MEI STRUCTURED MATHEMATICS   4762

Mechanics 2

| Friday | **27 JANUARY 2006** | Afternoon | 1 hour 30 minutes |

Additional materials:
   8 page answer booklet
   Graph paper
   MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

· Write your name, centre number and candidate number in the spaces provided on the answer booklet.

· Answer **all** the questions.

· You are permitted to use a graphical calculator in this paper.

· Final answers should be given to a degree of accuracy appropriate to the context.

· The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION FOR CANDIDATES

· The number of marks is given in brackets [ ] at the end of each question or part question.

· You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

· The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

1   When a stationary firework P of mass 0.4 kg is set off, the explosion gives it an instantaneous impulse of 16 N s vertically upwards.

   (i) Calculate the speed of projection of P. [2]

While travelling vertically upwards at $32 \, \text{m s}^{-1}$, P collides directly with another firework Q, of mass 0.6 kg, that is moving directly downwards with speed $u \, \text{m s}^{-1}$, as shown in Fig. 1. The coefficient of restitution in the collision is 0.1 and Q has a speed of $4 \, \text{m s}^{-1}$ vertically *upwards* immediately after the collision.
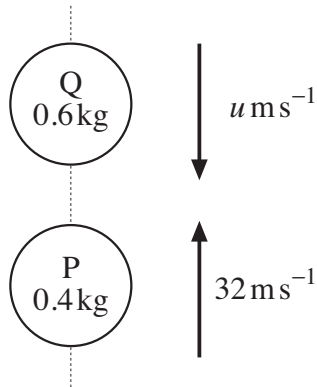

Fig. 1

   (ii) Show that $u = 18$ and calculate the speed and direction of motion of P immediately after the collision. [7]

Another firework of mass 0.5 kg has a velocity of $(-3.6\mathbf{i} + 5.2\mathbf{j}) \, \text{m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors, respectively. This firework explodes into two parts, C and D. Part C has mass 0.2 kg and velocity $(3\mathbf{i} + 4\mathbf{j}) \, \text{m s}^{-1}$ immediately after the explosion.

   (iii) Calculate the velocity of D immediately after the explosion in the form $a\mathbf{i} + b\mathbf{j}$. Show that C and D move off at 90° to one another. [8]

2   A uniform beam, AB, is 6 m long and has a weight of 240 N.

Initially, the beam is in equilibrium on two supports at C and D, as shown in Fig. 2.1. The beam is horizontal.
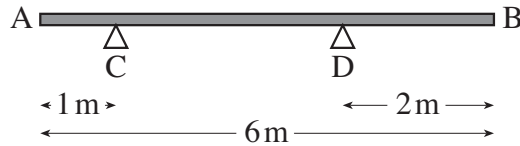
Fig. 2.1

(i) Calculate the forces acting on the beam from the supports at C and D.   [4]

A workman tries to move the beam by applying a force T N at A at 40° to the beam, as shown in Fig. 2.2. The beam remains in horizontal equilibrium but the reaction of support C on the beam is zero.
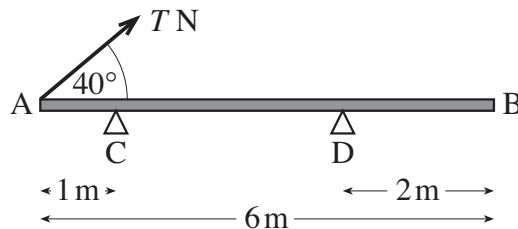
Fig. 2.2

(ii) (*A*) Calculate the value of T.   [4]

(*B*) Explain why the support at D cannot be smooth.   [1]

The beam is now supported by a light rope attached to the beam at A, with B on rough, horizontal ground. The rope is at 90° to the beam and the beam is at 30° to the horizontal, as shown in Fig. 2.3. The tension in the rope is P N. The beam is in equilibrium on the point of sliding.
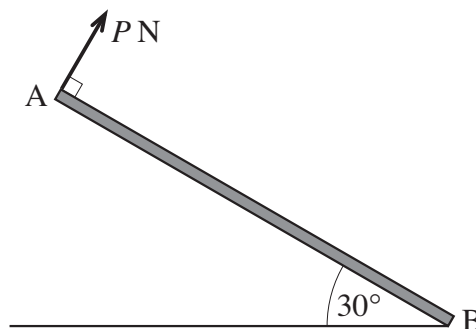
Fig. 2.3

(iii) (*A*) Show that $P = 60\sqrt{3}$ and hence, or otherwise, find the frictional force between the beam and the ground.   [5]

(*B*) Calculate the coefficient of friction between the beam and the ground.   [5]

[Turn over

3 **(a)** A uniform lamina made from rectangular parts is shown in Fig. 3.1. All the dimensions are centimetres. All coordinates are referred to the axes shown in Fig. 3.1.
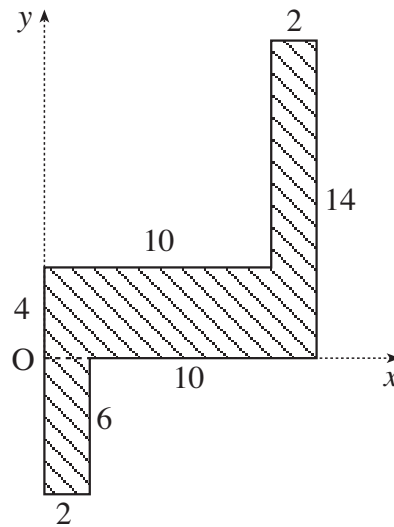
Fig. 3.1

(i) Show that the x-coordinate of the centre of mass of the lamina is 6.5 and find the y-coordinate. [5]

A square of side 2 cm is to be cut from the lamina. The sides of the square are to be parallel to the coordinate axes and the centre of the square is to be chosen so that the x-coordinate of the centre of mass of the new shape is 6.4.

(ii) Calculate the x-coordinate of the centre of the square to be removed. [3]

The y-coordinate of the centre of the square to be removed is now chosen so that the y-coordinate of the centre of mass of the final shape is as large as possible.

(iii) Calculate the y-coordinate of the centre of mass of the lamina with the square removed, giving your answer correct to three significant figures. [3]

(b) Fig. 3.2 shows a framework made from light rods of length 2 m freely pin-jointed at A, B, C, D and E. The framework is in a vertical plane and is supported at A and C. There are loads of 120 N at B and at E. The force on the framework due to the support at A is R N.

each rod is
2 m long
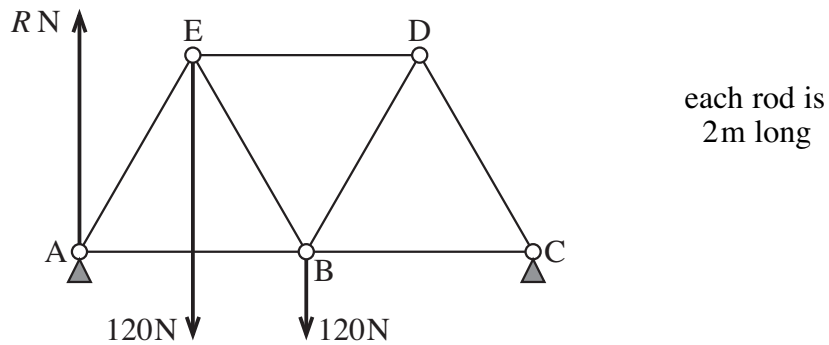
Fig. 3.2

(i) Show that $R = 150$. [2]

(ii) Draw a diagram showing all the forces acting at the points A, B, D and E, including the forces internal to the rods.

Calculate the internal forces in rods AE and EB, and determine whether each is a tension or a thrust. [You may leave your answers in surd form.] [6]

(iii) Without any further calculation of the forces in the rods, explain briefly how you can tell that rod ED is in thrust. [1]

[Question 4 is printed overleaf.]

4 A block of mass $20\,\text{kg}$ is pulled by a light, horizontal string over a rough, horizontal plane. During 6 seconds, the work done against resistances is 510 J and the speed of the block increases from $5\,\text{m s}^{-1}$ to $8\,\text{m s}^{-1}$.

  (i) Calculate the power of the pulling force. [4]

The block is now put on a rough plane that is at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The frictional resistance to sliding is $11g\,\text{N}$. A light string parallel to the plane is connected to the block. The string passes over a smooth pulley and is connected to a freely hanging sphere of mass $m\,\text{kg}$, as shown in Fig. 4.
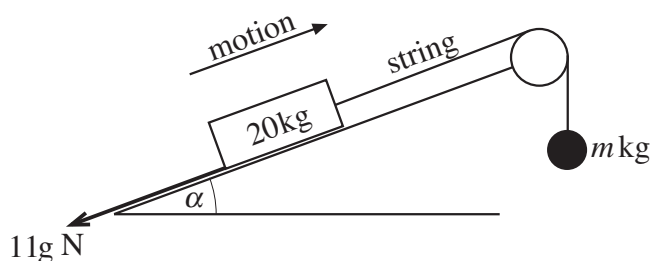
Fig. 4
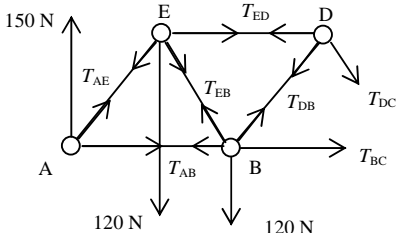
In parts **(ii)** and **(iii)**, the sphere is pulled downwards and then released when travelling at a speed of $4\,\text{m s}^{-1}$ vertically downwards. The block never reaches the pulley.

 (ii) Suppose that $m = 5$ and that after the sphere is released the block moves x m up the plane before coming to rest.

    (*A*) Find an expression in terms of x for the change in gravitational potential energy of the system, stating whether this is a gain or a loss. [4]

    (*B*) Find an expression in terms of x for the work done against friction. [1]

    (*C*) Making use of your answers to parts (*A*) and (*B*), find the value of x. [3]

(iii) Suppose instead that $m = 15$. Calculate the speed of the sphere when it has fallen a distance $0.5\,\text{m}$ from its point of release. [4]

Q 1		mark		Sub
(i)	$16 = 0.4v$ so 40 m s^{-1}	M1 A1	Use of $I = \Delta mv$	 2
(ii)	PCLM $\uparrow$ + ve $0.4 \times 32 - 0.6u = 0.4v_P + 0.6 \times 4$ NEL $\uparrow$ +ve $\dfrac{4 - v_P}{-u - 32} = -0.1$ Solving $u = 18$ $v_P = -1$ so 1 m s^{-1} downwards	M1 A1 M1 A1 E1 A1 A1	Use of PCLM Any form Use of NEL. Allow sign errors. Any form Must be obtained from a pair of correct equations. If given $u = 18$ used then $v_P = -1$ must be obtained from 1 equation and both values tested in the second equation cao. Accept use of given $u = 18$ cao	 7
(iii)	Considering the momenta involved $0.5\begin{pmatrix} -3.6 \\ 5.2 \end{pmatrix} = 0.2\begin{pmatrix} 3 \\ 4 \end{pmatrix} + 0.3\mathbf{v}_D$ $\mathbf{v}_D = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$ so $a = -8$ and $b = 6$ Gradients of the lines are $\dfrac{4}{3}$ and $\dfrac{6}{-8}$ Since $\dfrac{4}{3} \times \dfrac{6}{-8} = -1$, they are at 90°	M1 B1 B1 A1 A1 A1 M1 E1	PCLM applied. May be implied. LHS momentum of C correct Complete equation. Accept sign error. cao cao Any method for the angle Clearly shown	 8
				17

Q 2		mark		Sub
(i)	Moments about C $240 \times 2 = 3R_D$ $R_D = 160$ so 160 N Resolve vertically $R_C + R_D = 240$ $R_C = 80$ so 80 N	M1 A1 M1 F1	Moments about C or equivalent. Allow 1 force omitted Resolve vertically or moments about D or equivalent. All forces present. FT from **their** R_D only	 4
(ii) (A)	Moments about D $240 \times 1 = 4T \sin 40$ $T = 93.343\ldots$ so 93.3 N (3 s. f.)	M1 M1 A1 A1	Moments about D or equivalent Attempt at resolution for RHS RHS correct 	 4
(ii) (B)	In equilibrium so horizontal force needed to balance cpt of T. This must be friction and cannot be at C.		Need reference to horizontal force that must come from friction at D.	 1
(iii) (A)	Moments about B $3 \times 240 \times \cos 30 = 6P$ $P = 60\sqrt{3}$ (103.92…..) P inclined at 30° to vertical Resolve horizontally. Friction force F $F = P \sin 30$ so $F = 30\sqrt{3}$ (51.961…)	 M1 E1 B1 M1 A1	All terms present, no extras. Any resolution required attempted. Accept decimal equivalent Seen or equivalent or implied in (iii) (A) or (B). Resolve horizontally. Any resolution required attempted Any form	 5

(iii)				
(B)	Resolve vertically. Normal reaction R $$P\cos 30 + R = 240$$	M1	Resolve vertically. All terms present.and resolution attempted	
		A1		
	Using $F = \mu R$	M1		
	$$\mu = \frac{30\sqrt{3}}{240 - 60\sqrt{3} \times \frac{\sqrt{3}}{2}}$$	A1	Substitute **their expressions** for F and R	
	$$= \frac{30\sqrt{3}}{240 - 90} = \frac{\sqrt{3}}{5} = 0.34641 \text{ so } 0.346 \text{ (3 s. f.)}$$	A1	cao. Any form. Accept 2 s. f. or better	
				5
				19

Q 3		mark		Sub
(a) (i)	$$80\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=48\begin{pmatrix}6\\2\end{pmatrix}+12\begin{pmatrix}1\\-3\end{pmatrix}+20\begin{pmatrix}11\\9\end{pmatrix}$$	M1	Correct method for c.m.	
	$$80\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}520\\240\end{pmatrix}$$	B1	Total mass correct	
		B1	One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$\bar{x}=6.5$	E1		
	$\bar{y}=3$	A1	cao	5
(ii)	Consider x coordinate $520=76\times6.4+4x$	M1	Using additive principle o. e. on x cpts	
		B1	Areas correct. Allow FT from masses from (i)	
	so $x=8.4$	A1	cao	3
(iii)	y coordinate is 1 so we need	B1	Position of centre of square	
	$240=76\bar{y}+4\times1$ and $\bar{y}=3.10526...$	M1		
	so 3.11 (3 s. f.)	A1	cao	3
(b) (i)	Moments about C			
	$4R=120\times3+120\times2$	M1	Moments equation. All terms present	
	so $4R=600$ and $R=150$	E1		2
(ii)		B1		
	$A\uparrow$ $150+T_{AE}\cos30=0$	M1	Equilibrium at a pin-joint	
	$T_{AE}=-100\sqrt{3}$ so $100\sqrt{3}$ N (C)	A1	Any form. Sign correct. Neglect (C)	
	$E\downarrow$ $120+T_{AE}\cos30+T_{EB}\cos30=0$	M1	Equilibrium at E, all terms present	
	$T_{EB}=20\sqrt{3}$ so $20\sqrt{3}$ N (T)	F1	Any form. Sign follows working. Neglect (T).	
		F1	T/C consistent with answers	

				6
(iii)	Consider → at E, using (ii) gives ED as thrust	E1	Clearly explained. Accept 'thrust' correctly deduced from wrong answers to (ii).	
				1
				20

Q 4		mark		Sub
(i)	$\dfrac{0.5 \times 20 \times 8^2 - 0.5 \times 20 \times 5^2 + 510}{6}$ $= 150$ W	M1 B1 A1 A1	Use of $P = WD/t$ $\triangle$ KE. Accept ±390 soi All correct including signs	 4
(ii) (A)	$20g \times \dfrac{3}{5}x - 5gx$ $7gx$ (68.6x) gain	M1 B1 A1 A1	Use of mgh on both terms Either term (neglecting signs) ±$7gx$ in any form. cao	 4
(B)	$11gx$	B1		 1
(C)	$0.5 \times 25 \times 4^2 = 7gx + 11gx = 18gx$ $x = 1.13378\ldots$ so 1.13 m (3 s. f.)	M1 B1 A1	Use of work-energy equation. Allow 1 RHS term omitted. KE term correct cao. Except follow wrong sign for $7gx$ only.	 3
(iii)	**either** $0.5 \times 35 \times v^2 - 0.5 \times 35 \times 16$ $= 15g \times 0.5 - 11g \times 0.5 - 12g \times 0.5$ $v^2 = 13.76$ so $v = 3.70944\ldots$ so 3.71 m s^{-1} (3 s. f.) **or** $15g - T = 15a$ $T - 12g - 11g = 20a$ so $a = -2.24$ $v^2 = 4^2 + 2 \times (-2.24) \times 0.5$ so 3.71 m s^{-1} (3 s. f.)	 M1 B1 A1 A1 M1 A1 M1 A1	 Use of work-energy. KE, GPE and WD against friction terms present. $\triangle$ GPE correct inc sign (1.5g J loss) All correct cao N2L in 1 or 2 equations. All terms present cao Use of appropriate (sequence of) *uvast* cao	 4
				16