Thursday 12 June 2014 – Afternoon

AS GCE MATHEMATICS (MEI)

4761/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4761/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows the velocity-time graph of a cyclist travelling along a straight horizontal road between two sets of traffic lights. The velocity, v, is measured in metres per second and the time, t, in seconds. The distance travelled, s metres, is measured from when $t = 0$.

Fig. 1

(i) Find the values of s when $t = 4$ and when $t = 18$. **[3]**

(ii) Sketch the graph of s against t for $0 \leqslant t \leqslant 18$. **[3]**

2 The unit vectors **i** and **j** shown in Fig. 2 are in the horizontal and vertically upwards directions.

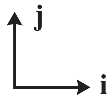


Fig. 2

Forces **p** and **q** are given, in newtons, by $\mathbf{p} = 12\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = 16\mathbf{i} + 1.5\mathbf{j}$.

(i) Write down the force $\mathbf{p} + \mathbf{q}$ and show that it is parallel to $8\mathbf{i} - \mathbf{j}$. **[3]**

(ii) Show that the force $3\mathbf{p} + 10\mathbf{q}$ acts in the horizontal direction. **[2]**

(iii) A particle is in equilibrium under forces $k\mathbf{p}$, $3\mathbf{q}$ and its weight $\mathbf{w}$.

Show that the value of k must be -4 and find the mass of the particle. **[3]**

3 Fig. 3 shows a smooth ball resting in a rack. The angle in the middle of the rack is 90°. The rack has one edge at angle α to the horizontal.

The weight of the ball is W N. The reaction forces of the rack on the ball at the points of contact are R N and S N.

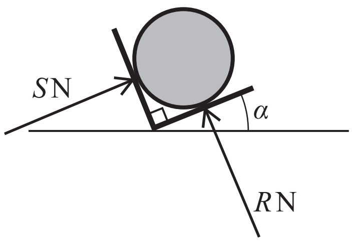


Fig. 3

(i) Draw a fully labelled triangle of forces to show the forces acting on the ball. Your diagram must indicate which angle is α. **[3]**

(ii) Find the values of R and S in terms of W and α. **[2]**

(iii) On the same axes draw sketches of R against α and S against α for $0^\circ \leqslant \alpha \leqslant 90^\circ$.

For what values of α is $R < S$? **[3]**

4 Fig. 4 illustrates a situation in which a film is being made. A cannon is fired from the top of a vertical cliff towards a ship out at sea. The director wants the cannon ball to fall just short of the ship so that it appears to be a near-miss. There are actors on the ship so it is important that it is not hit by mistake.

The cannon ball is fired from a height 75 m above the sea with an initial velocity of $20\,\mathrm{m\,s^{-1}}$ at an angle of 30° above the horizontal. The ship is 90 m from the bottom of the cliff.

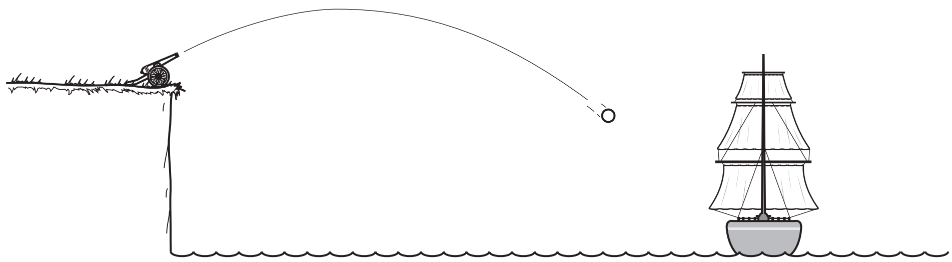


Fig. 4

(i) The director calculates where the cannon ball will hit the sea, using the standard projectile model and taking the value of g to be $10\,\mathrm{m\,s^{-2}}$.

Verify that according to this model the cannon ball is in the air for 5 seconds. Show that it hits the water less than 5 m from the ship. **[6]**

(ii) Without doing any further calculations state, with a brief reason, whether the cannon ball would be predicted to travel further from the cliff if the value of g were taken to be $9.8\,\mathrm{m\,s^{-2}}$. **[1]**

5 In a science fiction story a new type of spaceship travels to the moon. The journey takes place along a straight line. The spaceship starts from rest on the earth and arrives at the moon's surface with zero speed. Its speed, v kilometres per hour at time t hours after it has started, is given by

$$v = 37500(4t - t^2).$$

(i) Show that the spaceship takes 4 hours to reach the moon. **[1]**

(ii) Find an expression for the distance the spaceship has travelled at time t.

Hence find the distance to the moon. **[4]**

(iii) Find the spaceship's greatest speed during the journey. **[2]**

Section B (36 marks)

6 In this question the origin is a point on the ground. The directions of the unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ are east, north and vertically upwards.

Alesha does a sky-dive on a day when there is no wind. The dive starts when she steps out of a moving helicopter. The dive ends when she lands gently on the ground.

- During the dive Alesha can reduce the magnitude of her acceleration in the vertical direction by spreading her arms and increasing air resistance.
- During the dive she can use a power unit strapped to her back to give herself an acceleration in a horizontal direction.
- Alesha's mass, including her equipment, is 100 kg.
- Initially, her position vector is $\begin{pmatrix}-75\\90\\750\end{pmatrix}$ m and her velocity is $\begin{pmatrix}-5\\0\\-10\end{pmatrix}$ m s^{-1}.

(i) Calculate Alesha's initial speed, and the initial angle between her motion and the downward vertical. **[4]**

At a certain time during the dive, forces of $\begin{pmatrix}0\\0\\-980\end{pmatrix}$ N, $\begin{pmatrix}0\\0\\880\end{pmatrix}$ N and $\begin{pmatrix}50\\-20\\0\end{pmatrix}$ N are acting on Alesha.

(ii) Suggest how these forces could arise. **[3]**

(iii) Find Alesha's acceleration at this time, giving your answer in vector form, and show that, correct to 3 significant figures, its magnitude is 1.14 m s^{-2}. **[3]**

One suggested model for Alesha's motion is that the forces on her are constant throughout the dive from when she leaves the helicopter until she reaches the ground.

(iv) Find expressions for her velocity and position vector at time t seconds after the start of the dive according to this model. Verify that when $t = 30$ she is at the origin. **[6]**

(v) Explain why consideration of Alesha's landing velocity shows this model to be unrealistic. **[2]**

7 Fig. 7 illustrates a train with a locomotive, L, pulling two trucks, A and B.

The locomotive has mass 90 tonnes and is subject to a resistance force of 2000 N.

Each of the trucks A and B has mass 30 tonnes and is subject to a resistance force of 500 N.

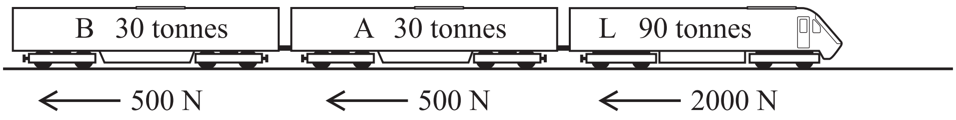


Fig. 7

Initially the train is travelling along a straight horizontal track. The locomotive is exerting a driving force of 12 000 N.

(i) Find the acceleration of the train. **[3]**

(ii) Find the tension in the coupling between trucks A and B. **[3]**

When the train is travelling at $10\,\mathrm{m\,s^{-1}}$, a fault occurs with truck A and the resistance to its motion changes from 500 N to 5000 N.

The driver reduces the driving force to zero and allows the train to slow down under the resistance forces and come to a stop.

(iii) Find the distance the train travels while slowing down and coming to a stop.

Find also the force in the coupling between trucks A and B while the train is slowing down, and state whether it is a tension or a thrust. **[7]**

The fault in truck A is repaired so that the resistance to its motion is again 500 N. The train continues and comes to a place where the track goes up a uniform slope at an angle of $\alpha°$ to the horizontal.

(iv) When the train is on the slope, it travels at uniform speed. The driving force remains at 12 000 N. Find the value of α. **[3]**

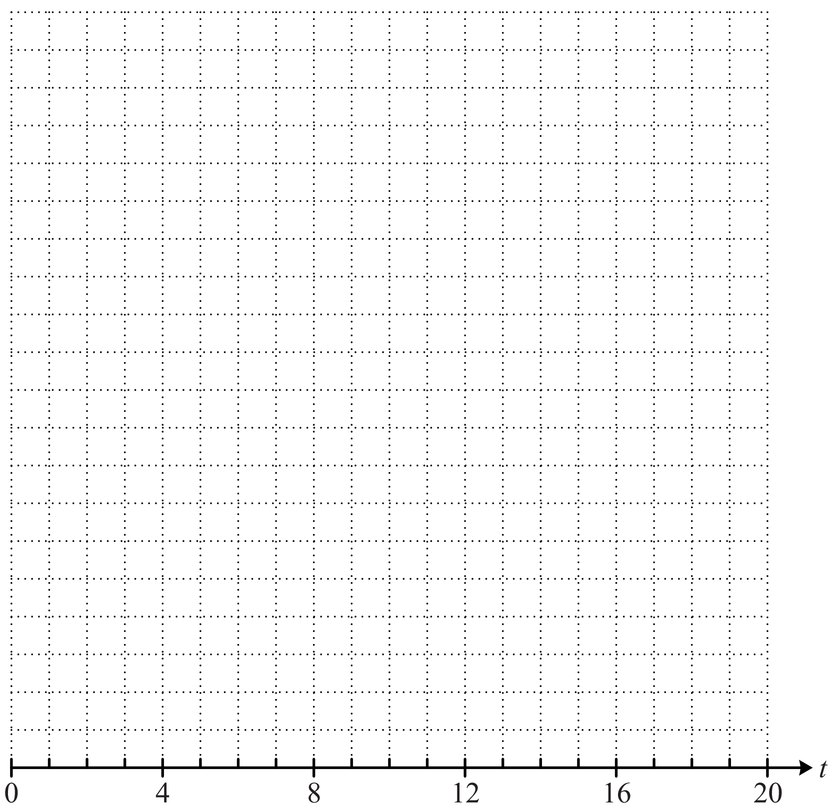
(v) Show that the force in the coupling between trucks A and B has the same value that it had in part **(ii)**. **[2]**

END OF QUESTION PAPER

Section A (36 marks)

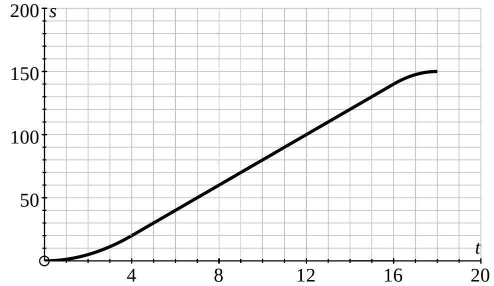
1 (i)

1 (ii)

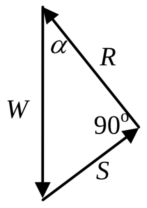
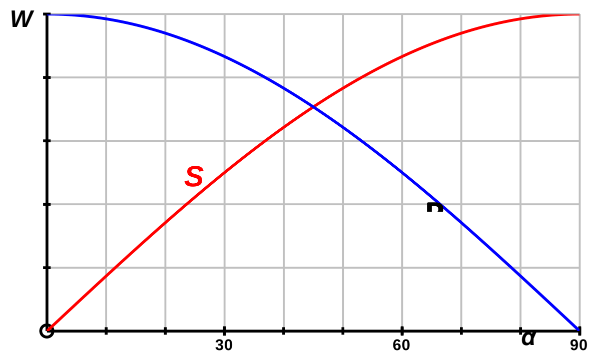


3 (ii)

3 (iii)

Question			Answer	Marks	Guidance
1	**(i)**		When $t = 4$, $s = \frac{1}{2}\times 4\times 10$		Finding the area of the triangle or equivalent.
			$s = 20$	B1	
			When $t = 18$, $s = \frac{1}{2}\times\ 18+12\ \times 10$	M1	A complete method of finding the area of the trapezium or equivalent.
			$s = 150$	A1	CAO
				[3]	
1	**(ii)**				
			Graph joining (0,0), (4,20) and (18, 150)	B1	Allow FT for their (4,20) and (18, 150) Condone extension to (20, 150) with a horizontal line.
			The graph goes through (16, 140)	B1	
					Allow SC1 for the first two marks if there is a consistent displacement from a correct scale, eg plotting (18,150) at (19, 150)
			Curves at both ends	B1	The sections from $t = 0$ to $t = 4$ and from $t = 16$ to $t = 18$ are both curves
				[3]	

Question			Answer	Marks	Guidance	
2	(i)		$\mathbf{p} + \mathbf{q} = 28\mathbf{i} - 3.5\mathbf{j}$	B1		
			$28\mathbf{i} - 3.5\mathbf{j} = k(8\mathbf{i} - \mathbf{j})$	M1	Or equivalent. k may be implied by going straight to 3.5	
			$k = 3.5$	A1		
			(So they are parallel)			
			Alternative			
			$\mathbf{p} + \mathbf{q} = 28\mathbf{i} - 3.5\mathbf{j}$	B1		
			$\mathbf{p} + \mathbf{q}$: $\tan\theta = \frac{-3.5}{28} \Rightarrow \theta = -7.13°$			
			$8\mathbf{i} - \mathbf{j}$: $\tan\theta = \frac{-1}{8} \Rightarrow \theta = -7.13°$	M1	Comparing the ratio of the components in each of the two vectors is sufficient, using any consistent sign convention. The angle does not need to be worked out, nor does tan have to be seen.	
			So they are parallel	A1	Both ratios the same and correct	
				[3]		
2	(ii)		$3\mathbf{p} + 10\mathbf{q} = (36 + 160)\mathbf{i} + (-15 + 15)\mathbf{j}$			
			$= 196\mathbf{i}$	B1		
			Zero $\mathbf{j}$ component so horizontal	B1	Or equivalent explanation. May be shown on a diagram	
				[2]		
2	(iii)		The horizontal component must be zero			
			So $12k + 3 \times 16 = 0 \Rightarrow k = -4$	B1	Substituting $k = -4$ and showing $\mathbf{i}$ component is zero is acceptable	
			$\mathbf{w} = -24.5\mathbf{j}$	B1	Award for 24.5 seen	
			$mg = 24.5 \Rightarrow m = 2.5$ The mass is 2.5 kg	B1	Award for 2.5 seen. FT from their weight.	
				[3]		

Question		Answer	Marks	Guidance
3	(i)		B1 B1 B1 **[3]**	Closed triangle with cycling arrows. Accept any consistent orientation. All forces labelled. Correct angles. The 90° may be implied. α may be shown between S and the horizontal (ie outside the triangle). SC1 Award for a force diagram with no extra forces and all labels and directions correct.
3	(ii)	$R = W\cos\alpha$ $S = W\sin\alpha$	B1 B1 **[2]**	Allow FT for sin-cos interchange following the wrong angle in the triangle being marked α in part (i) for both marks. SC1 if both S and R are given negative signs
3	(iii)	Sketch graph of R against α Correct sketch graph of S against α $45° < \alpha \ \ (\leq 90°)$	 B1 B1 B1 **[3]**	The answers in part (iii) must - either be fully correct - or they must all be consistent with those in part (ii) where the marks in part (ii) are FT from part (i). No credit should be given to forms other than $W\cos\alpha$ and $W\sin\alpha$. The curves must have the correct end points and lie within the correct range; no credit should be given for straight lines. Graphs must be correctly labelled. Unlabelled graphs get B0 B0. Condone no explicit vertical scale. Do not accept straight lines. Must be consistent with graph of R Condone $45° \leq \alpha$

Question			Answer	Marks	Guidance	
4	(i)		Vertical component of initial velocity $=20\sin 30°\ \ (=10)$	B1		
			Vertical motion $s=s_0+ut+\frac{1}{2}at^2$	M1	Substitution required. The sign of g must be correct. Condone no s_0	
			When it hits the sea $0=75+10t-5t^2$	A1		
			$75+10\times5-5\times5^2=0$ As required		Or equivalent, eg solving the quadratic equation.	
			This is satisfied when $t=5$	E1		
			Alternative			
			Vertical component of initial velocity $=20\sin 30°\ \ (=10)$	B1		
			Vertical motion $v=u+at$	M1	Complete method for finding t = 5 required.	
			At the top $0=10-10t \quad \Rightarrow \quad t=1$			
			It takes another 1 second to reach the level of the cliff top			
			At that point its speed is 10 m s^{-1} downwards			
			When it hits the sea $-75=-10t-5t^2$		Or equivalent finding the time (4 seconds) from the top (height 80 m) to hitting the sea	
			$t^2+2t-15=0 \quad \Rightarrow \quad t=3$	A1		
			Total time = 1 + 1 + 3 = 5 seconds	E1		
			Horizontal motion $x=20\times\cos 30°\times t$	M1		
			$t=5\Rightarrow 86.6$			
			It is 3.4 m from the ship so within 5 m	E1	Condone 3.5 m	
				[6]		
4	(ii)		It is longer in the air so it goes further	B1	Justification for travelling further is required for this mark.	
				[1]		

Question			Answer	Marks	Guidance	
5	(i)		$v = 0$ when it arrives $150\ 000(t - \frac{1}{4}t^2) = 0$ $\Rightarrow$ $t = 4$ (on arrival)	B1 **[1]**	Award this mark for substituting $t = 4$ to obtain $v = 0$ Condone omission of $t = 0$	
5	(ii)		Distance travelled $s = \int v \mathrm{d}t$	M1	Do not accept multiplication by t.	
			$s = 150\ 000\left[\frac{1}{2}t^2 - \frac{1}{12}t^3\right]$ $(+c)$	A1		
			When $t = 4$, $s = 400\ 000$	M1	Substituting their $t = 4$. This mark is dependent on the previous M mark.	
			The journey is 400 000 km	A1 **[4]**	If 400 000 seen award the previous mark	
5	(iii)		For maximum speed $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 0$ $\frac{\mathrm{d}v}{\mathrm{d}t} = 150\ 000(1 - \frac{1}{2}t)$ $\Rightarrow t = 2$	B1	$t = 2$ seen Accept a trial and error method	
			$v = 150\ 000(2 - \frac{1}{4} \times 2^2) = 150\ 000$ Maximum speed is 150 000 $\mathrm{km\,h^{-1}}$	B1 **[2]**	CAO	

Question			Answer	Marks	Guidance	
6	(i)		Speed = $\sqrt{(-5)^2+0^2+(-10)^2}$	M1	For use of Pythagoras. Accept $\sqrt{5^2+10^2}$.	
			$= 11.2 \text{ m s}^{-1}$ (11.18)	A1	Accept $\sqrt{125}$ or $5\sqrt{5}$	
			$\tan\theta = \frac{5}{10}$	M1	Complete method for correct angle; may use $\sin\theta = \frac{5}{11.2}$, $\cos\theta = \frac{10}{11.2}$.	
			$\theta = 26.6°$	A1	Allow 153.4°, 206.6°	
				[4]		
6	(ii)		$\begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix}$ her weight	B1	The descriptions should be linked to the forces, either by the layout of the answer or by suitable text. If not, assume that the forces they refer to are in the order given here (which is the same as the question).	
			$\begin{pmatrix} 0 \\ 0 \\ 880 \end{pmatrix}$ resistance to her vertical motion	B1	Accept "Air resistance", "Arms stretched out" and similar statements. Condone mention of a parachute.	
			$\begin{pmatrix} 50 \\ -20 \\ 0 \end{pmatrix}$ force from the power unit	B1		
				[3]		
6	(iii)		Resultant force = $\begin{pmatrix} 50 \\ -20 \\ -100 \end{pmatrix}$	B1	May be implied	
			Acceleration = $\begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix}$	B1	Newton's 2nd Law	
			Magnitude = $\sqrt{0.5^2+(-0.2)^2+1^2} = 1.1357...$ So 1.14 to 3 s.f.	B1	Answer given. Allow FT from sign errors. Accept $\lvert\mathbf{F}\rvert \div 100$	
				[3]		

Question			Answer	Marks	Guidance	
6	(iv)		$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	M1	FT their **a** for the first 5 marks of this part. Vectors must be seen or implied. Accept valid integration.	
			$\mathbf{v} = \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} + \begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix} t$	A1		
			$\mathbf{r} = \mathbf{r_0} + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	Vectors must be seen or implied. Accept valid integration. Condone no $\mathbf{r_0}$ for this M mark	
			$\mathbf{r} = \begin{pmatrix} -75 \\ 90 \\ 750 \end{pmatrix} + \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix} t^2$	A1		
			When $t = 30$	M1	Vectors must be seen or implied.	
			$\mathbf{r} = \begin{pmatrix} -75 - 150 + 225 \\ 90 + 0 - 90 \\ 750 - 300 - 450 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, as required	E1	CAO	
					SC1 to replace the first 4 marks of this section if the acceleration is taken to be **g** but the answer is otherwise correct.	
				[6]		
6	(v)		When $t = 30$, $\mathbf{v} = \begin{pmatrix} 10 \\ -6 \\ -40 \end{pmatrix}$	M1	There must be an attempt to work out at least the vertical component of the velocity at t = 30. This mark is not dependent on a correct answer.	
			The vertical component of the velocity is too fast for a safe landing	A1	Accept an argument based on speed derived from a vector.	
				[2]		

Question			Answer	Marks	Guidance	
7	(i)		Whole train: mass = 150 tonnes Total Resistance = 3000 N	B1	Both totals required.	
			$12000 - 3000 = 150000a$	M1	Correct elements must be present	
			$a = 0.06$ The acceleration is 0.06 m s^{-2}	A1	CAO. Errors with units (eg not converting tonnes to kilograms) are penalised here but condoned where possible for the remainder of the question.	
				[3]		
7	(ii)		Truck B: $T - 500 = 30000a$	M1	Correct elements must be present	
			$T - 500 = 30000 \times 0.06$	A1	Allow FT for a from part (i) if units are used consistently, for all the marks in this part	
			$T = 2300$ Between A and B, tension of 2300 N	A1		
				[3]		
			Alternative			
			Rest of train: 12 000 – 2500 – T = 120 000a	M1	Correct elements must be present	
			T = 12 000 –2500 – 120 000 × 0.06	A1		
			T = 2300	A1		
7	(iii)		Treating the train as a whole – 2000 – 5000 – 500 = 150 000a	M1		
			$a = -0.05$	A1	Allow FT for the remaining A marks in part (iii) from an error in a	
			$v^2 - u^2 = 2as$	M1		
			$0^2 - 10^2 = 2 \times (-0.05) \times s$			
			$s = 1000$ Stopping distance is 1000 m	A1		
			B: $T - 500 = 30000a$	M1	Correct elements must be present. Alternative for rest of train: – T – 5000 –2000 = 120 000 × – 0.05	
			$T = -1000$	A1	The sign of 1000 must be consistent with the direction of T.	
			Between A and B, thrust of 1000 N	A1	Dependent on previous M and A marks. Accept “compression”.	
				[7]		

Question			Answer	Marks	Guidance	
7	(iv)		Equilibrium parallel to the slope	M1	Correct elements must be present and there must be an attempt to resolve the weight. Condone omission of g for this mark.	
			$150000 \times 9.8 \times \sin\alpha + 3000 = 12000$	A1		
			$\alpha = 0.35°$	A1	CAO	
				[3]		
7	(v)		B: $T_2 - 500 - 30000 \times 9.8 \times \sin 0.35...° = 0$	M1	Correct elements must be present. Condone omission of g for this mark. Do not accept 1800 N for the component of the weight without justification. Alternative for rest of train: $12\,000 = T + 2500 + 120\,000 \times 9.8 \times \sin 0.35^{\circ}$	
			$T_2 = 2300$ Between A and B, tension of 2300 N, as in part (ii)	A1	This mark can only be awarded if the angle found in (iv) is correct.	
				[2]		