OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4762/01**

MATHEMATICS (MEI)

Mechanics 2

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

  [Turn over

1 **(a)** Disc A of mass 6 kg is on a smooth horizontal plane. The disc is at rest and then a constant force of magnitude 9 N acts on it for 2 seconds.

(i) Find the magnitude of the impulse of the force on the disc. Hence, or otherwise, find the speed of the disc after the two seconds. [2]

Without losing speed, disc A now collides directly with disc B of mass 2 kg which is also on the plane. Just before the collision, disc B is travelling at 1 m s^{-1} in the opposite direction to the motion of A, as shown in Fig. 1.1. On impact the two discs stick together to form the combined object AB.

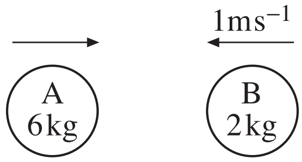


Fig. 1.1

(ii) Show that AB moves off with a speed of 2 m s^{-1} in the original direction of motion of disc A. [3]

(iii) Find the impulse that acts on disc B in the collision. [2]

The combined object AB now collides directly with disc C of mass 10 kg, which is moving on the plane at 1.8 m s^{-1} in the same direction as AB. After this collision the speed of AB is $v \text{ m s}^{-1}$ in the same direction as its speed before the impact, and disc C moves off with speed 1.9 m s^{-1}.

(iv) (*A*) Draw a diagram indicating the velocities before and after the collision. [1]

(*B*) Calculate the value of v. [3]

(*C*) Calculate the coefficient of restitution in the collision. [3]

(b) A small ball is thrown horizontally with a speed of 8 m s^{-1}. The point of projection is 10 m above a smooth horizontal plane, as shown in Fig. 1.2.

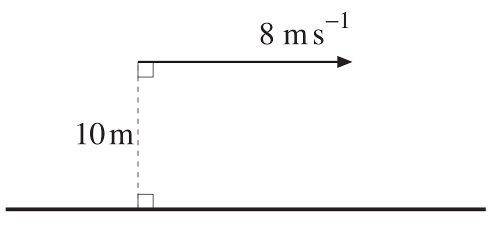
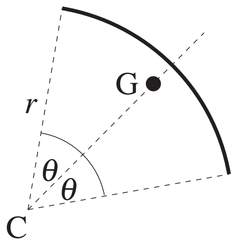


Fig. 1.2

The coefficient of restitution in the impact between the ball and the plane is $\frac{4}{7}$.

Calculate the vertical component of the velocity of the ball immediately after its first impact with the plane and also the angle at which the ball rebounds from the plane. [5]

2 The position of the centre of mass, G, of a uniform wire bent into the shape of an arc of a circle of radius r and centre C is shown in Fig. 2.1.



CG is $\frac{r\sin\theta}{\theta}$

where θ is in radians

Fig. 2.1

(i) Use this information to show that the centre of mass, G, of the uniform wire bent into the shape of a semi-circular arc of radius 8 shown in Fig. 2.2 has coordinates $\left(-\frac{16}{\pi}, 8\right)$. [3]

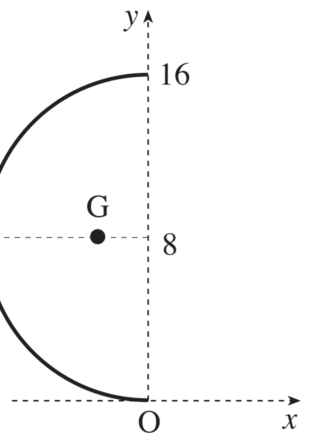


Fig. 2.2

A walking-stick is modelled as a uniform rigid wire. The walking-stick and coordinate axes are shown in Fig. 2.3. The section from O to A is a semi-circular arc and the section OB lies along the x-axis. The lengths are in centimetres.

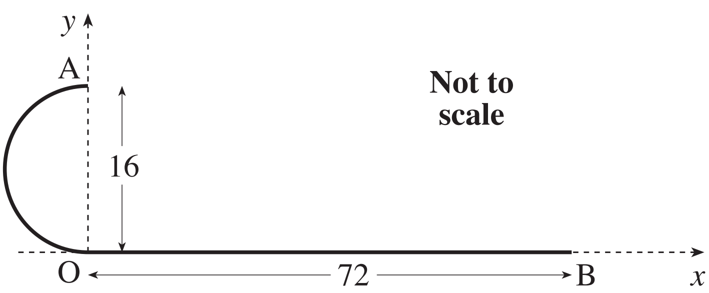


Fig. 2.3

(ii) Show that the coordinates of the centre of mass of the walking-stick are (25.37, 2.07), correct to two decimal places. [6]

The walking-stick is now hung from a shelf as shown in Fig. 2.4. The only contact between the walking-stick and the shelf is at A.

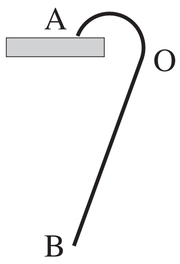


Fig. 2.4

(iii) When the walking-stick is in equilibrium, OB is at an angle α to the vertical.

Draw a diagram showing the position of the centre of mass of the walking-stick in relation to A.

Calculate α. [5]

(iv) The walking-stick is now held in equilibrium, with OB vertical and A still resting on the shelf, by means of a vertical force, F N, at B. The weight of the walking-stick is 12 N. Calculate F. [3]

3 A uniform plank is 2.8 m long and has weight 200 N. The centre of mass is G.

(i) Fig. 3.1 shows the plank horizontal and in equilibrium, resting on supports at A and B.

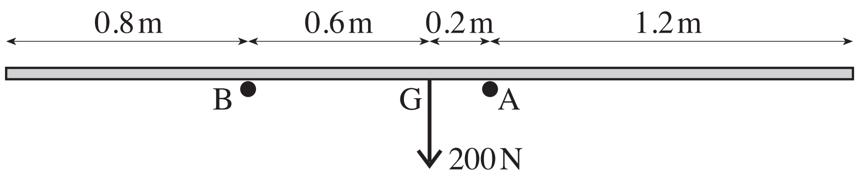


Fig. 3.1

Calculate the reactions of the supports on the plank at A and at B. [4]

(ii) Fig. 3.2 shows the plank horizontal and in equilibrium between a support at C and a peg at D.

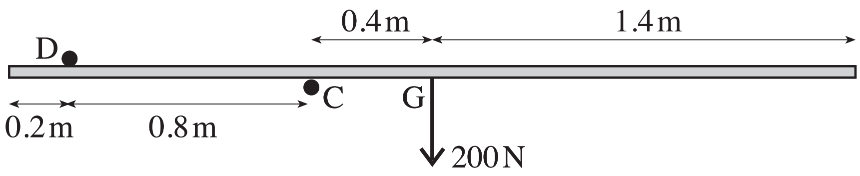


Fig. 3.2

Calculate the reactions of the support and the peg on the plank at C and at D, showing the directions of these forces on a diagram. [5]

Fig. 3.3 shows the plank in equilibrium between a support at P and a peg at Q. The plank is inclined at α to the horizontal, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$.

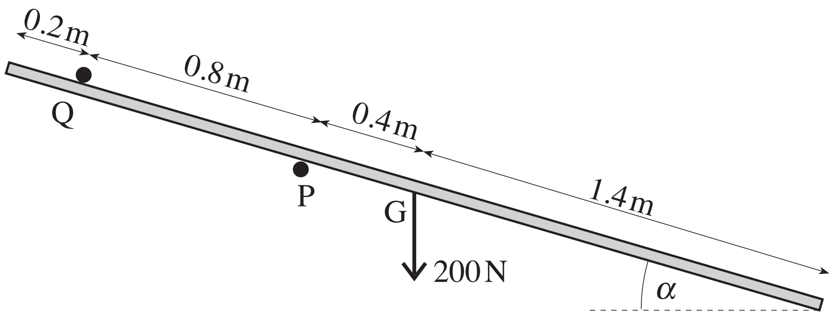


Fig. 3.3

(iii) Calculate the normal reactions at P and at Q. [6]

(iv) Just one of the contacts is rough. Determine which one it is if the value of the coefficient of friction is as small as possible. Find this value of the coefficient of friction. [4]

4 Jack and Jill are raising a pail of water vertically using a light inextensible rope. The pail and water have total mass 20 kg.

In parts **(i)** and **(ii)**, all non-gravitational resistances to motion may be neglected.

(i) How much work is done to raise the pail from rest so that it is travelling upwards at $0.5\,\mathrm{m\,s^{-1}}$ when at a distance of 4 m above its starting position? [4]

(ii) What power is required to raise the pail at a steady speed of $0.5\,\mathrm{m\,s^{-1}}$? [3]

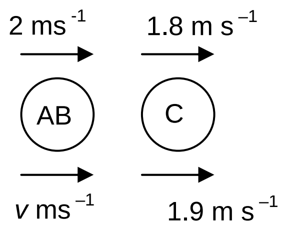
Jack falls over and hurts himself. He then slides down a hill.

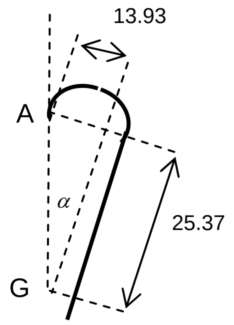
His mass is 35 kg and his speed increases from $1\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$ while descending through a vertical height of 3 m.

(iii) How much work is done against friction? [5]

In Jack's further motion, he slides down a slope at an angle α to the horizontal where $\sin\alpha = 0.1$. The frictional force on him is now constant at 150 N. For this part of the motion, Jack's initial speed is $3\,\mathrm{m\,s^{-1}}$.

(iv) How much further does he slide before coming to rest? [5]

Q 1				
(a) (i)	Impulse has magnitude $2 \times 9 = 18$ N s speed is $\frac{18}{6} = 3$ m s^{-1}.	B1 B1		2
(ii)	PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8v$ $v = 2$ so 2 m s^{-1} in orig direction of A	M1 A1 E1	Use of PCLM + combined mass RHS All correct Must justify direction (diag etc)	3
(iii)	$\rightarrow\ \ 2 \times 2 - 2 \times -1 = 6$ N s	M1 A1	Attempted use of $m\mathbf{v}$ - $m\mathbf{u}$ for 6 N s dir specified (accept diag)	2
(iv) (A)		B1	Accept masses not shown	1
(B)	PCLM $\rightarrow$ $2 \times 8 + 10 \times 1.8 = 8v + 10 \times 1.9$ $v = 1.875$	M1 A1 A1	PCLM. All terms present Allow sign errors only	3
(C)	NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ so $e = 0.125$	M1 A1 F1	Use of NEL with **their** v Any form. FT **their** v FT **their** v (only for $0 < e \leq 1$)	3
(b)	Using $v^2 = u^2 + 2as$ $v = \sqrt{2 \times 10 \times 9.8} = 14$ rebounds at $14 \times \frac{4}{7}$ = 8 m s^{-1} No change to the horizontal component Since both horiz and vert components are 8 m s^{-1} the angle is 45°	B1 M1 F1 B1 A1	Allow ± 14 Using **their** *vertical* component FT from **their** 14. Allow $\pm$ Need not be explicitly stated cao	5
		19		

Q 2				
(i)	$\theta = \frac{\pi}{2}$	B1		
	gives CG $= \frac{8\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{16}{\pi}$	E1		
	$\left(-\frac{16}{\pi}, 8\right)$ justified	E1		3
(ii)	$(8\pi + 72)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 8\pi\begin{pmatrix}-\frac{16}{\pi}\\ 8\end{pmatrix} + 72\begin{pmatrix}36\\ 0\end{pmatrix}$	M1	Method for c.m.	
		B1 A1 A1	Correct mass of 8π or equivalent 1st RHS term correct 2nd RHS term correct	
	$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}25.3673...\\ 2.06997...\end{pmatrix} = \begin{pmatrix}25.37\\ 2.07\end{pmatrix}$ (4 s. f.)	E1 E1		
			[If separate cpts award the A1s for *x*- and *y*- cpts correct on RHS]	6
(iii)		B1	General position and angle (lengths need not be shown)	
	$\tan\alpha = \frac{13.93}{25.37}$	M1	Angle or complement attempted. arctan or equivalent.	
		M1 A1	Attempt to get 16 – 2.0699… Obtaining 13.93… cao Accept use of 2.0699… but not 16.	
	α = 28.7700… so 28.8° (3 s. f.)	A1	cao	5
(iv)			[FT use of 2.0699…]	
	c. w. moments about A $12 \times 13.93 - 16F = 0$	M1 A1	Moments about any point, all forces present	
	so F = 10.4475…	A1	(1.5525... if 2.0699… used)	3
		17		

Q 3				
(i)	Moments c.w. about B $200 \times 0.6 - 0.8R_A = 0$ $R_A = 150$ so 150 N Resolve or moments $R_B = 50$ so 50 N	M1 A1 M1 F1	Accept about any point. Allow sign errors.	4
(ii)	Moments c.w. about D $-0.8R_C + 1.2 \times 200 = 0$ $R_C = 300 \uparrow$ Resolve or moments $R_D = 100 \downarrow$	M1 A1 M1 A1 E1	Or equiv. Accept about any point. All terms present. No extra terms. Allow sign errors. Neglect direction Or equiv. All terms present. No extra terms. Allow sign errors. Neglect direction Both directions clearly shown (on diag)	5
(iii)	Moments c.w. about P $0.4 \times 200\cos\alpha - 0.8R_Q = 0$ $R_Q = 96$ so 96 N resolve perp to plank $R_P = 200\cos\alpha + R_Q$ $R_P = 288$ so 288 N	M1 A1 A1 M1 A1 A1	Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working] Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working]	6
(iv)	Need one with greatest normal reaction So at P Resolve parallel to the plank $F = 200\sin\alpha$ so $F = 56$ $\mu = \frac{F}{R}$ $= \frac{56}{288} = \frac{7}{36}$ (= 0.194 (3 s. f.))	B1 B1 M1 A1	FT their reactions Must use **their** F and R cao	4
		19		

Q 4				
(i)	**either** $0.5 \times 20 \times 0.5^2 + 20 \times 9.8 \times 4$ = 786.5 J **or** $a = \frac{1}{32}$ $T - 20g = 20 \times \frac{1}{32}$ T = 196.625 WD is 4T = 786.5 so 786.5 J	 M1 B1 B1 A1 B1 M1 A1 A1	 KE or GPE terms KE term GPE term cao N2L. All terms present. cao	4
(ii)	$20g \times 0.5 = 10g$ so 98 W	M1 A1 A1	Use of $P = Fv$ or $\Delta WD / \Delta t$ All correct	3
(iii)	GPE lost is $35 \times 9.8 \times 3 = 1029$ J KE gained is $0.5 \times 35 \times (3^2 - 1^2) = 140$ J so WE gives WD against friction is 1029 – 140 = 889 J	B1 M1 A1 M1 A1	 ΔKE The 140 J need not be evaluated Use of WE equation cao	5
(iv)	**either** $0.5 \times 35 \times 3^2 + 35 \times 9.8 \times 0.1x = 150x$ x = 1.36127… so 1.36 m (3 S. F.) **or** $35g \times 0.1 - 150 = 35a$ a = – 3.3057… 0 = 9 – 2ax x = 1.36127… so 1.36 m (3 S. F.)	 M1 B1 B1 A1 A1 M1 A1 A1 M1 A1	 WE equation. Allow 1 missing term. No extra terms. One term correct (neglect sign) Another term correct (neglect sign) All correct except allow sign errors cao Use of N2L. Must have attempt at weight component. No extra terms. Allow sign errors, otherwise correct cao Use of appropriate *uvast* or sequence cao	5
		17		