OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

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1 **(a)** Two small spheres, P of mass 2 kg and Q of mass 6 kg, are moving in the same straight line along a smooth, horizontal plane with the velocities shown in Fig. 1.1.

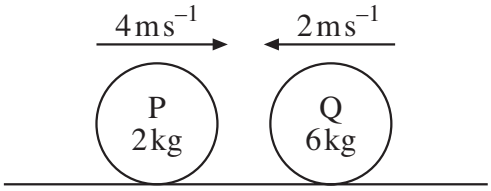

Fig. 1.1

Consider the direct collision of P and Q in the following two cases.

(i) The spheres coalesce on collision.

(*A*) Calculate the common velocity of the spheres after the collision. [3]

(*B*) Calculate the energy lost in the collision. [2]

(ii) The spheres rebound with a coefficient of restitution of $\frac{2}{3}$ in the collision.

(*A*) Calculate the velocities of P and Q after the collision. [6]

(*B*) Calculate the impulse on P in the collision. [2]

(b) A small ball bounces off a smooth, horizontal plane. The ball hits the plane with a speed of $26\,\text{m s}^{-1}$ at an angle of $\arcsin\frac{12}{13}$ to it. The ball rebounds at an angle of $\arcsin\frac{3}{5}$ to the plane, as shown in Fig. 1.2.

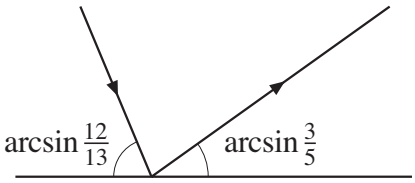

Fig. 1.2

Calculate the speed with which the ball rebounds from the plane.

Calculate also the coefficient of restitution in the impact. [6]

2 Two heavy rods AB and BC are freely jointed together at B and to a wall at A. AB has weight 90 N and centre of mass at P; BC has weight 75 N and centre of mass at Q. The lengths of the rods and the positions of P and Q are shown in Fig. 2.1, with the lengths in metres.

Initially, AB and BC are horizontal. There is a support at R, as shown in Fig. 2.1. The system is held in equilibrium by a vertical force acting at C.

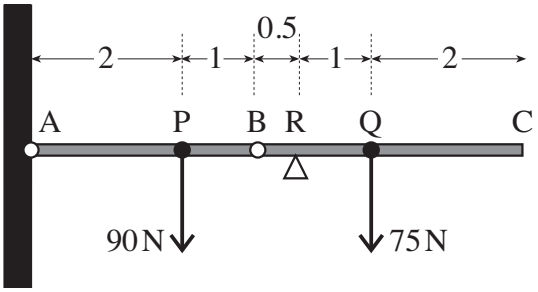

Fig. 2.1

(i) Draw diagrams showing all the forces acting on rod AB and on rod BC.

Calculate the force exerted on AB by the hinge at B and hence the force required at C. [6]

The rods are now set up as shown in Fig. 2.2. AB and BC are each inclined at 60° to the vertical and C rests on a rough horizontal table. Fig. 2.3 shows all the forces acting on AB, including the forces X N and Y N due to the hinge at A and the forces U N and V N in the hinge at B. The rods are in equilibrium.

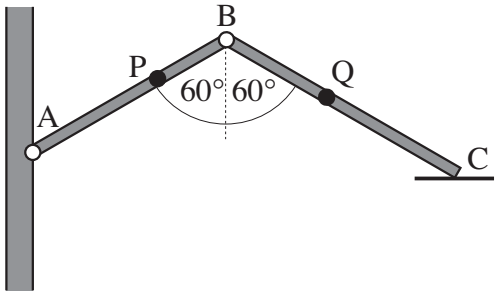

Fig. 2.2

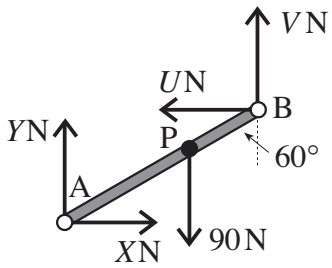

Fig. 2.3

(ii) By considering the equilibrium of rod AB, show that $60\sqrt{3} = U + V\sqrt{3}$. [3]

(iii) Draw a diagram showing all the forces acting on rod BC. [1]

(iv) Find a further equation connecting U and V and hence find their values. Find also the frictional force at C. [8]

3 **(a)** A car of mass 900 kg is travelling at a steady speed of $16\,\mathrm{m\,s^{-1}}$ up a hill inclined at $\arcsin 0.1$ to the horizontal. The power required to do this is 20 kW.

Calculate the resistance to the motion of the car. [4]

(b) A small box of mass 11 kg is placed on a uniform rough slope inclined at $\arccos \frac{12}{13}$ to the horizontal. The coefficient of friction between the box and the slope is μ.

(i) Show that if the box stays at rest then $\mu \geqslant \frac{5}{12}$. [3]

For the remainder of this question, the box moves on a part of the slope where $\mu = 0.2$.

The box is projected up the slope from a point P with an initial speed of $v\,\mathrm{m\,s^{-1}}$. It travels a distance of 1.5 m along the slope before coming instantaneously to rest. During this motion, the work done against air resistance is 6 joules per metre.

(ii) Calculate the value of v. [5]

As the box slides back down the slope, it passes through its point of projection P and later reaches its initial speed at a point Q. During this motion, once again the work done against air resistance is 6 joules per metre.

(iii) Calculate the distance PQ. [6]

[Question 4 is printed overleaf.]

4 Fig. 4.1 shows four uniform rods, OA, AB, BE and CD, rigidly fixed together to form a frame. The rods have weights proportional to their lengths and these lengths, in centimetres, are shown in Fig. 4.1.

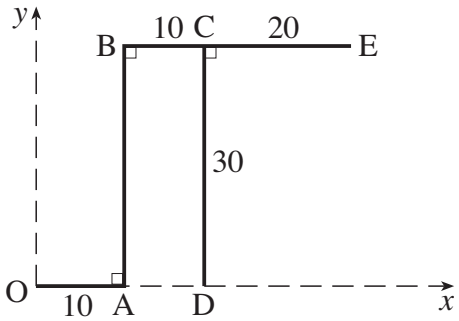

Fig. 4.1

(i) Calculate the coordinates of the centre of mass of the frame, referred to the axes shown in Fig. 4.1. [5]

The bracket shown in Fig. 4.2 is made of uniform sheet metal with cross-section the frame shown in Fig. 4.1. The bracket is 40 cm wide and its weight is 60 N. It stands on a horizontal plane containing O*x* and O*z*.

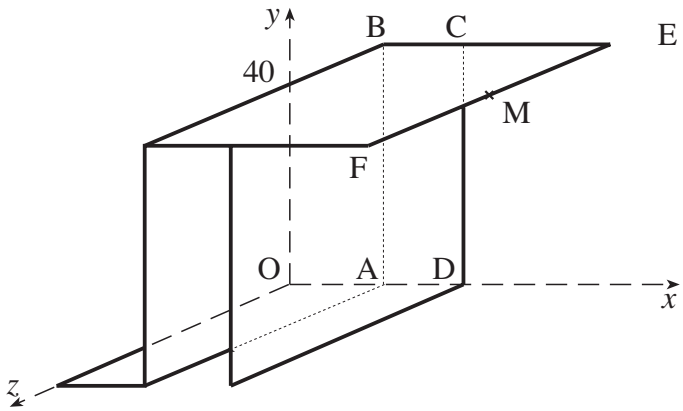

Fig. 4.2

(ii) Write down the coordinates of the centre of mass of the bracket, referred to the axes shown in Fig. 4.2. [2]

A force *P* N acts vertically downwards at the point M, shown in Fig. 4.2. M is the mid-point of EF. The bracket is on the point of tipping.

(iii) Calculate the value of *P*. [4]

In another situation, a horizontal force Q N acts through M parallel to EB and in the direction from E to B. The value of Q is increased from zero with the bracket in equilibrium at all times.

(iv) Draw a diagram showing the forces acting on the bracket when it is on the point of tipping. [1]

(v) If the limiting frictional force between the bracket and the plane is 30 N, does the bracket slide or tip first as Q is increased? [5]

Q 1		mark		Sub
(a) (i) (A)	PCLM $\rightarrow$ +ve $2\times4-6\times2=8v$ $v = -0.5$ so 0.5 m s^{-1} in opposite direction to initial motion of P	M1 A1 A1	Use of PCLM and correct mass on RHS Any form Direction must be negative and consistent or clear. Accept use of a diagram.	3
(B)	$0.5\times2\times4^2+0.5\times6\times2^2-0.5\times8\times(-0.5)^2$ = 27 J	M1 A1	Use of KE. Must sum initial terms. Must have correct masses FT **their** (A) only	2
(ii) (A)	PCLM $\rightarrow$ +ve $2\times4-6\times2=2v_P+6v_Q$ $v_P+3v_Q=-2$ NEL $\rightarrow$ +ve $\frac{v_Q-v_P}{-2-4}=-\frac{2}{3}$ $v_Q-v_P=4$ $v_Q=0.5$ so 0.5 m s^{-1} in orig direction of P $v_P=-3.5$ so 3.5 m s^{-1} in opp to orig dir of P	M1 A1 M1 A1 A1 A1	Use of PCLM Any form NEL Any form cao. Direction need not be made clear. cao. Direction must be negative and consistent or clear (e.g diag)	6
(B)	$\rightarrow$ +ve $2\times-3.5-2\times4=-15$ N s so 15 N s in opp to orig direction	M1 A1	Use of change in momentum with correct mass. FT (A). Dir must be clear (e.g. diag)	2
(b)	Let $\alpha=\arcsin(12/13)$ and $\beta=\arcsin(3/5)$ Parallel: $26\cos\alpha=u\cos\beta$ so $26\times\frac{5}{13}=u\times\frac{4}{5}$ and u = 12.5 Perp: $e=\frac{u\sin\beta}{26\sin\alpha}$ $=\frac{12.5\times\frac{3}{5}}{26\times\frac{12}{13}}=\frac{5}{16}$	 M1 A1 A1 M1 F1 F1	 PCLM parallel to plane attempted. At least one resolution correct NEL on normal components attempted. FT **their** u FT **their** u	6
				19

Q 2		mark		Sub
(i)	Diagrams	B1	Internal force at B must be shown	
	cw moments about A $2\times 90 - 3R_B = 0$	M1	1st moments equation attempted for either force.	
	$R_B = 60$ so 60 N upwards	A1	Accept direction not specified	
	cw moments about R: $T\downarrow$ $75\times 1 + 3T - 60\times 0.5 = 0$	M1	2nd moments equation for other force. All forces present. No extra forces.	
		A1	Allow only sign errors	
	$T = -15$ so 15 N upwards	A1	Direction must be clear (accept diag)	6
(ii)	cw moments about A $90\times 2\cos 30 - V\times 3\cos 30 - U\times 3\cos 60 = 0$	M1	Moments equation with resolution. Accept terms missing	
		A1	All correct. Allow only sign errors.	
	giving $60\sqrt{3} = U + V\sqrt{3}$	E1	Clearly shown	3
(iii)	Diagram	B1	*U* and *V* correct with labels and arrows	1
(iv)	ac moments about C $75\times 2\cos 30 + 3.5V\cos 30 - 3.5U\cos 60 = 0$	M1	Moments equation with resolution. Accept term missing	
		B1	At least two terms correct (condone wrong signs)	
	$\frac{300}{7}\sqrt{3} = U - V\sqrt{3}$	A1	Accept any form	
	Solving for *U* and *V*	M1	Any method to eliminate one variable	
	$U = \frac{360\sqrt{3}}{7}$ (= 89.0768…)	A1	Accept any form and any reasonable accuracy	
	$V = \frac{60}{7}$ (= 8.571428…)	F1	Accept any form and any reasonable accuracy [Either of *U* and *V* is cao. FT the other]	
	Resolve $\rightarrow$ on BC *F* = *U*	M1		
	so frictional force is $\frac{360\sqrt{3}}{7}$ N (= 89.1 N (3 s. f.))	F1		8
				18

Q 3		mark		Sub
(a)	$20000 = (R + 900g \times 0.1) \times 16$	M1	Use of $P = Fv$, may be implied.	
		B1	Correct weight term	
		A1	All correct	
	R = 368 so 368 N	A1		4
(b) (i)	$F_{\max} = \mu mg\cos\alpha$	B1	Correct expression for $F_{\max}$ **or** wt cpt down slope (may be implied and in any form)	
	Force down slope is weight cpt $mg\sin\alpha$	B1	Identifying $\sin\alpha$ as $5/13$ or equivalent	
	Require $\mu mg\cos\alpha \geq mg\sin\alpha$			
	so $\mu \geq \tan\alpha = \frac{5}{12}$	E1	Proper use of $F \leq \mu R$ or equivalent.	
			[$\mu = \tan\alpha$ used WW; SC1]	3
(ii)	**either**			
	$0.5 \times 11 \times v^2$	M1	Use of work energy with at least three required terms attempted	
	$= 11g \times 1.5 \times \frac{5}{13} + 0.2 \times 11g \times 1.5 \times \frac{12}{13} + 9$	B1	Any term RHS. Condone sign error.	
		B1	Another term RHS. Condone sign error.	
		A1	All correct . Allow if trig consistent but wrong	
	$v^2 = 18.3717...$			
	v = 4.2862… so 4.29 m s^{-1} (3 s. f.)	A1	cao	
	or			5
	+ ve up the slope			
	$-11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$	M1	Use of N2L	
		B1	Any correct term on LHS	
	a = - 6.1239 m s^{-2}	**A1**		
	v^2 = - 3a	**M1**	use of appropriate *uvast*	
	v = 4.286 m s^{-1}	**A1**	c.a.o.	
(iii)	**continued overleaf**			

3	**continued**			
(iii)	**either** Extra GPE balances WD against resistances	M1	Or equivalent	
	$mgx\sin\alpha$	B1		
	$= 6(x+3) + 0.2\times 11g\times\cos\alpha(x+3)$	B1	One of 1st three terms on RHS correct	
		B1	Another of 1st 3 terms on RHS correct	
		A1	All correct. FT **their** *v* if used.	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	6
	or	M1	Allow 1 term missing	
	$0.5\times 11\times 18.3717...$	B1	KE. FT **their** *v*	
	$= (1.5+x)\times 11g\times\frac{5}{13} - 6(1.5+x)$	B1	Use of 1.5 + *x* (may be below)	
	$-(1.5+x)\times 0.2\times 11g\times\frac{12}{13}$	B1	WD against friction	
		A1	All correct	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	
	or + ve down the slope			
	$11g\times {}^{5}\!/_{13} - 0.2\times 11g\times {}^{12}\!/_{13} - 6 = 11a$	M1	N2L with all terms present	
		A1	all correct except condone sign errors	
	$a = 1.4145...$ m s^{-2}	**A1**		
	$4.286^2 = 2a(1.5+x)$	**M1**	use of appropriate *uvast*	
		B1	for (1.5 + *x*) (may be implied)	
	x = 4.99	A1	c.a.o.	
				18

Q 4		mark		Sub
(i)	$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=10\begin{pmatrix}5\\0\end{pmatrix}+30\begin{pmatrix}10\\15\end{pmatrix}+30\begin{pmatrix}20\\15\end{pmatrix}+30\begin{pmatrix}25\\30\end{pmatrix}$	M1	Correct method for c.m.	
		B1	Total mass correct	
		B1	One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}1700\\1800\end{pmatrix}$			
	$\bar{x}=17$	A1	cao	
	$\bar{y}=18$	A1	cao. [Allow SC 4/5 for $\bar{x}=18$ and $\bar{y}=17$]	5
(ii)	$(17,18,20)$	B1	*x*- and *y*- coordinates. FT from (i).	
		B1	*z* coordinate	2
(iii)	cw moments about horizontal edge thro' D	M1	Or equivalent with all forces present	
	x component			
	$P\times 20-60\times(20-17)=0$	B1	One moment correct (accept use of mass or length)	
		B1	correct use of **their** $\bar{x}$ in a distance	
	P = 9	A1	FT only **their** $\bar{x}$	4
(iv)	Diagram	B1	Normal reaction must be indicated acting vertically upwards at edge on O*z* and weight be in approximately the correct place.	1
(v)	On point of toppling ac moments about edge along O*z*	M1	Or equivalent with all forces present	
	$30\times Q-60\times 17=0$	B1	Any moment correct (accept use of mass or length)	
	Q = 34	F1	FT only **their** $\bar{x}$	
	Resolving horizontally *F* = *Q*	B1		
	As 34 > 30, slips first	B1	FT **their** Q correctly argued.	5
				17