

**ADVANCED GCE UNIT
MATHEMATICS (MEI)**

Mechanics 2

WEDNESDAY 10 JANUARY 2007

4762/01

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

- 1** A sledge and a child sitting on it have a combined mass of 29.5 kg. The sledge slides on horizontal ice with negligible resistance to its movement.

- (i) While at rest, the sledge is hit directly from behind by a ball of mass 0.5 kg travelling horizontally at 10 m s^{-1} . The coefficient of restitution in the collision is 0.8. After the impact the speeds of the sledge and the ball are $V_1 \text{ m s}^{-1}$ and $V_2 \text{ m s}^{-1}$ respectively.

Calculate V_1 and V_2 and state the direction in which the ball is travelling after the impact. [7]

- (ii) While at rest, the sledge is hit directly from behind by a snowball of mass 0.5 kg travelling horizontally at 10 m s^{-1} . The snowball sticks to the sledge.

(A) Calculate the velocity with which the combined sledge and snowball start to move. [3]

(B) The child scoops up the 0.5 kg of snow and drops it over the back of the sledge. What happens to the velocity of the sledge? Give a reason for your answer. [2]

- (iii) In another situation, the sledge is travelling over the ice at 2 m s^{-1} with 10.5 kg of snow on it (giving a total mass of 40 kg). The child throws a snowball of mass 0.5 kg from the sledge, parallel to the ground and in the positive direction of the motion of the sledge. Immediately after the snowball is thrown, the sledge has a speed of $V \text{ m s}^{-1}$ and the snowball and sledge are separating at a speed of 10 m s^{-1} .

Draw a diagram showing the velocities of the sledge and snowball before and after the snowball is thrown.

Calculate V . [5]

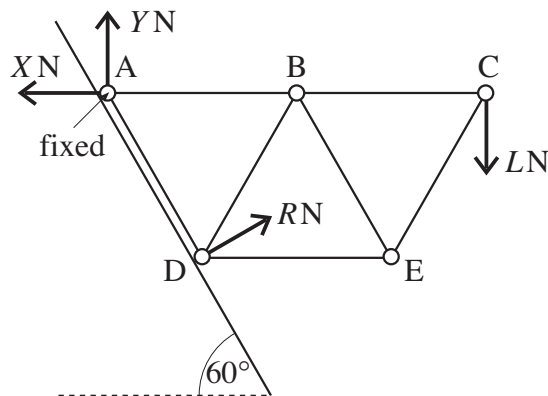


Fig. 2

Fig. 2 shows a framework in a vertical plane made from the equal, light, rigid rods AB, BC, AD, BD, BE, CE and DE. [The triangles ABD, BDE and BCE are all equilateral.]

The rods AB, BC and DE are horizontal.

The rods are freely pin-jointed to each other at A, B, C, D and E.

The pin-joint at A is also fixed to an inclined plane. The plane is smooth and parallel to the rod AD. The pin-joint at D rests on this plane.

The following external forces act on the framework: a vertical load of LN at C; the normal reaction force RN of the plane on the framework at D; the horizontal and vertical forces XN and YN , respectively, acting at A.

- (i) Write down equations for the horizontal and vertical equilibrium of the framework. [3]
- (ii) By considering moments, find the relationship between R and L . Hence show that $X = \sqrt{3}L$ and $Y = 0$. [4]
- (iii) Draw a diagram showing all the forces acting on the pin-joints, including the forces internal to the rods. [2]
- (iv) Show that the internal force in the rod AD is zero. [2]
- (v) Find the forces internal to AB, CE and BC in terms of L and state whether each is a tension or a thrust (compression). [You may leave your answers in surd form.] [7]
- (vi) Without calculating their values in terms of L , show that the forces internal to the rods BD and BE have equal magnitude but one is a tension and the other a thrust. [2]

- 3 A box is to be assembled in the shape of the cuboid shown in Fig. 3.1. The lengths are in centimetres. All the faces are made of the same uniform, rigid and thin material. All coordinates refer to the axes shown in this figure.

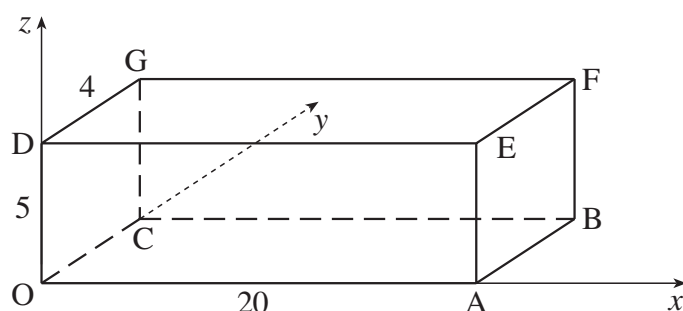


Fig. 3.1

- (i) The four vertical faces OAED, ABFE, FGCB and CODG are assembled first to make an open box without a base or a top. Write down the coordinates of the centre of mass of this open box. [1]

The base OABC is added to the vertical faces.

- (ii) Write down the x - and y -coordinates of the centre of mass of the box now. Show that the z -coordinate is now 1.875. [5]

The top face FGDE is now added. This is a lid hinged to the rest of the box along the line FG. The lid is open so that it hangs in a vertical plane touching the face FGCB.

- (iii) Show that the coordinates of the centre of mass of the box in this situation are (10, 2.4, 2.1). [6]

[This question is continued on the facing page.]

The box, with the lid still touching face FGCB, is now put on a sloping plane with the edge OA horizontal and the base inclined at 30° to the horizontal, as shown in Fig. 3.2.

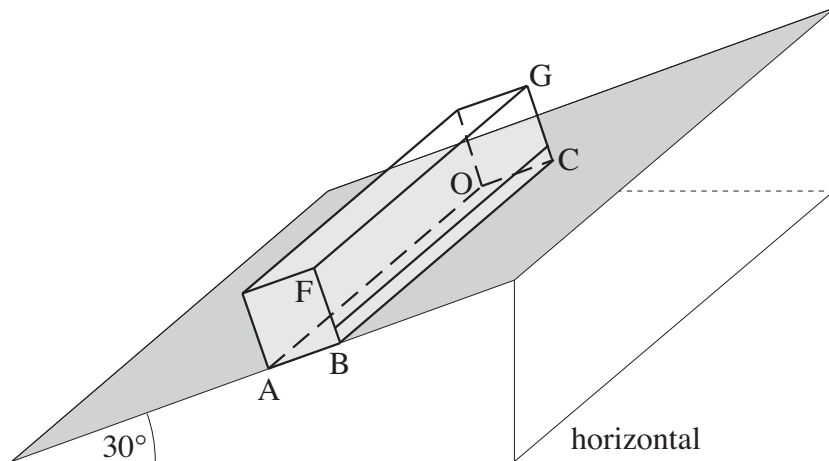


Fig. 3.2

The weight of the box is 40 N. A force P N acts parallel to the plane and is applied to the mid-point of FG at 90° to FG. This force tends to push the box down the plane. The box does not slip and is on the point of toppling about the edge AO.

- (iv) Show that the clockwise moment of the weight of the box about the edge AO is about 0.411 Nm. [4]
- (v) Calculate the value of P . [2]

[Question 4 is printed overleaf.]

- 4 Some tiles on a roof are being replaced. Each tile has a mass of 2 kg and the coefficient of friction between it and the existing roof is 0.75. The roof is at 30° to the horizontal and the bottom of the roof is 6 m above horizontal ground, as shown in Fig. 4.

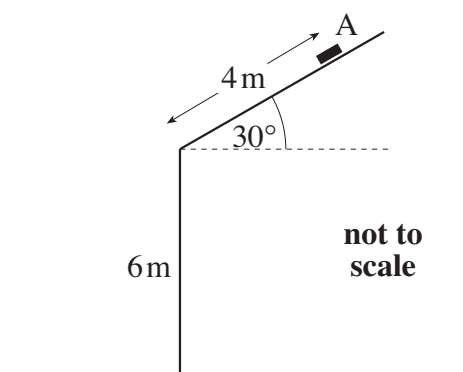


Fig. 4

- (i) Calculate the limiting frictional force between a tile and the roof.

A tile is placed on the roof. Does it slide? (Your answer should be supported by a calculation.) [5]

- (ii) The tiles are raised 6 m from the ground, the only work done being against gravity. They are then slid 4 m up the roof and placed at the point A shown in Fig. 4.

(A) Show that each tile gains 156.8 J of gravitational potential energy. [3]

(B) Calculate the work done against friction per tile. [2]

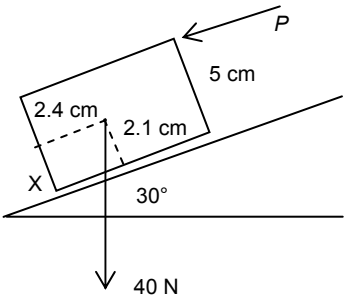
(C) What average power is required to raise 10 tiles per minute from the ground to A? [2]

- (iii) A tile is kicked from A directly down the roof. When the tile is at B, x m from the edge of the roof, its speed is 4 m s^{-1} . It subsequently hits the ground travelling at 9 m s^{-1} . In the motion of the tile from B to the ground, the work done against sliding and other resistances is 90 J.

Use an energy method to find x . [5]

Q 1		mark		sub
(i)	<div><div><div>before</div><div><div><div><div>10 m s⁻¹</div><div>→</div></div><div>0.5 kg</div></div><div>after</div><div><div><div><div>v₂ m s⁻¹</div><div>→</div></div><div>29.5 kg</div></div><div><div><div><div>0</div><div>→</div></div><div>v₁ m s⁻¹</div></div></div></div><div>$10 \times 0.5 = 0.5v_2 + 29.5v_1$</div><div>$\frac{v_1 - v_2}{0 - 10} = -0.8$</div><div>$v_1 = 0.3$ so $V_1 = 0.3$</div><div>$v_2 = -7.7$ so $V_2 = 7.7 \text{ m s}^{-1}$</div><div>in opposite to original direction</div></div></div><td>M1 A1 M1 A1 A1 A1 F1</td><td>PCLM and two terms on RHS All correct. Any form. NEL Any form Speed. Accept ± . Must be correct interpretation of clear working</td><td>7</td></div>	M1 A1 M1 A1 A1 A1 F1	PCLM and two terms on RHS All correct. Any form. NEL Any form Speed. Accept ± . Must be correct interpretation of clear working	7
(ii) (A)	<div>$10 \times 0.5 = 30V$</div> <div>so $V = \frac{1}{6}$</div>	M1 A1 A1	PCLM and coalescence All correct. Any form. Clearly shown. Accept decimal equivalence. Accept no direction.	3
(B)	Same velocity No force on sledge in direction of motion	E1 E1	Accept speed	2
(iii)	<div><div><div>before</div><div><div><div><div>2 m s⁻¹</div><div>→</div></div><div>39.5 kg</div></div><div>after</div><div><div><div><div>V</div><div>→</div></div><div>0.5 kg</div></div><div><div><div><div>2 m s⁻¹</div><div>→</div></div><div>u</div></div></div></div><div>$2 \times 40 = 0.5u + 39.5V$</div><div>$u - V = 10$</div><div>Hence $V = 1.875$</div></div></div><td>B1 M1 A1 B1 A1</td><td>PCLM, masses correct Any form May be seen on the diagram. Accept no reference to direction.</td><td>5</td></div>	B1 M1 A1 B1 A1	PCLM, masses correct Any form May be seen on the diagram. Accept no reference to direction.	5
		17		

Q 2		mark	comment	sub
(i)	$X = R \cos 30$ (1) $Y + R \sin 30 = L$ (2)	B1 M1 A1	Attempt at resolution	3
(ii)	ac moments about A $R - 2L = 0$ Subst in (1) and (2) $X = 2L \frac{\sqrt{3}}{2}$ so $X = \sqrt{3}L$ $Y + 2L \times \frac{1}{2} = L$ so $Y + L = L$ and $Y = 0$	B1 M1 E1 E1	Subst their $R = 2L$ into their (1) or (2) Clearly shown Clearly shown	4
(iii)	(Below all are taken as tensions e. g. T_{AB} in AB)	B1 B1	Attempt at all forces (allow one omitted) Correct. Accept internal forces set as tensions or thrusts or a mix	2
(iv)	\downarrow A $T_{AD} \cos 30 (-Y) = 0$ so $T_{AD} = 0$	M1 E1	Vert equilibrium at A attempted. $Y = 0$ need not be explicit	2
(v)	Consider the equilibrium at pin-joints A $\rightarrow T_{AB} - X = 0$ so $T_{AB} = \sqrt{3}L$ (T) C $\downarrow L + T_{CE} \cos 30 = 0$ so $T_{CE} = \frac{-2L}{\sqrt{3}}$ so $\frac{2L}{\sqrt{3}} \left(= \frac{2L\sqrt{3}}{3} \right)$ (C) C $\leftarrow T_{BC} + T_{CE} \cos 60 = 0$ so $T_{BC} = - \left(-\frac{2\sqrt{3}L}{3} \right) \times \frac{1}{2} = \frac{\sqrt{3}L}{3}$ (T)	M1 B1 B1 B1 B1 B1 F1	At least one relevant equilib attempted (T) not required Or equiv from their diagram Accept any form following from their equation. (C) not required. Or equiv from their diagram FT their T_{CE} or equiv but do not condone inconsistent signs even if right answer obtained. (T) not required. T and C consistent with their answers and their diagram	7
(vi)	\downarrow B $T_{BD} \cos 30 + T_{BE} \cos 30 = 0$ so $T_{BD} = -T_{BE}$ so mag equal and opp sense	M1 E1	Resolve vert at B A statement required	2
		20		

Q 3		mark		sub
(i)	(10, 2, 2.5)	B1		1
(ii)	By symmetry $\bar{x} = 10$, $\bar{y} = 2$ $(240 + 80)\bar{z} = 80 \times 0 + 240 \times 2.5$ so $\bar{z} = 1.875$	B1 B1 B1 M1 A1	Total mass correct Method for c.m. Clearly shown	5
(iii)	$\bar{x} = 10$ by symmetry $(320 + 80) \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = 320 \begin{pmatrix} 10 \\ 2 \\ 1.875 \end{pmatrix} + 80 \begin{pmatrix} 10 \\ 4 \\ 3 \end{pmatrix}$ $\bar{y} = 2.4$ $\bar{z} = 2.1$	E1 M1 B1 B1 E1 E1	Could be derived Method for c.m. y coord c.m. of lid z coord c.m. of lid shown shown	6
(iv)	 <p>c.w moments about X $40 \times 0.024 \cos 30 - 40 \times 0.021 \sin 30$ $= 0.41138 \dots$ so 0.411 N m (3 s. f.)</p>	B1 B1 E1	Award for correct use of dimensions 2.1 and 2.4 or equivalent 1 st term o.e. (allow use of 2.4 and 2.1) 2 nd term o.e. (allow use of 2.4 and 2.1) Shown [Perpendicular method: M1 Complete method: A1 Correct lengths and angles E1 Shown]	4
(v)	$0.41138 \dots - 0.05P = 0$ $P = 8.22768 \dots$ so 8.23 (3 s. f.)	M1 A1	Allow use of 5 Allow if cm used consistently	2
		18		

Q 4		mark		sub
(i)	$F_{\max} = \mu R$ $R = 2g \cos 30$ so $F_{\max} = 0.75 \times 2 \times 9.8 \times \cos 30 = 12.730 \dots$ so 12.7 N (3 s. f.) either Weight cpt down plane is $2g \sin 30 = 9.8$ N so no as $9.8 < 12.7$ or Slides if $\mu < \tan 30$ But $0.75 > 0.577 \dots$ so no	M1 B1 A1 B1 E1 B1 E1	Must have attempt at R with mg resolved [Award 2/3 retrospectively for limiting friction seen below] The inequality must be properly justified The inequality must be properly justified	5
(ii) (A)	Increase in GPE is $2 \times 9.8 \times (6 + 4 \sin 30) = 156.8$ J	M1 B1 A1	Use of mgh $6 + 4 \sin 30$	3
(B)	WD against friction is $4 \times 0.75 \times 2 \times 9.8 \times \cos 30 = 50.9222 \dots$ J	M1 A1	Use of $WD = Fd$	2
(C)	Power is $10 \times (156.8 + 50.9222 \dots) / 60$ $= 34.620 \dots$ so 34.6 W (3 s. f.)	M1 A1	Use $P = WD/t$	2
(iii)	$0.5 \times 2 \times 9^2$ $= 2 \times 9.8 \times (6 + x \sin 30)$ $+ 0.5 \times 2 \times 4^2$ $- 90$ so $x = 3.8163 \dots$ so 3.82 (3 s. f.)	M1 B1 A1 A1 A1	Equating KE to GPE and WD term. Allow sign errors and one KE term omitted. Allow 'old' friction as well. Both KE terms. Allow wrong signs. All correct but allow sign errors All correct, including signs. cao	5
		17		