendicular to OAIJ; ABCDEFH is folded along AH so that it is perpendicular to OAIJ and on the same side of OAIJ as LMJK. The folded object is placed on a horizontal table with the edges KL and FED in contact with the table. A plan view and a 3D representation are shown in Fig. 3.2.

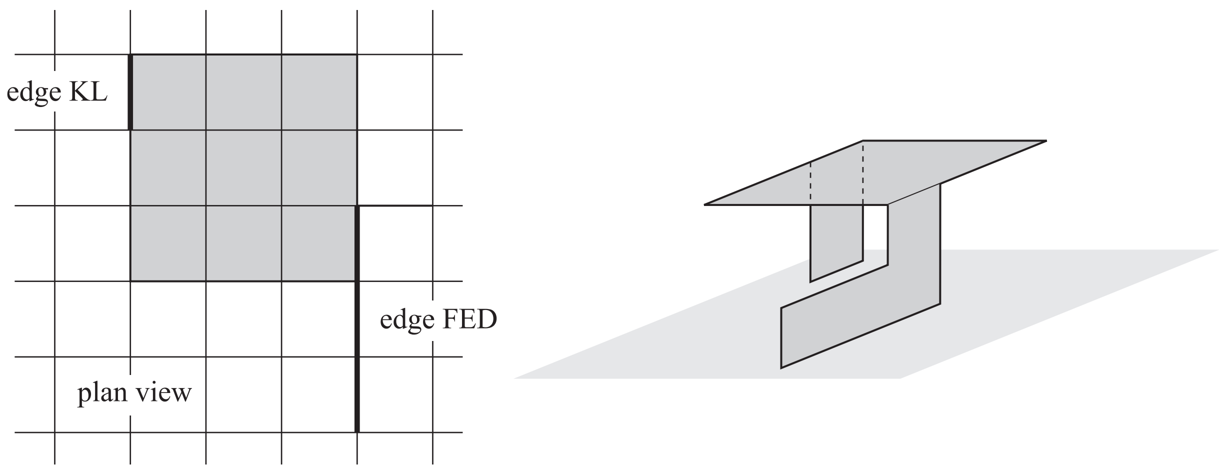


Fig. 3.2

(iv) On the plan, indicate the region corresponding to positions of the centre of mass for which the folded object is stable.

You are given that the x-coordinate of the centre of mass of the folded object is 1.7. Determine whether the object is stable. **[4]**

4 A rigid thin uniform rod AB with length 2.4 m and weight 30 N is used in different situations.

(i) In the first situation, the rod rests on a small support 0.6 m from B and is held horizontally in equilibrium by a vertical string attached to A, as shown in Fig. 4.1.

Fig. 4.1

Calculate the tension in the string and the force of the support on the rod. **[4]**

(ii) In the second situation, the rod rests in equilibrium on the point of slipping with end A on a horizontal floor and the rod resting at P on a fixed block of height 0.9 m, as shown in Fig. 4.2. The rod is perpendicular to the edge of the block on which it rests and is inclined at θ to the horizontal.

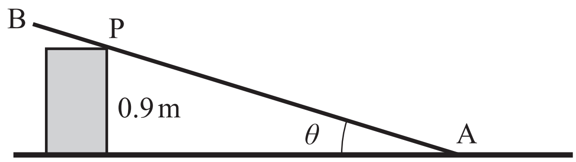


Fig. 4.2

(*A*) Suppose that the contact between the block and the rod is rough with coefficient of friction 0.6 and contact between the end A and the floor is smooth.

Show that $\tan\theta = 0.6$. **[5]**

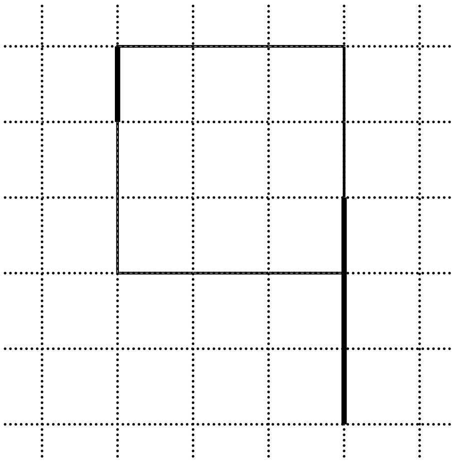
(*B*) Suppose instead that the contact between the block and the rod is smooth and the contact between the end A and the floor is rough. The rod is now in limiting equilibrium at a different angle θ such that the distance AP is 1.5 m.

Calculate the normal reaction of the block on the rod.

Calculate the coefficient of friction between the rod and the floor. **[9]**

3 (iv)

plan view (with shading removed)



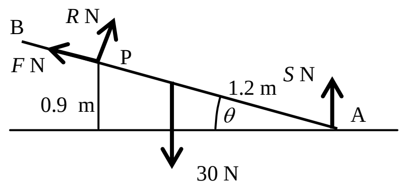
Question			Answer	Marks	Guidance
1	(a)		Take **j** north and **i** east velocity: before 5**i** - 5√3**j** (after 3**i**) **I** = m(**v** – **u**) so **I** = 120 000 000(– 2**i** + 5√3**j**) Modulus is 120 000 000 × 8.888194… = 1.0665… × 10^9 N s so 1.07 × 10^9 N s (to 3 s. f.)	B1 M1 A1 A1 **[4]**	Resolving initial velocity (may be implied). Allow 5**i** +5√3**j** **or** 5**i** - 5√3**j** oe May be implied Allow if only one direction considered or both combined without vectors. Must include an attempt to resolve 10 Accept mass of 120 000 cao Alternative method using a diagram, cos and sine rules
1	(b)	(i)	PCLM 0.4 × 6 = 0.5 V V = 4.8 ms^{-1} direction is opposite to that of P	M1 A1 **[2]**	Implied by 4.8 or -4.8 Allow -4.8 as the speed
1	(b)	(ii)	P travels 6 × $\frac{2}{3}$ = 4 m before the collision so Q travels 4 – 2 × 0.75 = 2.5 m in $\frac{2}{3}$ s $2.5 = \frac{(4.8 + v_Q)}{2} \times \frac{2}{3}$ Hence v_Q = 2.7 ms^{-1}	B1 B1 M1 E1 **[4]**	Or find $t = \frac{13}{24}$ for time from edge to collision AND $d = 3.25$ 3.25 - 0.75 = 2.5 Using appropriate *suvat* FT their 2.5 Answer given

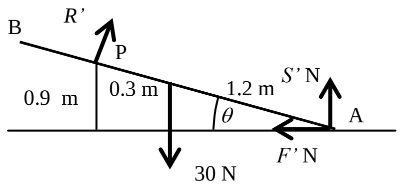
Question			Answer	Marks	Guidance
1	(b)	(iii)	Suppose friction on Q is F $-F \times \frac{2}{3} = 0.5(2.7 - 4.8)$ so $F = 1.575$ $1.575 = \mu \times 0.5 \times 9.8$ $\mu = 0.32142...$ so 0.321 (3 s. f.)	 B1 M1 A1 A1 **[4]**	 Using $Ft = m(v - u)$ or find $a = -3.15$ and use $F = ma$. FT **their** 2.7 $F = \mu R$ R correct (4.9) cao Note: F and R need not be explicit: $F{=}ma$ and $R{=}mg$ give $\mu = \frac{a}{g}$ (M1A1). Find a = -3.15(B1) gives 0.321 (A1)
1	(b)	(iv)	Let the speeds after be V_P and V_Q. PCLM $0.4 \times 6 + 0.5 \times 2.7 = 0.4\,V_P + 0.5V_Q$ so $4V_P + 5V_Q = 37.5$ NEL $\frac{V_Q - V_P}{2.7 - 6} = -\frac{1}{8}$ so $V_Q - V_P = 0.4125$ $V_Q = 4.35$ so 4.35 m s^{-1}	 M1 A1 M1 A1 A1 **[5]**	 PCLM. FT **their** 2.7 from (ii). Award M1A0 for use of their 4.8 from (i) instead of 2.7 FT **their** 2.7 from (ii). Accept any form NEL. FT **their** 2.7 from (ii). Award M1A0 for use of their 4.8 from (i) instead of 2.7 FT **their** 2.7 from (ii). Accept any form cao

Question			Answer	Marks	Guidance
2		**(i)**	$455 = 0.01 \times 80 \times 9.8 \times \cos 4 \times 12 + \text{WD}$ WD = 361.149… so 361 J (3 s. f.)	M1 B1 A1 A1 **[4]**	Use of *Fx* rolling friction force correct (7.82) 12 not needed All correct terms in an equation (allow sign errors) cao SC B1B1 for final answer 30.1 seen
2		**(ii)**	$0.5 \times 80 \times v^2 - 0.5 \times 80 \times 2^2$ $= 80 \times 9.8 \times 12 \times \sin 4 - 455$ $v = 3.0052..$ so 3.01 m s^{-1} (3 s. f.)	M1 B1 B1 A1 A1 **[5]**	Use of W-E equation. Must include GPE, at least one KE and the WD Either KE term GPE term (656.27) All correct terms in an equation (allow sign errors) cao
2		**(iii)**	Using N2L with driving force *S* $S - (15 + 0.01 \times 80 \times 9.8 \times \cos 5)$ $- 80 \times 9.8 \times \sin 5$ $= 80 \times 1.5$ $S = 211.1402\ldots$ $405 = Sv$ so $v = 1.918\ldots$ so 1.92 m s^{-1} (3 s. f.)	M1 B1 B1 A1 A1 M1 A1 **[7]**	N2L with at most one force term missing Both resistance terms seen (15 and 7.81) Condone wrong sign (68.33) All correct terms present; allow sign errors May be implicit Use of Power = *Sv* with any *S* calculated using N2L FT their *S* Note: missing out one term in N2L can earn 4/7 (M1B1B0A0A0M1A1)

Question			Answer	Marks	Guidance
3		(i)	$15\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=2\begin{pmatrix}-1\\2.5\end{pmatrix}+9\begin{pmatrix}1.5\\1.5\end{pmatrix}+2\begin{pmatrix}4\\0.5\end{pmatrix}+2\begin{pmatrix}4.5\\-1\end{pmatrix}$	M1	A systematic method for at least 1 cpt
				A1	*Either* all x or all y values correct *or* 2 vector terms correct on RHS
				A1	Completely correct expressions seen for all components
			$=\begin{pmatrix}28.5\\17.5\end{pmatrix}$		Need not be explicit
			so $\bar{x}=1.9$	A1	Accept any form
			$\bar{y}=\frac{7}{6}$	A1	Accept any form (1.17, 1.2) but not 1.16
				[5]	
3		(ii)	Referred to Fig 3.1 with c.m. G, G is 2 + 1.9 to the right of K and $3-\frac{7}{6}=\frac{11}{6}$ below K	B1	FT from (i) May be implied
			When hanging, G is vertically below K	B1	May be implied
			Angle is $\arctan\left(\frac{\frac{11}{6}}{3.9}\right)$	M1	o.e. FT their values but must be attempting to find the appropriate angle
			= 25.1775… so 25.2° (3 s. f.)	A1	cao
				[4]	
3		(iii)	New c.m. is at (1.5, 1.5) & mass of object is 0.3 kg	B1	Do not penalise below if mass of lamina is taken to be 15
			For $\bar{x}$:	M1	Recognising need first to produce an equation in terms of m for the x-component
			$(0.3 + m) \times 1.5 = 0.3 \times 1.9 + m \times 0$	M1	Must be 0 not x
			$m = 0.08$	A1	FT their 1.9 from (i). If 15 used, accept $m = 4$
			For $\bar{y}$: $(0.3 + 0.08) \times 1.5 = 0.3 \times \frac{7}{6} + 0.08y$	M1	
			so particle should be at (0, 2.75)	A1	cao. Condone no reference to x component. Allow obtained using 15. Allow 2.74, 2.7375 (from 1.17), 2.775 (from 1.16), 2.625 (from 1.2)
				[6]	

Question			Answer	Marks	Guidance
3		(iv)	The c.m. must lie inside KFDL as seen in the plan in Fig. 3.2	E1	Some indication of this is what is required. Accept a closed region with KF correct and sides parallel to KL and FD.
				E1	Correct. Accept freehand.
			The c.m. shown to be in this region	M1	Recognition that com is at (1.7, their $\bar{y}$) and is related to their critical region even if region is incorrect or calculation with at least 1 correct equation ($3y+2x=9$ and $3y+4x=6$) Do NOT award simply for a recalculation of com as (1.7, 7/6)
				E1	Properly established including a statement. (i.e. correct region, correct com marked and statement of stability)
				[4]	

Question			Answer	Marks	Guidance
4		**(i)**	Let vertical force from support be R N and tension in string T N. moments about A $30 \times 0.5 \times 2.4 - R \times (2.4 - 0.6) = 0$ $R = 20$ so force from block is 20 N $\uparrow$ $R + T - 30 = 0$ $T = 10$ so tension is 10 N	 M1 A1 M1 F1 **[4]**	 Use of moments with all relevant moments attempted (FT from T if T found first) FT from R
4	**(ii)**	(A)	 $\rightarrow \quad R\sin\theta - F\cos\theta = 0$ As on the point of slipping $F = 0.6R$ so $R\sin\theta = 0.6R\cos\theta$ so $\sin\theta = 0.6\cos\theta$ and $\tan\theta = 0.6$ OR $\quad F = mg\sin\theta - S\sin\theta$ $\quad R = mg\cos\theta - S\cos\theta$ As on the point of slipping $F = 0.6R$ $\frac{F}{R} = \frac{(mg - S)\sin\theta}{(mg - S)\cos\theta} = \frac{\sin\theta}{\cos\theta}$ $\tan\theta = 0.6$	M1 A1 M1 M1 E1 **[5]** M1 A1 M1 M1 E1 **[5]**	 Must be consideration of a force at A F and R must be identified, e.g. on a diagram Complete argument Resolve parallel and perpendicular to rod Both correct F and R must be identified, e.g. on a diagram Divide factored expressions with S included

Question			Answer	Marks	Guidance
4	(ii)	(*B*)			
			AP is 1.5 gives sin θ = 0.6 or cos θ = 0.8	B1	oe. or $\theta = 36.9^\circ$
			c. w. moments about A		
			$1.5R' - 30 \times 1.2 \times \cos\theta = 0$	M1	Moments and all terms present. Accept $\cos\theta$ or 0.8
			R' = 19.2 so 19.2 N	A1	cao
			$\uparrow \quad S' + R'\cos\theta - 30 = 0$	M1	An equilibrium equation with all relevant forces, resolved appropriately, e.g. $R' + S'\cos\theta = 30\cos\theta + F'\sin\theta$. Allow $\sin \leftrightarrow \cos$
			(*S'* = 14.64)	A1	Correct equation involving only *S'*. Numerical answer not required
			$\rightarrow \quad R'\sin\theta - F' = 0$	M1	Second equilibrium equation with all relevant forces, resolved appropriately. e.g. $F'\cos\theta + S'\sin\theta = 30\sin\theta$. Allow $\sin \leftrightarrow \cos$
			(*F'* = 11.52)	A1	Correct equation involving only *F'*. Numerical answer not required
			$\mu = \frac{11.52}{14.64}$	M1	Use of $F' = \mu S'$ for a calculated *F'* and *S'*
			= 0.78688… so 0.787 (3 s. f.)	A1 **[9]**	cao

4762 Mechanics 2

General Comments

The quality of the responses of candidates on many topics was again of a pleasingly high standard. Candidates seem confident when they are on familiar territory, with questions on centres of mass, conservation of momentum, Newton's experimental law and basic resolution of forces and moments. On this paper, there were two more unusual requests in Question 3(iv) and Question 4(ii)(*A*) and candidates struggled to apply their knowledge to unfamiliar situations. In the case of Question 4, many candidates put themselves at great disadvantage by attempting to proceed without the use of a diagram. It cannot be emphasised enough that a diagram is crucial when tackling a question involving the equilibrium of forces. Having drawn a diagram, candidates are then advised to take a moment to think about ALL of the forces that are involved, and the direction in which each acts.

Comments on Individual Questions

1 Momentum, impulse and collisions

(a) A significant number of candidates did not understand the concept of momentum as a vector quantity and worked only with the 'horizontal' component. Of those candidates who did use vectors, only a minority went on to find the magnitude of the impulse. A common error by many candidates was to treat the mass of the tanker as 120 000 kg rather than 120 000 tonnes.

(b)(i) The majority of candidates understood that the directions of motion after the collision had to be opposite, in order that momentum was conserved. There seemed to be confusion, however, between speeds and velocities.

(ii) Candidates offered a large variety of different solutions, often very convoluted, but still successful. Those candidates who did not consider the distances travelled by P and Q before their collision rarely earned any marks.

(iii) There were many concise and accurate solutions. Any loss of marks was usually due to an incorrect calculation for the frictional force.

(iv) Candidates were on very familiar territory here with a routine application involving the principle of conservation of linear momentum and Newton's experimental law. A few candidates made arithmetical and sign errors or used incorrect velocities, but the majority produced neat accurate solutions.

2 Work and energy

(i) The majority of candidates produced solutions that indicated a good understanding of the relationship between forces and work done. Others seemed to confuse themselves and gave a force rather than the work done by the force as the final answer.

(ii) Again, there were many good solutions, demonstrating good understanding of energy and work. Any errors were usually due to the omission of the gravitational potential term in the energy equation.

(iii) A significant number of candidates omitted either the weight component or part or all of the resistive term in their application of Newton's second law. Almost all, having found a force, used the formula for power as force times distance and thereby gained follow through marks. The modal mark for this question was 4/7.

3 Centres of mass and stability

(i) The vast majority of candidates scored full marks with solutions displaying very clear systematic approaches to the problem.

(ii) Again, a high proportion of candidates scored well on this part of the question. The minority of candidates who fared less well usually attempted to solve the problem without the aid of a diagram. As always in requests of this type, a clear diagram with relevant lengths marked would have been invaluable.

(iii) Solutions to this more searching request were of a pleasingly good standard, with many candidates displaying a sound understanding of the principles involved. A surprising number of candidates did, however, repeat unnecessarily their calculations from part (i). The range of the final answers given by candidates varied considerably, largely because of some premature or incorrect rounding errors in the value of the *y*-coordinate the initial centre of mass. Candidates should be encouraged to work with exact values wherever possible.

(iv) This was an unusual question that seemed to throw the vast majority of candidates. Few were able to visualise the physical situation being described and the shading indicating the region of stability of the folded object often appeared to be either vague and random or non-existent. A significant number of candidates did seem to appreciate that the *y*-coordinate of the centre of mass was unchanged, although this was often deduced from yet another recalculation of work already done in part (i). However, few candidates plotted the position of the centre of mass and concluded that the object was stable because the point was within their shaded region. A few candidates worked out equations for the lines bordering the region and argued algebraically that their centre of mass was in their region.

4 Forces and equilibrium

(i) Most candidates earned full marks on this simple application of moments. A minority of candidates found only the tension in the string.

(ii) Candidates seemed to struggle with the two parts of this question. A major reason for this was the lack of a diagram. It is difficult to imagine how a candidate might hope to resolve and take moments for an equilibrium situation when they do not have a diagram with all the relevant forces marked on it. The evidence suggests very strongly that a diagram was absolutely key to any meaningful progress.

(ii)(a) Only a very small minority of candidates scored well. The majority of candidates, many working without a diagram, filled the page with a selection of equations, resulting from resolving and taking moments with largely unidentified forces. Of those candidates who drew a diagram, the normal reaction between the rod and the floor at A was often omitted or assumed to be perpendicular to the rod rather than the floor. Many solutions suggested that the candidate was confusing the given situation with the more familiar one of a block resting on an inclined plane.

(ii)(b) Again, a diagram with all the forces labelled was crucial. There seemed to be a lot of confusion about the directions of the friction and the normal reaction at A and the normal reaction at P. Candidates are also advised to think more carefully about the most appropriate directions in which to resolve and the points about which to take moments, before embarking on filling the page with equations. An apparently trivial point is to note that not all normal reactions have to be referred to as R. This assumption led to some simpler but totally erroneous attempts at solutions.