ADVANCED GCE

MATHEMATICS (MEI) 4762

Mechanics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4762
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 **(a)** Sphere P, of mass 10 kg, and sphere Q, of mass 15 kg, move with their centres on a horizontal straight line and have no resistances to their motion. P, Q and the positive direction are shown in Fig. 1.1.

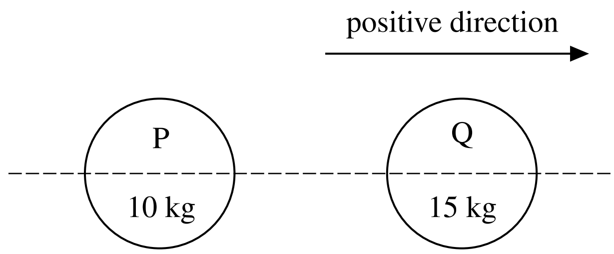


Fig. 1.1

Initially, P has a velocity of $-1.75\ \mathrm{m\,s^{-1}}$ and is acted on by a force of magnitude 13 N acting in the direction PQ.

After T seconds, P has a velocity of $4.75\ \mathrm{m\,s^{-1}}$ and has not reached Q.

(i) Calculate T. **[3]**

The force of magnitude 13 N is removed. P is still travelling at $4.75\ \mathrm{m\,s^{-1}}$ when it collides directly with Q, which has a velocity of $-0.5\ \mathrm{m\,s^{-1}}$.

Suppose that P and Q coalesce in the collision to form a single object.

(ii) Calculate their common velocity after the collision. **[2]**

Suppose instead that P and Q separate after the collision and that P has a velocity of $1\ \mathrm{m\,s^{-1}}$ afterwards.

(iii) Calculate the velocity of Q after the collision and also the coefficient of restitution in the collision. **[6]**

(b) Fig. 1.2 shows a small ball projected at a speed of $14\ \mathrm{m\,s^{-1}}$ at an angle of $30°$ below the horizontal over smooth horizontal ground.

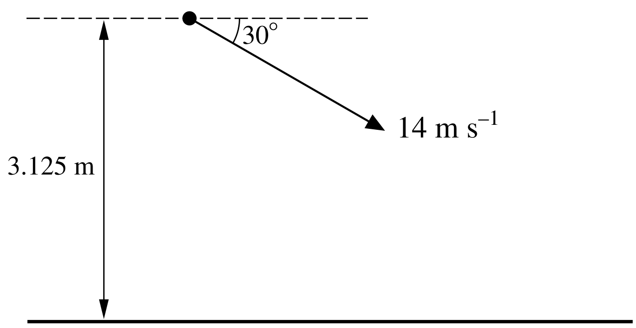


Fig. 1.2

The ball is initially 3.125 m above the ground. The coefficient of restitution between the ball and the ground is 0.6.

Calculate the angle with the horizontal of the ball's trajectory immediately after the **second** bounce on the ground. **[8]**

2 Any non-exact answers to this question should be given correct to four significant figures.

A thin, straight beam AB is 2 m long. It has a weight of 600 N and its centre of mass G is 0.8 m from end A. The beam is freely pivoted about a horizontal axis through A.

The beam is held horizontally in equilibrium.

Initially this is done by means of a support at B, as shown in Fig. 2.1.

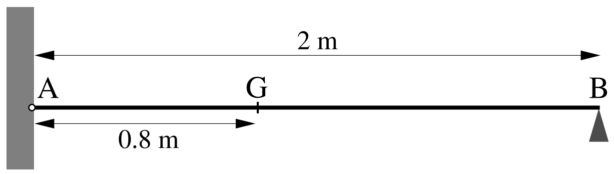


Fig. 2.1

(i) Calculate the force on the beam due to the support at B. **[3]**

The support at B is now removed and replaced by a wire attached to the beam 0.3 m from A and inclined at 50° to the beam, as shown in Fig. 2.2. The beam is still horizontal and in equilibrium.

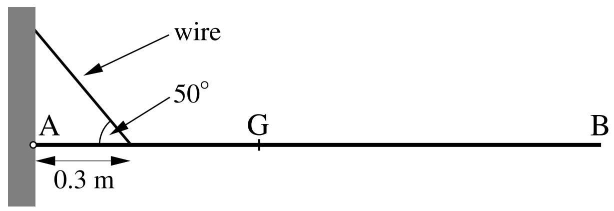


Fig. 2.2

(ii) Calculate the tension in the wire. **[5]**

The forces acting on the beam at A due to the hinge are a horizontal force X N in the direction AB, and a downward vertical force Y N, which have a resultant of magnitude R at α to the horizontal.

(iii) Calculate X, Y, R and α. **[7]**

The dotted lines in Fig. 2.3 are the lines of action of the tension in the wire and the weight of the beam. These lines of action intersect at P.

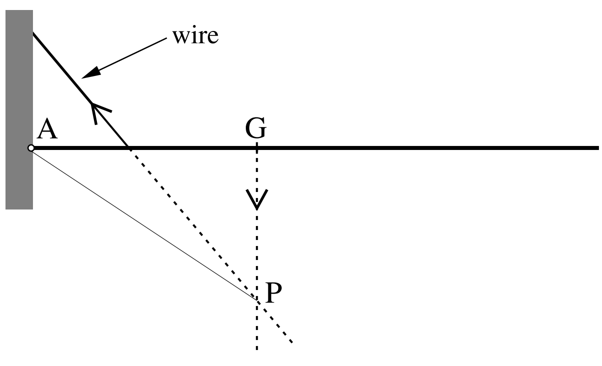


Fig. 2.3

(iv) State with a reason the size of the angle GAP. **[2]**

3 A bracket is being made from a sheet of uniform thin metal. Firstly, a plate is cut from a square of the sheet metal in the shape OABCDEFHJK, shown shaded in Fig. 3.1. The dimensions shown in the figure are in centimetres; axes Ox and Oy are also shown.

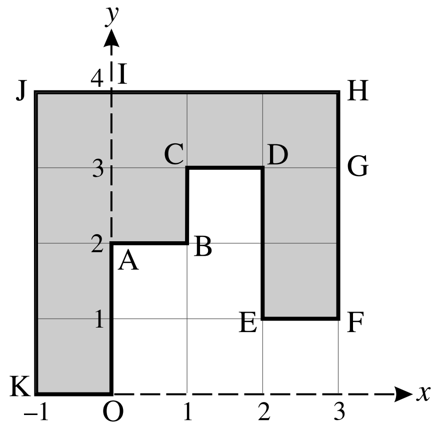


Fig. 3.1

(i) Show that, referred to the axes given in Fig. 3.1, the centre of mass of the plate OABCDEFHJK has coordinates (0.8, 2.5). **[4]**

The plate is hung using light vertical strings attached to J and H. The edge JH is horizontal and the plate is in equilibrium. The weight of the plate is 3.2 N.

(ii) Calculate the tensions in each of the strings. **[5]**

The plate is now bent to form the bracket. This is shown in Fig. 3.2: the rectangle IJKO is folded along the line IA so that it is perpendicular to the plane ABCGHI; the rectangle DEFG is folded along the line DG so it is also perpendicular to the plane ABCGHI but on the other side of it. Fig. 3.2 also shows the axes Ox, Oy and Oz.

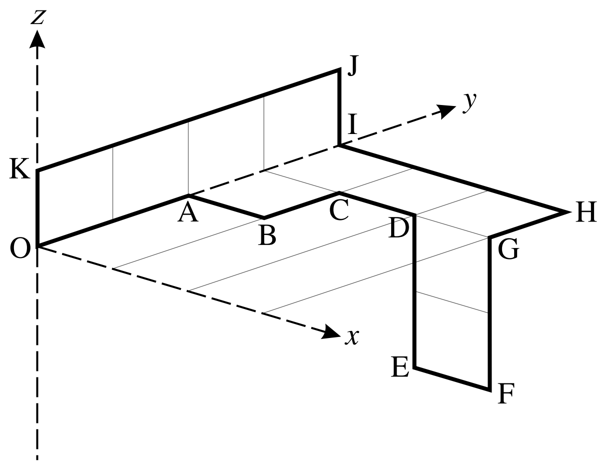


Fig. 3.2

(iii) Show that, referred to the axes given in Fig. 3.2, the centre of mass of the bracket has coordinates (1, 2.7, 0). **[5]**

The bracket is now hung freely in equilibrium from a string attached to O.

(iv) Calculate the angle between the edge OI and the vertical. **[4]**

4 **(a)** A parachutist and her equipment have a combined mass of 80 kg. During a descent where the parachutist loses 1600 m in height, her speed reduces from $V\,\mathrm{m\,s^{-1}}$ to $6\,\mathrm{m\,s^{-1}}$ and she does 1.3×10^6 J of work against resistances.

Use an energy method to calculate the value of V. **[5]**

(b) A vehicle of mass 800 kg is climbing a hill inclined at θ to the horizontal, where $\sin\theta = 0.1$. At one time the vehicle has a speed of $8\,\mathrm{m\,s^{-1}}$ and is accelerating up the hill at $0.25\,\mathrm{m\,s^{-2}}$ against a resistance of 1150 N.

(i) Show that the driving force on the vehicle is 2134 N and calculate its power at this time. **[7]**

The vehicle is pulling a sledge, of mass 300 kg, which is sliding up the hill. The sledge is attached to the vehicle by a light, rigid coupling parallel to the slope. The force in the coupling is 900 N.

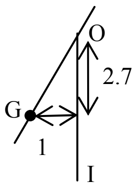
(ii) Assuming that the only resistance to the motion of the sledge is due to friction, calculate the coefficient of friction between the sledge and the ground. **[6]**

Q 1		mark	notes
(a) (i)	$13T = 10(4.75 - (-1.75))$	M1	Use of $I = Ft$. Allow sign errors
		A1	Signs correct on RHS
	so $T = 5$. So 5 s.	A1	cao
	OR: $13 = 10a$	B1	N2L
	$T = \dfrac{4.75-(-1.75)}{1.3} = 5$	M1	Use of *suvat*
		A1	cao
		3	
(ii)	PCLM: $10\times 4.75 - 15\times 0.5 = 25v_{P+Q}$	M1	PCLM with combined mass. Allow sign errors
	$v_{P+Q} = 1.6$ so 1.6 m s^{-1} in +ve direction	A1	No need for reference to direction
		2	
(iii)	PCLM: $10\times 4.75 - 15\times 0.5 = 10\times 1 + 15v_Q$	M1	PCLM with all correct terms. Allow sign errors
		A1	Any form
	Hence $v_Q = 2$ and Q has velocity 2 m s^{-1}	A1	Accept no direct reference to direction
	NEL: $\dfrac{v_Q - 1}{-0.5 - 4.75} = -e$	M1	NEL. Accept **their** v_Q and any sign errors. Fraction must be correct way up
		A1	Any form. FT **their** v_Q.
	so $e = 0.19047\ldots$ so 0.190 (3 s. f.)	A1	cao accept 0.19 , 4/21 accept 0.2 only if 0.19 seen earlier
		6	

(b)	Initial vert cpt is 14sin30 = 7	B1	
	1st hits ground at v given by		
	$v^2 = 7^2 + 2\times 9.8\times 3.125$	M1	Appropriate *suvat*. Allow ±9.8 etc Condone u =14
	v = 10.5	A1	
	Vert cpt after 2nd bounce		
	10.5×0.6^2	M1	**their** 10.5×0.6^n for n = 1, 2 or 3 Condone use of their initial vertical component. Do not award if horiz component is also multiplied by 0.6
		B1	use of $\times\ 0.6^2$ or attempt at two bounces with 0.6 used each time
	Horiz cpt is unchanged throughout ($14\cos 30$)	B1	Award even if value wrong or not given
	Angle is $\arctan\left(\dfrac{10.5\times 0.6^2}{14\cos 30}\right) = 17.31586...$	M1	FT their horiz and vert components. oe. Fraction must be for correct angle.
	so 17.3° (3 s. f.)	A1 8	cao SC answer of 11.7 will usually earn 5/8
		19	

Q 2		mark	notes
			Penalise answers to fewer than 4sf only once
(i)	cw moments about A Let force be S $600 \times 0.8 - S \times 2 = 0$ $S = 240$ so 240 N vertically upwards	 M1 A1 A1 3	 Moments. All forces. No extras Need statement of direction or diagram
(ii)	cw moments about A Let tension be T $600 \times 0.8 - T\sin 50 \times 0.3 = 0$ $T = 2088.65\ldots$ $\left(\frac{1600}{\sin 50}\right)$ so 2089 N (4 s. f.)	M1 M1 A1 A1 A1 5	Moments. All forces. No extras. Attempt at moment of T (need not be resolved) Note that mmts about B needs forces at hinge. Correct method for moment of T. Allow length errors and $s \leftrightarrow c$ Moment of T correct (allow sign error) All correct cao
(iii)	Resolve $\rightarrow$ $X - T\cos 50 = 0$ so $X = 1342.55\ldots$. = 1343 (4 s. f.) Resolve $\downarrow$ $Y - T\sin 50 + 600 = 0$ so $Y = 1000$ Method for either R or α $R = \sqrt{1600^2 \cot^2 50 + 1000^2} = 1674.05..$ so 1674 (4 s. f.) $\alpha = \arctan \dfrac{1000}{1600 \cot 50}$ $\alpha = 36.6804\ldots$ so 36.68° (4 s. f.)	M1 F1 M1 F1 M1 F1 F1 7	Resolving horiz. Allow sign error. T must be resolved, allow $s \leftrightarrow c$ FT **their** T only. Allow 1600cot50 NB other methods possible FT **their** T only M dependent on attempts at X and Y using moments/resolution FT **their** X and Y Numerical value only FT **their** X and Y Numerical value only Accept 36.67
(iv)	Angle GAP is α above so 36.68° (4 s. f.) Weight, T and R are the only forces acting on the beam which is in equilibrium. Hence they are concurrent. Or geometrical calculation	B1 E1 2	 Must be clear
		17	

Q 3		mark	notes
(i)	$10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=4\begin{pmatrix}-\frac{1}{2}\\2\end{pmatrix}+2\begin{pmatrix}\frac{1}{2}\\3\end{pmatrix}+\begin{pmatrix}1\frac{1}{2}\\3\frac{1}{2}\end{pmatrix}+3\begin{pmatrix}2\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}$	M1	Correct method clearly indicated for x or y component.
		B1	If 2D method, at least 1 mass + cm correct for a region. If separate cpts, at least 2 mass + cm correct for one of the cpts
	$=\begin{pmatrix}-2+1+1\frac{1}{2}+7\frac{1}{2}\\8+6+3\frac{1}{2}+7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}8\\25\end{pmatrix}$	E1	Working shown. Either expression shown oe
	so $\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}0.8\\2.5\end{pmatrix}$ and c.m. is (0.8, 2.5)	E1	Both
		4	
(ii)	c.w. moments about J $3.2\times1.8-T_{\mathrm{H}}\times4=0$	B1	Use of 1.8 oe
		M1	A moments equation with all relevant forces. Allow use of 10 instead of 3.2
	so $T_{\mathrm{H}}=1.44$ and the force at H is 1.44 N	A1	
	Resolving ↑	M1	Or moments again
	force at J is 3.2 – 1.44 = 1.76 N	F1	Only FT if positive final answer
		5	
(iii)	below		

(iii)	$10\begin{pmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{pmatrix}=4\begin{pmatrix}0\\2\\\frac{1}{2}\end{pmatrix}+2\begin{pmatrix}\frac{1}{2}\\3\\0\end{pmatrix}+2\begin{pmatrix}2\\3\frac{1}{2}\\0\end{pmatrix}+2\begin{pmatrix}2\frac{1}{2}\\3\\-1\end{pmatrix}$	M1 B1 B1	Dealing with 3D Dealing correctly with one folded part Dealing with the other folded part
	$=\begin{pmatrix}0+1+4+5\\8+6+7+6\\2+0+0-2\end{pmatrix}=\begin{pmatrix}10\\27\\0\end{pmatrix}$	E1	Working shown. Either expression shown oe
	so $\begin{pmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{pmatrix}=\begin{pmatrix}1\\2.7\\0\end{pmatrix}$ and c.m. is (1, 2.7, 0)	E1 5	All three components
(iv)		B1	Recognising that cm is vertically below O (may be implied)
	Let angle IOG be θ $\tan\theta=\frac{1}{2.7}$ so angle is 20.323... so 20.3° (3 s. f.)	B1 M1 A1 4	Correctly identifying the angle Accept $\tan\theta=\frac{2.7}{1}$ oe Do NOT isw
		18	

Q 4		mark	notes
(a)	$\frac{1}{2}\times 80\times\left(6^2 - V^2\right)$ $= 80 \times 9.8 \times 1600 - 1300000$ so $V =$ 34.29285… so 34.3 m s^{-1}, (3 s. f.)	M1 B1 B1 A1 A1 5	WE equation. Allow GPE OR init KE term omitted or wrong. Allow sign errors. There must be 3 terms one of which is the WD term KE terms correct (accept $40 \times (V^2 - 6^2)$) GPE term. Allow sign error All terms present. Accept only sign errors, but not the 1300000 and 80x9.8x1600 terms with same sign Cao accept $14\sqrt{6}$
(b) (i)	N2L up the slope. Driving force is S N $S - 1150 - 800 \times 9.8 \times 0.1 = 800 \times 0.25$ $S = 2134$ Power is 2134×8 = 17072 so 17.1 kW (3 s. f.)	M1 B1 M1 A1 E1 M1 A1 7	N2L. Allow either resistance or weight cpt omitted. Allow weight not resolved and sign errors. RHS correct Attempt at weight cpt ($800g\sin\theta$ is sufficient) Allow missing $\boldsymbol{g}$ Weight cpt correct (numerical) May be implied Use of $P = Fv$
(ii)	Let resistance on sledge be F N N2L up slope for sledge $900 - F - 300 \times 9.8 \times 0.1 = 300 \times 0.25$ so $F = 531$ normal reaction is $300g\cos\theta$ Use $\cos\theta = \sqrt{0.99}$ or cos5.7 $\mu = \dfrac{531}{300\times 9.8\times\sqrt{0.99}}$ = 0.181522… so 0.182 (3 s. f.)	M1 A1 B1 B1 M1 A1 6	Need non-zero accn, correct mass and 900. Allow weight missing or unresolved and allow sign errors. Do not award if 2134 included In context Use of $F = \mu R$ for any F and R but not F=900 cao
		18	