

Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4762 Mechanics 2

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4762
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $gm s^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 (a) A stone of mass 0.6 kg falls vertically 1.5 m from A to B against resistance. Its downward speeds at A and B are 5.5 m s^{-1} and 7.5 m s^{-1} respectively.
- (i) Calculate the change in kinetic energy and the change in gravitational potential energy of the stone as it falls from A to B. [3]
- (ii) Calculate the work done against resistance to the motion of the stone as it falls from A to B. [2]
- (iii) Assuming the resistive force is constant, calculate the power with which the resistive force is retarding the stone when it is at A. [4]
- (b) A uniform plank is inclined at 40° to the horizontal. A box of mass 0.8 kg is on the point of sliding down it. The coefficient of friction between the box and the plank is μ .

- (i) Show that $\mu = \tan 40^\circ$. [4]

The plank is now inclined at 20° to the horizontal.

- (ii) Calculate the work done when the box is pushed 3 m up the plank, starting and finishing at rest. [5]

- 2 The rigid object shown in Fig. 2.1 is made of thin non-uniform rods. ABC is a straight line; BC, BE and ED form three sides of a rectangle. The centre of mass of the object is at G. The lengths are in centimetres. The weight of the object is 15 N.

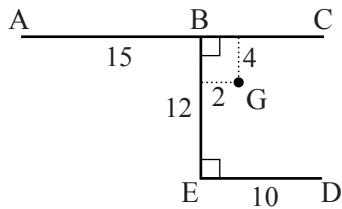


Fig. 2.1

Initially, the object is suspended by light vertical strings attached to B and to C and hangs in equilibrium with AC horizontal.

- (i) Calculate the tensions in each of the strings.

[4]

In a new situation the strings are removed. The object can rotate freely in a vertical plane about a fixed horizontal axis through A and perpendicular to ABCDE. The object is held in equilibrium with AC horizontal by a force of magnitude T N in the plane ABCDE acting at C at an angle of 30° to CA. This situation is shown in Fig. 2.2.

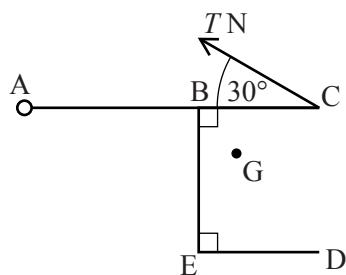


Fig. 2.2

- (ii) Calculate T .

Calculate also the magnitude of the force exerted on the object by the axis at A.

[6]

The object is now placed on a rough horizontal table and is in equilibrium with ABCDE in a vertical plane and DE in contact with the table. The coefficient of friction between the edge DE and the table is 0.65. A force of slowly increasing magnitude (starting at 0 N) is applied at A in the direction AB. Assume that the object remains in a vertical plane.

- (iii) Determine whether the object slips before it tips.

[6]

- 3 (a) You are given that the position of the centre of mass, G, of a right-angled triangle cut from thin uniform material in the position shown in Fig. 3.1 is at the point $(\frac{1}{3}a, \frac{1}{3}b)$.

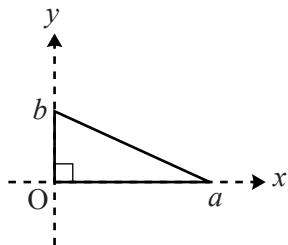


Fig. 3.1

A plane thin uniform sheet of metal is in the shape OABCDEFHIJO shown in Fig. 3.2. BDEA and CDIJ are rectangles and FEH is a right angle. The lengths of the sides are shown with each unit representing 1 cm.

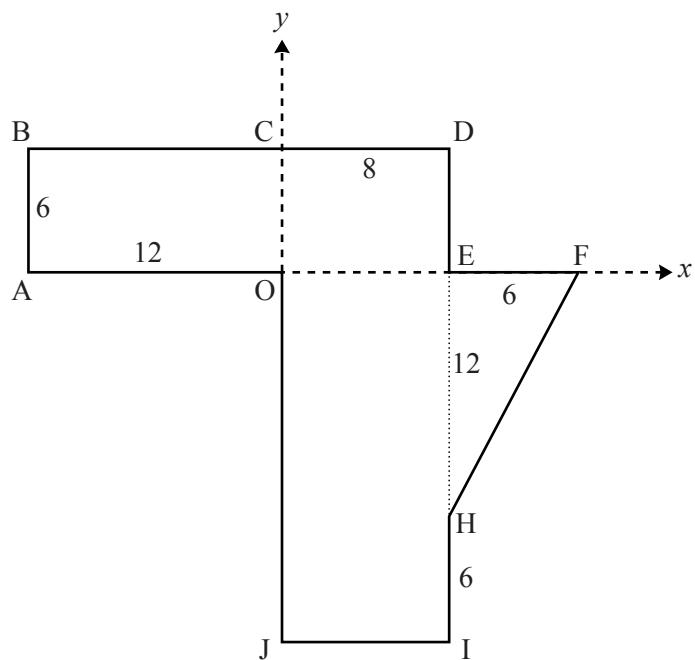


Fig. 3.2

- (i) Calculate the coordinates of the centre of mass of the metal sheet, referred to the axes shown in Fig. 3.2. [5]

The metal sheet is freely suspended from corner B and hangs in equilibrium.

- (ii) Calculate the angle between BD and the vertical. [4]

- (b) Part of a framework of light rigid rods freely pin-jointed at their ends is shown in Fig. 3.3. The framework is in equilibrium.

All the rods meeting at the pin-joints at A, B and C are shown. The rods connected to A, B and C are connected to the rest of the framework at P, Q, R, S and T.

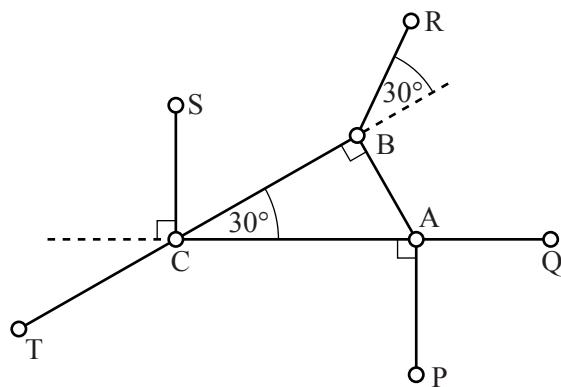


Fig. 3.3

There is a tension of 18 N in rod AP and a thrust (compression) of 5 N in rod AQ.

- (i) Show the forces internal to the rods acting on the pin-joints at A, B and C. [2]
- (ii) Calculate the forces internal to the rods AB, BC and CA, stating whether each rod is in tension or compression. [You may leave your answers in surd form. Your working in this part should be consistent with your diagram in part (i).] [7]

- 4 P and Q are circular discs of mass 3 kg and 10 kg respectively which slide on a smooth horizontal surface. The discs have the same diameter and move in the line joining their centres with no resistive forces acting on them. The surface has vertical walls which are perpendicular to the line of centres of the discs. This information is shown in Fig. 4 together with the direction you should take as being positive.

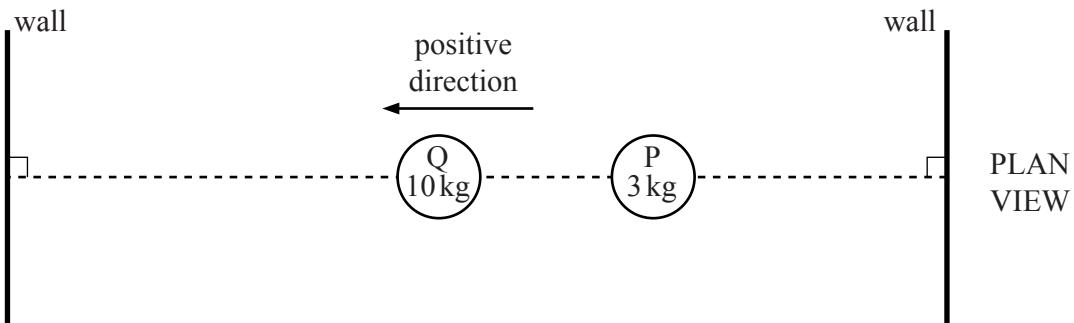


Fig. 4

- (i) For what time must a force of 26 N act on P to accelerate it from rest to 13 m s^{-1} ? [2]

P is travelling at 13 m s^{-1} when it collides with Q, which is at rest. The coefficient of restitution in this collision is e .

- (ii) Show that, after the collision, the velocity of P is $(3 - 10e) \text{ m s}^{-1}$ and find an expression in terms of e for the velocity of Q. [7]

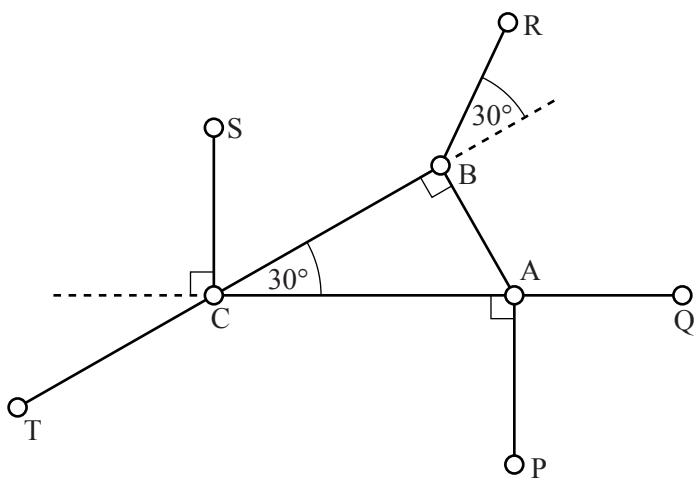
- (iii) For what set of values of e does the collision cause P to reverse its direction of motion? [2]

- (iv) Determine the set of values of e for which P has a greater speed than Q immediately after the collision. [4]

You are now given that $e = \frac{1}{2}$. After P and Q collide with one another, each has a perfectly elastic collision with a wall. P and Q then collide with one another again and in this second collision they stick together (coalesce).

- (v) Determine the common velocity of P and Q. [4]

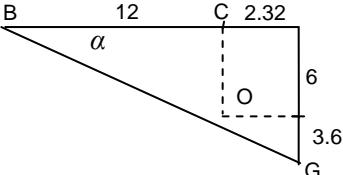
- (vi) Determine the impulse of Q on P in this collision. [1]

3 (b) (i)**3 (b) (ii)**

(answer space continued on next page)

Question		Answer	Marks	Guidance
1	(a) (i)	KE change: $\frac{1}{2} \times 0.6 \times (7.5^2 - 5.5^2)$ = 7.8 J GPE change: $0.6 \times 9.8 \times 1.5 = 8.82$ J	M1 A1 B1 [3]	Difference of two KE terms Allow -8.82J
1	(a) (ii)	W is work done against resistance $7.8 = 8.82 - W$ so $W = 1.02$ J	M1 A1 [2]	W-E all terms. Allow sign errors FT (i) only. Also FT only if mod (their KE) < mod (their PE) -1.02 gets M1A0; 16.62 gets M1A0
1	(a) (iii)	Average resistance is F so $F \times 1.5 = 1.02$ so $F = 0.68$ Power is 0.68×5.5 = 3.74 so 3.74 W	M1 A1 M1 A1 [4]	Use of $WD = Fs$ OR find $a = 8.667$ and use $F = 0.6g - 0.6 \times 8.667$ May be implied. FT (ii) Use of $P = Fv$ any calculated F cao
1	(b) (i)	$R = mg\cos 40$ $F_{\max} = mgs\sin 40$ $F_{\max} = \mu R$ so $\mu = \frac{mg \sin 40}{mg \cos 40} = \tan 40$	B1 B1 M1 E1 [4]	Seen or implied Seen or implied Use of $F = \mu R$: substitute F and R This is the minimum amount of working needed to earn the E1 Must see explicit evidence of method Note: g omitted, treat as MR
1	(b) (ii)	EITHER $\tan 40 \times 0.8 \times 9.8 \times \cos 20$ $\times 3 (= 18.545)$ $(+)0.8 \times 9.8$ $\times 3 \sin 20 (= 8.044)$ $= 26.5897\dots$ so 26.6 J (3 s.f.)	B1 M1 B1 M1 A1	Use of $F_{\max} = \mu R$ with $\tan 40$ and $\cos 20$ Use of $WD = Fs$ NOTE: This mark may be awarded here or for use in PE term Use of mgh Allow $\sin \leftrightarrow \cos$ interchange Two relevant terms added Cao Allow 26.7 Allow 27 Omission of g can get B0M1B1M1A0

Question		Answer	Marks	Guidance
		OR $\tan 40 \times 0.8 \times 9.8 \times \cos 20 (= 6.182)$ $(+) 0.8 \times 9.8 \times \sin 20 (= 2.68)$ $(= 8.8632444\dots)$ WD is $3 \times 8.8632444\dots$ $= 26.5897\dots$ so 26.6 J (3 s.f.)	B1 B1 M1 M1 A1 [5]	Use of $F_{\max} = \mu R$ with $\tan 40$ and $\cos 20$ Allow $\sin \leftrightarrow \cos$ interchange Two relevant forces added Use of $WD = Fs$ (for at least one of forces) cao Omission of g can get B0B1M1M1A0
2	(i)	a.c. moments about B $10T_C - 15 \times 2 = 0$ so $T_C = 3$. Tension at C is 3 N $\uparrow T_C + T_B - 15 = 0$ so $T_B = 12$. Tension at B is 12 N	M1 A1 M1 F1 [4]	Moments with all forces present, no extra forces. May take moments again
2	(ii)	a.c. moments about A $25T \sin 30 - 15 \times 17 = 0$ so $T = 20.4$ At A Let force \uparrow be Y N $\uparrow Y + T \sin 30 - 15 = 0$ so $Y = 4.8$ $\rightarrow X = T \cos 30 = 17.6669\dots$ N $\sqrt{4.8^2 + (T \cos 30)^2}$ $= 18.3073755\dots$ so 18.3 N (3 s.f.)	M1 A1 B1 B1 M1 A1 [6]	Attempt at moments with resolution; allow $\cos \leftrightarrow \sin$ error. All forces present, no extra forces cao FT (can take moments about C) FT Need not be evaluated cao
2	(iii)	Let force be P . a.c. moments about D. $8 \times 15 - 12 \times P = 0$ so $P = 10$ on point of tipping Using $F_{\max} = \mu R$ on point of slipping with $R = 15$ gives $F_{\max} = 0.65 \times 15 = 9.75$ so slips first	M1 A1 M1 B1 A1 E1 [6]	Moments about D with all forces present, no extra forces cao cao cao and WWW

Question			Answer	Marks	Guidance
3	(a)	(i)	$300 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 72 \begin{pmatrix} -6 \\ 3 \end{pmatrix} + 192 \begin{pmatrix} 4 \\ -6 \end{pmatrix} + 36 \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 696 \\ -1080 \end{pmatrix}$ so $\bar{x} = 2.32$ $\bar{y} = -3.6$	B1 M1 B1 A1 A1 [5]	Correctly identifying the position of the c.m. of triangle EFH (10, -4) A systematic method for at least 1 cpt <i>Either</i> all x or all y values correct <i>or</i> 2 vector terms correct <i>or</i> allow one common error in both components, e.g. one wrong mass, misunderstanding of c.m. of triangle Allow FT for either if only error is common to both
3	(a)	(ii)	 centre of mass is at G $\tan \alpha = \frac{9.6}{14.32}$ so $\alpha = 33.8376\dots$ so 33.8° (3 s.f.)	M1* B1 M1dep* A1 [4]	Identifying correct angle. May be implied At least 1 relevant distance found. FT (i) Use of $\arctan \frac{9.6}{14.32}$ or $\arctan \frac{14.32}{9.6}$ o.e. cao or $180^\circ - 33.8^\circ$
3	(b)	(i)	Marking given tension and thrust Marking all other forces internal to rods acting on A, B and C (as T or C)	B1 B1 [2]	Each labelled with magnitude and correct direction Need ALL forces at A, B and C. Need pairs of arrows on AB, AC and BC

Question		Answer	Marks	Guidance
3	(b) (ii)	<p>Equilibrium at A ↑ $T_{AB} \cos 30 - 18 = 0$ $T_{AB} = 12\sqrt{3}$. Force in AB: $12\sqrt{3}$ N (T) A ← $T_{AC} + T_{AB} \cos 60 + 5 = 0$ $T_{AC} = -(5 + 6\sqrt{3})$. Force in AC: $(5 + 6\sqrt{3})$ N (C) At B in direction AB $T_{BR} \cos 60 - T_{AB} = 0$ so $T_{BR} = 24\sqrt{3}$ At B in direction BC $T_{BC} - T_{BR} \cos 30 = 0$ $T_{BC} = 36$. Force in BC: 36 N (T)</p>	M1 A1 M1 F1 M1 F1 A1 [7]	<p>Equilibrium at one pin-joint 20.8 Sign consistent with tension on their diagram -15.39 FT their T_{AB} Allow FT Other methods are possible, but award this M1 only for a complete method that would lead to T_{BC} cao WWW T/C all correct</p>
4	(i)	$26t = 3 \times 13$ $t = 1.5$ so 1.5 s	M1 A1 [2]	Use of $Ft = m(v - u)$ or N2L to find a ($= 26/3$) and use $v = u + at$ cao
4	(ii)	PCLM $10 \times 0 + 3 \times 13 = 10v_Q + 3v_P$ $39 = 10v_Q + 3v_P$ NEL $\frac{v_Q - v_P}{0 - 13} = -e$ $v_Q - v_P = 13e$ $v_Q = 3(1 + e)$ $v_P = 3 - 10e$	M1 A1 M1 A1 M1 B1 E1 [7]	Use of PCLM Any form Use of NEL. Allow sign errors but not inversion Any form Eliminating one of v_Q or v_P OR allow substitution of given result in one equation and check both answers in other equation cao; aef Properly shown

Question		Answer	Marks	Guidance
4	(iii)	Need $v_P < 0$ so $3 - 10e < 0$ Hence $\frac{3}{10} < e \leq 1$	M1 A1 [2]	Accept \leq cao (Allow $e \leq 1$ omitted) Correct answer www gets 2/2
4	(iv)	When $e > \frac{3}{10}$, its speed is $10e - 3$ We require $(10e - 3) > 3(1 + e)$ so $7e > 6$ and so $\frac{6}{7} < e \leq 1$	M1 M1 A1 A1 [4]	FT their v_Q SC1 for $(3 - 10e) > \pm 3(1 + e)$ FT their v_Q cao. Allow $e > \frac{6}{7}$ (0.857) Correct answer www gets 4/4
4	(v)	Either $v_Q = 4.5$ and $v_P = -2$ When they collide the speed of Q is -4.5 and of P is 2 PCLM $10 \times -4.5 + 3 \times 2 = 13V$ so $V = -3$ and velocity is -3 m s^{-1}	M1 M1 M1 A1 [4]	Substitute $e = 0.5$; FT their v_Q Change signs of their velocities Use of PCLM Allow sign errors cao; OR 3 m s^{-1} to the right or use argument about final LM is $-ve$ of original LM
		Or $10(-3(1+e)) + 3(10e-3) = 13V$ $-39 = 13V$ so $V = -3$ and velocity is -3 m s^{-1}	M1 M1 M1 A1 [4]	Use of PCLM; Allow sign errors ; FT their v_Q Change signs of their velocities Simplify cao; OR 3 m s^{-1} to the right
4	(vi)	$3(-3 - 2) = -15 \text{ N s}$	B1 [1]	FT $3(\text{their}(v) - 2)$ Using $10(-3 + 4.5) = 15$ gets B0 until it leads to correct answer

4762 Mechanics 2

General Comments

The general standard of the work was pleasing. Many candidates showed a sound understanding of the methods and techniques involved and most presented their solutions with clarity. It is worth emphasising, however, that a good clear diagram is often the key to a successful solution. It is also important that candidates read the question carefully. When there is a supporting diagram, information may be given in the text or on the diagram, or both. Particularly careful reading is required when a scenario changes in the later parts of questions to ensure that only relevant information is carried forward.

Comments on Individual Questions

- 1) (a) Work, energy and power
 - (i) The vast majority of candidates were able to state and use the formulae for kinetic and potential energy and scored full marks. A minority found only the kinetic energy change.
 - (ii) Most candidates who found the energy changes in part (i) were able to combine them appropriately to find the work done against resistance. A few made sign errors. Those candidates who did not find the potential energy change in part (i) usually gave it as the answer to this part of the question.
 - (iii) The most concise solutions to this request involved the use of the two formulae, 'work done = force x distance' and 'power = force x velocity.' Many candidates, however, pursued an incorrect method, using *suvat* to calculate a time and then dividing their answer for the work done from part (ii) by this calculated time. This resulted in an 'average' power, which was indicative of a lack of understanding of the fact that power changes with velocity. Other candidates calculated a force, often the weight of the stone, and multiplied this by 5.5.
- (b) Frictional force
 - (i) This was answered well by almost all candidates. A few solutions were rather too brief and candidates should be aware that they need to give adequate working to support a given answer. A small minority of candidates either omitted g in their calculations or lost accuracy when using numerical values for the trigonometric functions.
 - (ii) Only a minority of candidates scored full marks on this question. Many offered incomplete solutions, including only one of the two required terms, usually the potential energy term. A significant number of candidates changed the value of the coefficient of friction because the angle of inclination of the plane was changed.
- 2) Rigid body in equilibrium
 - (i) Most candidates scored full marks. A small percentage of candidates misread the question and attached the strings to *A* and *C* instead of *B* and *C*.
 - (ii) Most candidates were able to calculate T correctly, by taking moments about *A*. The vast majority of candidates then went on to calculate just *one* component of the force exerted on the object by the axis at *A* usually the horizontal component. Of those candidates who calculated both components, a significant number did not proceed to find the magnitude of the resultant force.

- (iii) There were some very concise fully correct solutions to this part of the question, displaying a sound understanding of the principles involved. Candidates who did not score full marks seemed confident in considering the slipping situation, but were unsure about how to get a condition for tipping. They attempted to take moments, but often not about the point D . A significant minority of candidates did not read the question carefully and retained the force T , from part (ii).
- 3) (a) Centre of mass
- (i) Both the presentation and the accuracy of the solutions to this part of the question were very good. Some candidates lost marks because they used the distance from OJ of the centre of mass of the triangular part of the metal sheet as the x -coordinate in their calculations.
- (ii) The majority of candidates realised which angle they needed to calculate and did so accurately from their answer in part (i). Other candidates assumed that triangle BDG , where G is the centre of mass of the whole sheet, was right-angled at G . The minority of candidates who did not attempt to draw a diagram were rarely successful.
- (b) Light framework
- (i) Almost all candidates marked the forces internal to the rods AB , BC and CA , but many candidates omitted showing any forces on the rods CT , CS , and BR . Others put arrows in the wrong direction for each of the given forces in AP and AQ .
- (ii) The majority of candidates were able to resolve horizontally at A to find the force in the rod AB and then vertically to find the force in the rod AC . A significant number of candidates made a sign error in the vertical resolution. Finding the tension in the rod BC proved difficult for many candidates and it was common to see attempts at resolutions at B and C with forces missing and incorrect signs. A minority of candidates attempted to find the forces in all of the rods shown in the framework.
- 4) Momentum and impulse
- (i) The vast majority of candidates scored full marks.
- (ii) Most candidates were able to apply the Principle of conservation of linear momentum and Newton's experimental law effectively and then solve the resulting simultaneous equations to find the two required velocities. A small number of candidates made algebraic or arithmetic slips.
- (iii) Again, most candidates scored full marks.
- (iv) Only a minority of candidates scored more than a single mark on this part of the question. The common error was in not realising that the direction of P must have been reversed in the collision. This resulted in the inequality $e < 0$, contradicting the fact that e is a positive quantity, but this rarely alerted candidates to think again about their solution.
- (v) The majority of candidates realised that the velocities of P and Q needed to be reversed in direction when applying the Principle of conservation of linear momentum.
- (vi) Although candidates knew that the impulse was equal to the change in momentum, only a minority dealt successfully with the signs involved.

GCE Mathematics (MEI)											
			Max Mark	90% cp	a	b	c	d	e	u	
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper		Raw 100	72 18 18 100	66 16 16 90	60 15 15 80	53 13 13 70	47 11 11 60	41 9 9 50	34 8 8 40	0 0 0 0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark		Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework		UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics		Raw UMS	72 100	66 90	61 80	53 70	46 60	39 50	32 40	0 0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics		Raw UMS	72 100	61 90	54 80	47 70	40 60	34 50	28 40	0 0

4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw 18	72	68	63	57	51	45	39	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw 18	16	15	13	11	9	8	0	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw 100	18	16	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS 100	90	80	70	60	50	40	0	0
4762/01 (M2) MEI Mechanics 2	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4763/01 (M3) MEI Mechanics 3	Raw 100	72	67	63	56	50	44	38	0
	UMS 100	90	80	70	60	50	40	0	0
4764/01 (M4) MEI Mechanics 4	Raw 100	72	63	56	49	42	35	29	0
	UMS 100	90	80	70	60	50	40	0	0
4767/01 (S2) MEI Statistics 2	Raw 100	72	66	61	55	49	43	38	0
	UMS 100	90	80	70	60	50	40	0	0
4768/01 (S3) MEI Statistics 3	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4769/01 (S4) MEI Statistics 4	Raw 100	72	63	56	49	42	35	28	0
	UMS 100	90	80	70	60	50	40	0	0
4772/01 (D2) MEI Decision Mathematics 2	Raw 100	72	62	56	50	44	39	34	0
	UMS 100	90	80	70	60	50	40	0	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw 100	72	52	46	40	34	29	24	0
	UMS 100	90	80	70	60	50	40	0	0
4777/01 (NC) MEI Numerical Computation	Raw 100	72	63	55	47	39	32	25	0
	UMS 100	90	80	70	60	50	40	0	0