MEI STRUCTURED MATHEMATICS   **4762**

Mechanics 2

Tuesday   **7 JUNE 2005**   Afternoon   1 hour 30 minutes

Additional materials:
    Answer booklet
    Graph paper
    MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** Roger of mass $70\,\text{kg}$ and Sheuli of mass $50\,\text{kg}$ are skating on a horizontal plane containing the standard unit vectors **i** and **j**. The resistances to the motion of the skaters are negligible. The two skaters are locked in a close embrace and accelerate from rest until they reach a velocity of $2\mathbf{i}\,\text{m s}^{-1}$, as shown in Fig. 1.1.
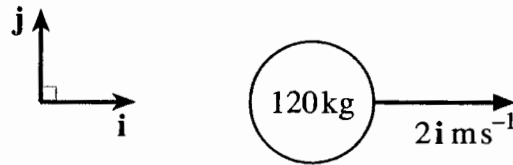

Fig. 1.1

    (i) What impulse has acted on them? [1]

During a dance routine, the skaters separate on three occasions from their close embrace when travelling at a constant velocity of $2\mathbf{i}\,\text{m s}^{-1}$.

    (ii) Calculate the velocity of Sheuli after the separation in the following cases.

        (A) Roger has velocity $\mathbf{i}\,\text{m s}^{-1}$ after the separation.

        (B) Roger and Sheuli have equal speeds in opposite senses after the separation, with Roger moving in the **i** direction.

        (C) Roger has velocity $4(\mathbf{i}+\mathbf{j})\,\text{m s}^{-1}$ after the separation. [6]

(b) Two discs with masses $2\,\text{kg}$ and $3\,\text{kg}$ collide directly in a horizontal plane. Their velocities just before the collision are shown in Fig. 1.2. The coefficient of restitution in the collision is 0.5.
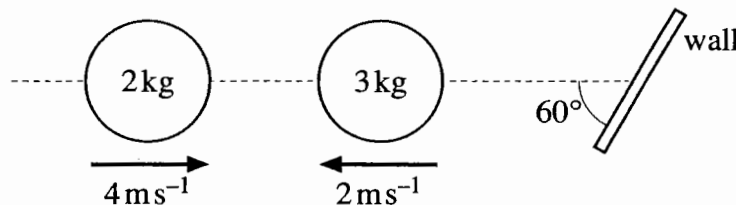

Fig. 1.2

    (i) Calculate the velocity of each disc after the collision. [6]

The disc of mass $3\,\text{kg}$ moves freely after the collision and makes a perfectly elastic collision with a smooth wall inclined at $60°$ to its direction of motion, as shown in Fig. 1.2.

    (ii) State with reasons the speed of the disc and the angle between its direction of motion and the wall after the collision. [4]

2 A car of mass 850 kg is travelling along a road that is straight but not level.

On one section of the road the car travels at constant speed and gains a vertical height of 60 m in 20 seconds. Non-gravitational resistances to its motion (e.g. air resistance) are negligible.

   (i) Show that the average power produced by the car is about 25 kW. [2]

On a *horizontal* section of the road, the car develops a constant power of exactly 25 kW and there is a constant resistance of 800 N to its motion.

   (ii) Calculate the maximum possible steady speed of the car. [3]

   (iii) Find the driving force and acceleration of the car when its speed is $10 \,\mathrm{ms}^{-1}$. [3]

When travelling along the horizontal section of road, the car accelerates from $15 \,\mathrm{m\,s}^{-1}$ to $20 \,\mathrm{m\,s}^{-1}$ in 6.90 seconds with the same constant power and constant resistance.

   (iv) By considering work and energy, find how far the car travels while it is accelerating. [6]

When the car is travelling at $20 \,\mathrm{m\,s}^{-1}$ up a constant slope inclined at $\arcsin(0.05)$ to the horizontal, the driving force is removed. Subsequently, the resistance to the motion of the car remains constant at 800 N.

   (v) What is the speed of the car when it has travelled a further 105 m up the slope? [5]

[Turn over

3   Fig. 3.1 shows an object made up as follows. ABCD is a uniform lamina of mass 16 kg. BE, EF, FG, HI, IJ and JD are each uniform rods of mass 2 kg. ABCD, BEFG and HIJD are squares lying in the same plane. The dimensions in metres are shown in the figure.

Fig. 3.1

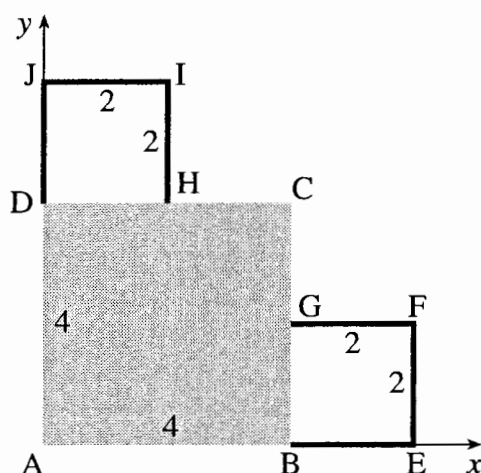(i) Find the coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.1.
                                                                                                    [5]

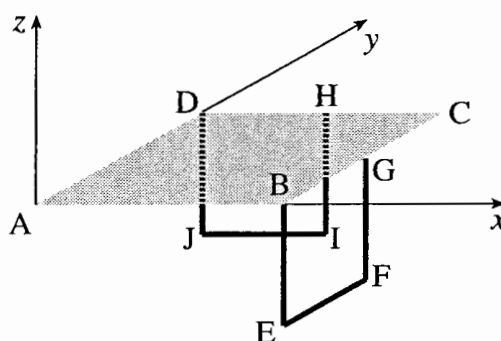The rods are now re-positioned so that BEFG and HIJD are perpendicular to the lamina, as shown in Fig. 3.2.

Fig. 3.2

(ii) Find the x-, y- and z-coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.2. Calculate the distance of the centre of mass from A.   [8]

The object is now freely suspended from A and hangs in equilibrium with AC at $\alpha°$ to the vertical.

(iii) Calculate α.   [4]

4 **(a)** A framework is made from light rods AB, BC and CA. They are freely hinged to each other at A, B and C and to a vertical wall at A. The hinge at B rests on a smooth, horizontal support. The rod AC is horizontal. A vertical load of L N acts at C. This information is shown in Fig. 4.1 together with the dimensions of the framework and the external forces U N, V N and R N acting on the framework.
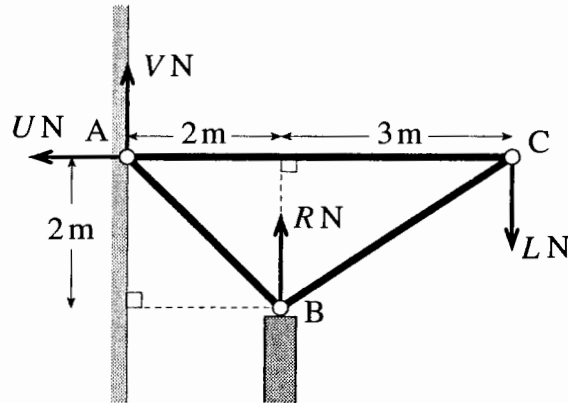

Fig. 4.1

(i) Show that $R = 2.5L$, $U = 0$ and $V = -1.5L$. [4]

(ii) Calculate the internal forces in the rods AB, AC and BC in terms of L, stating whether each of these rods is in tension or thrust (compression). [8]

(b) Fig. 4.2 shows a plank of weight W resting at the points A and B on two fixed supports. The plank is at an angle θ to the horizontal. Its centre of mass, G, is such that AG is 2 m and GB is 1 m.
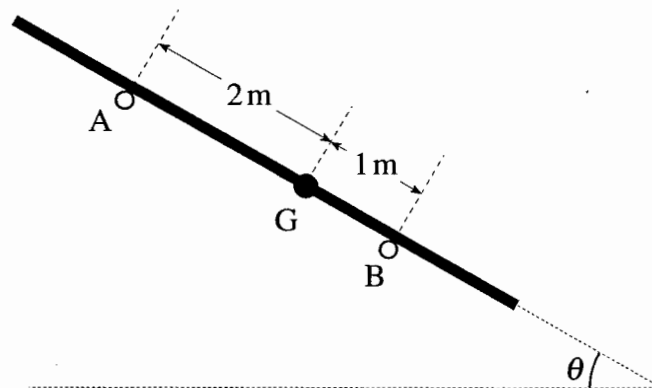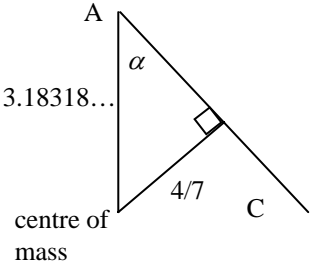

Fig. 4.2
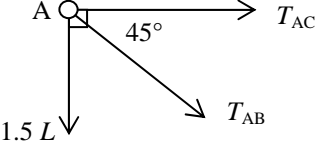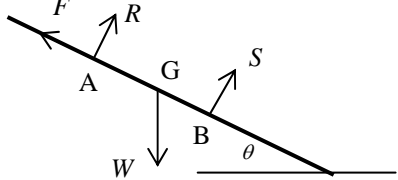
The contact between the plank and the support at A is rough, but that at B is smooth. The plank is on the point of slipping.

(i) Draw a diagram showing all the forces acting on the plank. [1]

(ii) By taking moments about a suitable point, find an expression in terms of W and θ for the normal reaction at A of the support on the plank. [3]

(iii) Find an expression in terms of θ for the coefficient of friction between the plank and the rough support. [3]

Q 1		mark		Sub
(a) (i)	$240\,\mathbf{i}$ N s $\rightarrow$	B1		1
(ii) (A)	$240\,\mathbf{i} = 70\,\mathbf{i} + 50\,\mathbf{v}$ so $\mathbf{v} = 3.4\,\mathbf{i}$ m s^{-1}	M1 A1	Equating to **their** $240\,\mathbf{i}$ in this part FT $240\,\mathbf{i}$	
(B)	$240\,\mathbf{i} = 70u\,\mathbf{i} - 50u\,\mathbf{i}$ $u = 12$ so $\mathbf{v} = -12\,\mathbf{i}$ m s^{-1}	M1 A1	Must have u in both RHS terms and opposite signs FT $240\,\mathbf{i}$	
(C)	$240\,\mathbf{i} = 280(\,\mathbf{i} + \mathbf{j}\,) + 50\mathbf{v}_B$ so $\mathbf{v}_B = (\text{-}0.8\,\mathbf{i} - 5.6\,\mathbf{j})$ m s^{-1}	M1 A1	FT $240\,\mathbf{i}$ Must have all terms present cao	6
(b) (i)	 NEL $\dfrac{v_2 - v_1}{-2 - 4} = -0.5$ so $v_2 - v_1 = 3$ PCLM $8 - 6 = 2v_1 + 3v_2$ Solving $v_2 = 1.6$ so 1.6 m s^{-1} $\rightarrow$ $v_1 = -1.4$ so 1.4 m s^{-1} $\leftarrow$	M1 A1 M1 A1 A1 A1	NEL Any form PCLM Any form Direction must be clear (accept diagram) Direction must be clear (accept diagram). [Award A1 A0 if v_1 & v_2 correct but directions not clear]	6
(ii)	1.6 m s^{-1} at $60°$ to the wall (glancing angles both $60°$) No change in the velocity component parallel to the wall as no impulse No change in the velocity component perpendicular to the wall as perfectly elastic	B1 B1 E1 E1	FT **their** 1.6 Must give reason Must give reason	4
	total	17		

Q 2		mark		Sub
(i)	We need $\dfrac{mgh}{t} = \dfrac{850 \times 9.8 \times 60}{20} = 24\,990$ so approx 25 kW	M1 E1	Use of $\dfrac{mgh}{t}$ Shown	 2
(ii)	Driving force – resistance = 0 $25000 = 800v$ so $v = 31.25$ and speed is 31.25 m s^{-1}	B1 M1 A1	May be implied Use of $P = Fv$	 3
(iii)	Force is $\dfrac{25000}{10} = 2500$ N N2L in direction of motion $2500 - 800 = 850a$ $a = 2$ so 2 m s^{-2}	B1 M1 A1	 Use of N2L with all terms	 3
(iv)	$0.5 \times 850 \times 20^2 = 0.5 \times 850 \times 15^2$ $\qquad\qquad +25000 \times 6.90$ $\qquad\qquad -800x$ $x = 122.6562\ldots$ so 123 m (3 s. f.)	M1 B1 B1 B1 A1 A1	W-E equation with KE and power term One KE term correct Use of Pt. Accept wrong sign WD against resistance. Accept wrong sign All correct cao	 6
(v)	**either** $0.5 \times 850 \times v^2 = 0.5 \times 850 \times 20^2$ $\qquad\qquad -850 \times 9.8 \times \dfrac{105}{20}$ $\qquad\qquad -800 \times 105$ $v^2 = 99.452\ldots$ so 9.97 m s^{-1} **or** N2L + ve up plane $-(800 + 850g \times 0.05) = 850a$ $a = -1.43117\ldots$ $v^2 = 20^2 + 2 \times (-1.43117\ldots) \times 105$ $v^2 = 99.452\ldots$ so 9.97 m s^{-1}	M1 M1 A1 B1 A1 M1 A1 M1 A1 A1	W-E equation inc KE, GPE and WD GPE term with attempt at resolution Correct. Accept expression. Condone wrong sign. WD term. Neglect sign. cao N2L. All terms present. Allow sign errors. Accept $\pm$ Appropriate *uvast*. Neglect signs. All correct including consistent signs. Need not follow sign of a above. cao	 5
		19		

Q 3		mark		Sub
(i)	$28\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=16\begin{pmatrix}2\\2\end{pmatrix}+2\begin{pmatrix}5\\0\end{pmatrix}+2\begin{pmatrix}6\\1\end{pmatrix}+2\begin{pmatrix}5\\2\end{pmatrix}$ $+2\begin{pmatrix}0\\5\end{pmatrix}+2\begin{pmatrix}1\\6\end{pmatrix}+2\begin{pmatrix}2\\5\end{pmatrix}$ $\bar{x}=2.5$ $\bar{y}=2.5$	M1 B1 B1 A1 A1	Complete method Total mass correct 3 c. m. correct (or 4 x- or y-values correct) [Allow A0 A1 if only error is in total mass] [If $\bar{x}=\bar{y}$ claimed by symmetry and only one component worked replace final A1, A1 by B1 explicit claim of symmetry A1 for the 2.5]	5
(ii)	$\bar{x}=\bar{y}$ $28\bar{x}=16\times2+6\times4+2\times0+2\times1+2\times2$ $\bar{x}=\dfrac{31}{14}$ (2.21428…) $\bar{z}=\dfrac{8\times(-1)+4\times(-2)}{28}=-\dfrac{4}{7}$ (- 0.57142…) Distance is $\sqrt{\left(\dfrac{31}{14}\right)^2+\left(\dfrac{31}{14}\right)^2+\left(\dfrac{4}{7}\right)^2}$ $=3.18318..$ so 3.18 m (3 s. f.)	B1 M1 A1 A1 A1 A1 M1 F1	Or by direct calculation Dealing with 'folded' parts for $\bar{x}$ or for $\bar{z}$ At least 3 terms correct for $\bar{x}$ All terms correct allowing sign errors Use of Pythagoras in 3D on **their** c.m.	8
(iii)	$\sin\alpha=\dfrac{4}{7}/3.18318..$ so $\alpha=10.3415…$ so 10.3° (3 s. f.)	M1 B1 M1 A1	c.m. clearly directly below A Diagram showing α and known lengths (or equivalent). FT their values. Award if final answer follows **their** values. Appropriate expression for α. FT **their** values. cao	4
	total	17		

Q 4		mark		Sub
(a) (i)	Moments c.w. about A			
	$2R = 5L$ so $R = 2.5L$	E1		
	Resolve $\rightarrow$ $U = 0$	E1		
	Resolve $\uparrow$ $V + R = L$	M1	Resolve vertically or take moments about B (or C)	
	so $V = -1.5L$	E1		4
(ii)				
		M1	Equilibrium at a pin-joint	
	For equilibrium at A	M1	Attempt at equilibrium at A or C including resolution with correct angle	
	$\uparrow$ $T_{AB}\cos 45 + 1.5L = 0$			
	so $T_{AB} = -\dfrac{3\sqrt{2}L}{2}$ so $\dfrac{3\sqrt{2}L}{2}$ N (C) in AB	A1	(2.12L (3 s. f.))	
	$\rightarrow$ $T_{AC} + T_{AB}\cos 45 = 0$			
	so $T_{AC} = \dfrac{3L}{2}$ so $\dfrac{3L}{2}$ N (T) in AC	F1	(1.5L)	
	At C $\downarrow$ $L + T_{BC}\cos\theta = 0$	M1	Must include attempt at angle	
	$\tan\theta = 3/2 \Rightarrow \cos\theta = 2/\sqrt{13}$	B1		
	so $T_{BC} = -\dfrac{\sqrt{13}L}{2}$ so $\dfrac{\sqrt{13}L}{2}$ N (C) in BC	A1	(1.80 L (3 s. f.))	
		F1	Award for T/C correct from their internal forces. Do not award without calcs	8
(b) (i)		B1	All forces present with arrows and labels. Angles and distances not required.	1
(ii)	c.w. moments about B			
	$R \times 3 - W \times 1 \cos\theta = 0$	M1	If moments about other than B, then need to resolve perp to plank as well	
		A1	Correct	
	so $R = \dfrac{1}{3}W\cos\theta$	A1		3
(iii)	Resolve parallel to plank			
	$F = W\sin\theta$	B1		
	$\mu = \dfrac{F}{R} = \dfrac{W\sin\theta}{\dfrac{1}{3}W\cos\theta} = 3\tan\theta$	M1	Use of $F = \mu R$ and their F and R	
		A1	Accept any form.	3
	total	19		