OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

AS GCE MATHEMATICS (MEI)

4761/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4761/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows a pile of four uniform blocks in equilibrium on a horizontal table. Their masses, as shown, are 4 kg, 5 kg, 7 kg and 10 kg.

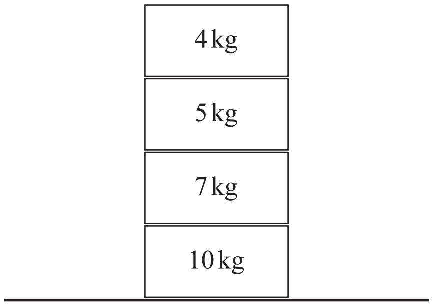


Fig. 1

Mark on the diagram the magnitude and direction of each of the forces acting on the 7 kg block. **[3]**

2 In this question, air resistance should be neglected.

Fig. 2 illustrates the flight of a golf ball. The golf ball is initially on the ground, which is horizontal.

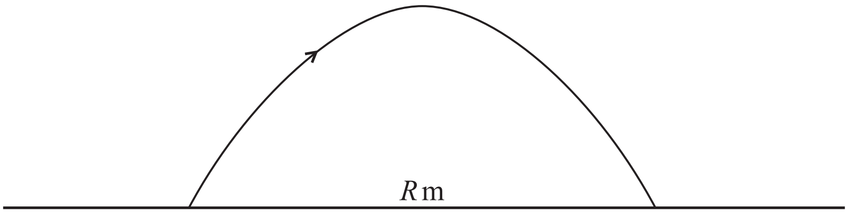


Fig. 2

It is hit and given an initial velocity with components of $15\,\mathrm{m\,s^{-1}}$ in the horizontal direction and $20\,\mathrm{m\,s^{-1}}$ in the vertical direction.

(i) Find its initial speed. **[1]**

(ii) Find the ball's flight time and range, R m. **[4]**

(iii) (A) Show that the range is the same if the components of the initial velocity of the ball are $20\,\mathrm{m\,s^{-1}}$ in the horizontal direction and $15\,\mathrm{m\,s^{-1}}$ in the vertical direction. **[1]**

(B) State, justifying your answer, whether the range is the same whenever the ball is hit with the same initial speed. **[2]**

3 **In this question take $g = 10$.**

The directions of the unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ are east, north and vertically upwards.

Forces **p**, **q** and **r** are given by $\mathbf{p} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}$ N, $\mathbf{q} = \begin{pmatrix}-1\\-4\\2\end{pmatrix}$ N and $\mathbf{r} = \begin{pmatrix}2\\5\\0\end{pmatrix}$ N.

(i) Find which of **p**, **q** and **r** has the greatest magnitude. **[2]**

(ii) A particle has mass 0.4 kg. The forces acting on it are **p**, **q**, **r** and its weight.

Find the magnitude of the particle's acceleration and describe the direction of this acceleration. **[4]**

4 The directions of the unit vectors **i** and **j** are east and north.

The velocity of a particle, $\mathbf{v}$ m s^{-1}, at time t s is given by

$$\mathbf{v} = (16 - t^2)\mathbf{i} + (31 - 8t)\mathbf{j}.$$

Find the time at which the particle is travelling on a bearing of 045° and the speed of the particle at this time. **[6]**

5 Fig. 5 shows blocks of mass 4 kg and 6 kg on a smooth horizontal table. They are connected by a light, inextensible string. As shown, a horizontal force F N acts on the 4 kg block and a horizontal force of 30 N acts on the 6 kg block.

The magnitude of the acceleration of the system is 2 m s^{-2}.

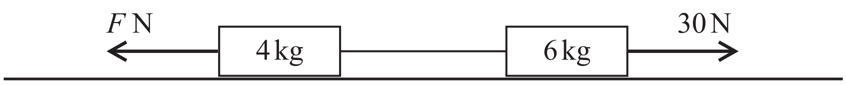


Fig. 5

(i) Find the two possible values of F. **[4]**

(ii) Find the tension in the string in each case. **[3]**

6 A particle moves along a straight line through an origin O. Initially the particle is at O.

At time t s, its displacement from O is x m and its velocity, v m s^{-1}, is given by

$$v = 24 - 18t + 3t^2.$$

(i) Find the times, T_1 s and T_2 s (where $T_2 > T_1$), at which the particle is stationary. **[2]**

(ii) Find an expression for x at time t s.

Show that when $t = T_1$, $x = 20$ and find the value of x when $t = T_2$. **[4]**

Section B (36 marks)

7 Abi and Bob are standing on the ground and are trying to raise a small object of weight 250 N to the top of a building. They are using long light ropes. Fig. 7.1 shows the initial situation.

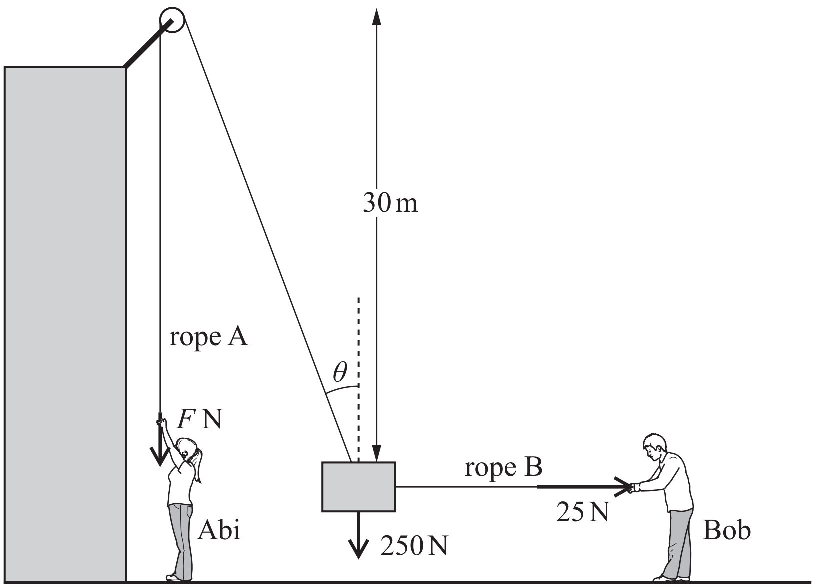


Fig. 7.1

Abi pulls vertically downwards on the rope A with a force F N. This rope passes over a small smooth pulley and is then connected to the object. Bob pulls on another rope, B, in order to keep the object away from the side of the building.

In this situation, the object is stationary and in equilibrium. The tension in rope B, which is horizontal, is 25 N. The pulley is 30 m above the object. The part of rope A between the pulley and the object makes an angle θ with the vertical.

(i) Represent the forces acting on the object as a fully labelled triangle of forces. **[3]**

(ii) Find F and θ.

Show that the distance between the object and the vertical section of rope A is 3 m. **[5]**

[Question 7 is continued on the next page.]

Abi then pulls harder and the object moves upwards. Bob adjusts the tension in rope B so that the object moves along a vertical line.

Fig. 7.2 shows the situation when the object is part of the way up. The tension in rope A is S N and the tension in rope B is T N. The ropes make angles α and β with the vertical as shown in the diagram. Abi and Bob are taking a rest and holding the object stationary and in equilibrium.

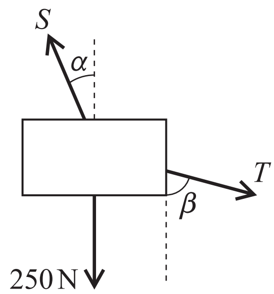


Fig. 7.2

(iii) Give the equations, involving S, T, α and β, for equilibrium in the vertical and horizontal directions. **[3]**

(iv) Find the values of S and T when $\alpha = 8.5°$ and $\beta = 35°$. **[4]**

(v) Abi's mass is 40 kg.

Explain why it is not possible for her to raise the object to a position in which $\alpha = 60°$. **[3]**

[Question 8 is printed overleaf.]

8 Fig. 8.1 shows a sledge of mass 40 kg. It is being pulled across a horizontal surface of deep snow by a light horizontal rope. There is a constant resistance to its motion.

The tension in the rope is 120 N.

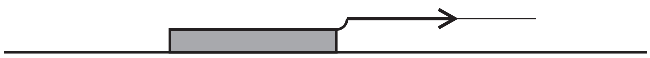


Fig. 8.1

The sledge is initially at rest. After 10 seconds its speed is $5\,\text{m}\,\text{s}^{-1}$.

(i) Show that the resistance to motion is 100 N. **[4]**

When the speed of the sledge is $5\,\text{m}\,\text{s}^{-1}$, the rope breaks.

The resistance to motion remains 100 N.

(ii) Find the speed of the sledge

(A) 1.6 seconds after the rope breaks, **[3]**

(B) 6 seconds after the rope breaks. **[1]**

The sledge is then pushed to the bottom of a ski slope. This is a plane at an angle of 15° to the horizontal.

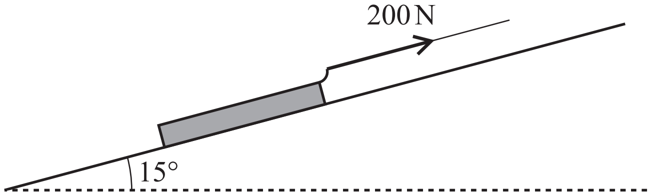


Fig. 8.2

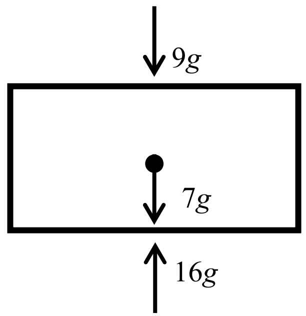
The sledge is attached by a light rope to a winch at the top of the slope. The rope is parallel to the slope and has a constant tension of 200 N. Fig. 8.2 shows the situation when the sledge is part of the way up the slope.

The ski slope is smooth.

(iii) Show that when the sledge has moved from being at rest at the bottom of the slope to the point when its speed is $8\,\text{m}\,\text{s}^{-1}$, it has travelled a distance of 13.0 m (to 3 significant figures). **[4]**

When the speed of the sledge is $8\,\text{m}\,\text{s}^{-1}$, this rope also breaks.

(iv) Find the time between the rope breaking and the sledge reaching the bottom of the slope. **[6]**

Question			Answer	Marks	Guidance
1				B1 B1 B1 **[3]**	One mark for each force with correct magnitude and direction Deduct 1 mark only for *g* missing 16g ↑ 7g ↓ 9g ↓ If all three forces are correct but there is at least one extra force, deduct 1 mark and so give 2 marks. Otherwise ignore extra forces. Note For 16*g* ↑ 16*g* ↓ Award B1 B0 B0
2	(i)		Initial speed is 25 $\mathrm{m\,s^{-1}}$	B1 **[1]**	

Question			Answer	Marks	Guidance
2	(ii)		Vertical motion: $y = 20t - 4.9t^2$	M1	Forming an equation or expression for vertical motion
			When $y = 0$,	M1	Finding t when the height is 0
			$T = (0 \text{ or}) \dfrac{20}{4.9} = 4.08$ s	A1	
			$R = 15 \times 4.08... = 61.22$	F1	Allow $15\times$ their T Note If horizontal and vertical components of the initial velocity are interchanged treat it as a misread; if no other errors are present this gives 3 marks.
				[4]	
			Alternative Using time to maximum height		
			Vertical motion: $v = 20 - 9.8t$	M1	Forming an equation or expression for vertical motion
			Flight time = 2 × Time to top	M1	Using flight time is twice time to maximum height or equivalent for range.
			$T = 2 \times \dfrac{20}{9.8} = 4.08$ s	A1	
			$R = 15 \times 4.08... = 61.22$	F1	Allow $15\times$ their T
			Alternative Using formulae		
			Finding angle of projection $\alpha = \arctan\left(\dfrac{20}{15}\right) = 53.1°$	M1	Only award this mark if there is a clear intention to use this method
			$R = \dfrac{2u^2 \sin\alpha\cos\alpha}{g} = \dfrac{2\times 25^2 \times \sin 53.1° \times \cos 53.1°}{9.8}$	M1	Allow the alternative form $R = \dfrac{u^2 \sin 2\alpha}{g}$ with substitution
			$R = 61.2$	A1	
			$T = \dfrac{2u\sin\alpha}{g} = 4.08$	A1	

Question			Answer	Marks	Guidance
2	(iii)	(*A*)	Flight time $=\dfrac{15}{4.9}$ Range $=20\times\dfrac{15}{4.9}=61.22$	B1 **[1]**	Allow FT from part (ii) for a correct argument that they should be the same
2	(iii)	(*B*)	No eg angle of projection 45°	M1 A1 **[2]**	Attempt at disproof or counter-example. There must be some reference to the angle. Complete argument

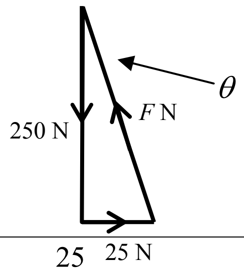
Question			Answer	Marks	Guidance
3	(i)		**p** $\sqrt{(-1)^2+(-1)^2+5^2}=\sqrt{27}$ **q** $\sqrt{(-1)^2+(-4)^2+2^2}=\sqrt{21}$ **r** $\sqrt{2^2+5^2+0^2}=\sqrt{29}$	M1	Use of Pythagoras Note Magnitudes are 5.196, 4.583 and 5.385 respectively
			Greatest magnitude: **r**	A1	
				[2]	
3	(ii)		Weight $=\begin{pmatrix}0\\0\\-4\end{pmatrix}$	B1	Condone $g=9.8$ giving weight is $\begin{pmatrix}0\\0\\-3.92\end{pmatrix}$ N. Accept 4↓.
			$\mathbf{p}+\mathbf{q}+\mathbf{r}+\textbf{weight}=\begin{pmatrix}0\\0\\3\end{pmatrix}$		$g=9.8$ gives $\begin{pmatrix}0\\0\\3.08\end{pmatrix}$
			$0.4\mathbf{a}=\begin{pmatrix}0\\0\\3\end{pmatrix}$	B1	Relevant attempt at Newton's 2nd Law. The total force must be expressed as a vector in some form. For this mark allow the weight to be missing, in the wrong component or to have the wrong sign. Condone *mg* in place of *m* for this mark only.
			Magnitude of acceleration is 7.5 m s^{-2}	B1	CAO apart from using $g=9.8 \Rightarrow a=7.7$
			Direction is vertically upwards	B1	
				[4]	

Question			Answer	Marks	Guidance
4			Equate **i** and **j** components of **v**	M1	The candidate recognises that the **i** and **j** components must be equal.
			$16-t^2=31-8t$	A1	An equation is formed.
			$t^2-8t+15=0$		
			$(t-3)(t-5)=0$		
			$t=3$ or 5	A1	May be implied by later working.
			When $t=3$, $\mathbf{v}=7\mathbf{i}+7\mathbf{j}$	B1	
			Speed when $t=3$ is $7\sqrt{2}=9.9$ m s^{-1}	B1	
			The values of the **i** and **j** components must both be positive for the bearing to be 045°.	B1	This mark is dependent on obtaining A1 for the result $t=3$ or 5. It is awarded if the speed for the case when $t=5$ is not included (since $t=5 \Rightarrow \mathbf{v}=-9\mathbf{i}-9\mathbf{j}$ and the bearing is 225°). Note Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.
				[6]	

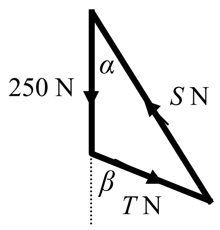
Question			Answer	Marks	Guidance
4			**Alternative Trial and error**		
			The i and j components of v must be equal	M1	The candidate recognises that the **i** and **j** components must be equal.
			The **i** and **j** components of **v** must both be positive for the bearing to be 045°.	B1	This can be demonstrated during the question either by a suitable convincing diagram including 45°, or by a suitable convincing argument
			At least one value of t is substituted	A1	Trial and error is used
			$t=3$	A1	$t=3$ is found by trial and error
			When $t=3$, $\mathbf{v}=7\mathbf{i}+7\mathbf{j}$	B1	
			Speed when $t=3$ is $7\sqrt{2}=9.9$ m s^{-1}	B1	
					Note Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.
				[6]	

Question			Answer	Marks	Guidance
5	(i)		If the acceleration is to the right		
			Overall $30 - F = (4+6) \times 2$	M1	Newton's 2nd Law in one direction. No extra forces allowed and signs must be correct.
			$F = 10$	A1	
			If the acceleration is to the left	M1	For considering second direction. No extra forces allowed and signs must be correct.
			$F = 50$	A1	
				[4]	
5	(ii)		6 kg block $30 - T = 6 \times 2$	M1	Newton's 2nd law with correct elements on either block
			$\Rightarrow T = 18$	A1	CAO No follow through from part (i)
			In the other case $T = 42$	A1	CAO No follow through from part (i)
				[3]	

Question			Answer	Marks	Guidance
6	(i)		$v=0\Rightarrow 3(t-2)(t-4)=0$	M1	Setting $v=0$ (may be implied)
			$T_1=2,\ T_2=4$	A1	Accept $t=2$ and $t=4$
				[2]	
6	(ii)		$x=\int v\,\mathrm{d}t$	M1	Use of integration
			$x=24t-9t^2+t^3+c:\ c=0$	A1	Condone omission of c
			$t=2\Rightarrow x=48-36+8=20$	E1	CAO
			$t=4\Rightarrow x=96-144+64=16$	A1	CAO
				[4]	

Question			Answer	Marks	Guidance
7	(i)		 **Or equivalent**	B1 B1 B1 **[3]**	Shape of triangle; ignore position of θ if marked in diagram 2 marks -1 per error but penalise no arrows only once and penalise no labels only once. Condone T written for F. In the case of a force diagram showing F, 25 and 250 allow maximum of 2 marks with -1 per error but penalise no arrows only once and penalise no labels only once
7	(ii)		$\tan\alpha = \frac{25}{250}$	M1	M1 for recognising and using α in the triangle
			$\Rightarrow \alpha = 5.7°$	A1	
			$F = \sqrt{25^2 + 250^2}$	M1	Use of Pythagoras
			$F = 251.2$	A1	At least 3 significant figures required
			Distance $= 30\tan\alpha = 30 \times 0.1 = 3$ m	B1	CAO
				[5]	
			Alternative $F\cos\theta = 250$ $F\sin\theta = 25$		
			$\tan\theta = \frac{25}{250}$	M1	
			$\Rightarrow \theta = 5.7°$	A1	
			$F\cos 5.7° = 250$	M1	
			$F = 251.2$	A1	At least 3 significant figures required
			Distance $= 30\tan\alpha = 30 \times 0.1 = 3$ m	B1	CAO

Question			Answer	Marks	Guidance
7	(iii)		Vertical equilibrium	M1	M1 for attempt at resolution in an equation involving both S and T; condone sin-cos errors for the M mark only
			$\uparrow\ S\cos\alpha = T\cos\beta + 250\ \downarrow$	A1	
			Horizontal equilibrium $S\sin\alpha = T\sin\beta$	A1	
				[3]	
7	(iv)		$S\sin 8.5° = T\sin 35° \Rightarrow S = 3.8805T$	M1	Using one equation to make S or T the subject in terms of the other
			$(3.8805T)\cos 8.5° = T\cos 35° + 250$	M1	Substituting in the other equation
			$T = 82.8$	A1	CAO
			$S = 321.4$	A1	CAO
				[4]	
			Alternative		**Use of linear simultaneous equations**
			$S\sin 8.5° - T\sin 35° = 0$ $S\cos 8.5° - T\cos 35° = 250$		
			$S\sin 8.5°\cos 35° - T\sin 35°\cos 35° = 0$ $S\cos 8.5°\sin 35° - T\cos 35°\sin 35° = 250\sin 35°$		
			$S(-\sin 8.5°\cos 35° + \cos 8.5°\sin 35°) = 250\sin 35°$	M1	Valid method that has eliminated terms in either S or T (execution need not be perfect)
			$S = 321.4$	A1	CAO First answer
			Substituting in either equation	M1	Substituting to find the second answer
			$\Rightarrow T = 82.8$	A1	CAO Second answer

Question			Answer	Marks	Guidance
7	(iv)		Alternative Triangle of forces		
				M1	Either Drawing and using a triangle of forces Or Quoting and using Lami's Theorem
			$\frac{S}{\sin 145°} = \frac{T}{\sin 8.5°} = \frac{250}{\sin 26.5°}$	M1	Correct form of these equations
			$S = 321.4$	A1	CAO
			$T = 82.8$	A1	CAO

Question			Answer	Marks	Guidance
7	(v)		Abi's weight is $40g = 392$ N	M1	Consideration of Abi's weight
			When $\alpha = 60°$, $S\cos 60° > 250 \Rightarrow S > 500$	M1	Consideration of vertical forces on the object. Condone no mention of Bob's rope
			The tension in rope A would be greater than Abi's weight and so she would be lifted off the ground	A1	The argument must be of high quality and must include consideration of the tension in Bob's rope
				[3]	
			Alternative		
			If Abi is on the ground, the maximum possible tension in rope A is Abi's weight of 392 N	M1	Consideration of Abi's weight
			So the maximum upward force on the object is $392 \times \cos 60° = 192$ N		
			This is less than the weight of the object, and the tension in Bob's rope is pulling the box down.	M1	Consideration of vertical forces on the object. Condone no mention of Bob's rope Or the box accelerated downwards
			So Abi would be lifted off the ground	A1	The argument must be of high quality and must include consideration of the tension in Bob's rope

Question			Answer	Marks	Guidance
8	**(i)**		$v = u + at$	M1	Use of a suitable constant acceleration formula
			$5 = 0 + a \times 10 \Rightarrow a = 0.5$	A1	Notice The value of *a* is not required by the question so may be implied by subsequent working
			$F = ma \Rightarrow 120 - R = 40 \times 0.5$	M1	Use of Newton's 2nd Law with correct elements
			$R = 100\text{ N}$	E1	
				[4]	
8	**(ii)**	(A)	$F = ma \Rightarrow -100 = 40a$	M1	Equation to find *a* using Newton's 2nd Law
			$\Rightarrow a = -2.5$	A1	
			When $t = 1.6$ $v = 5 + (-2.5) \times 1.6 = 1\text{ ms}^{-1}$	A1	CAO
				[3]	
8	**(ii)**	(B)	When $t = 6$, it is stationary. $v = 0\text{ ms}^{-1}$	B1	
				[1]	

Question			Answer	Marks	Guidance
8	(iii)		Motion parallel to the slope:	B1	Component of the weight down the slope, ie $40g\sin 15°$ (= 101.457…)
			$200 - 40g\sin 15° = 40a$	M1	Equation of motion with the correct elements present. No extra forces.
			$a = 2.463...$		This result is not asked for in the question
			$v^2 - u^2 = 2as \Rightarrow 8^2 = 2\times 2.46...\times s$	M1	Use of a suitable constant acceleration formula, or combination of formulae. Dependent on previous M1.
			$\Rightarrow s = 12.989...$ rounding to 13.0 m	E1	Note If the rounding is not shown for s the acceleration must satisfy $2.452... < a < 2.471...$
				[4]	
8	(iv)		Let a be acceleration up the slope		
			$-40\times 9.8\times\sin 15° = 40a$	M1	Use of Newton's 2nd Law parallel to the slope
			$a = -2.536...$, ie 2.536 m s^{-2} down the slope	A1	Condone sign error
			$s = ut + \frac{1}{2}at^2$		
			$-12.989... = 8t + \frac{1}{2}\times(-2.536...)t^2$	M1	Dependent on previous M1. Use of a suitable constant acceleration formula (or combination of formulae) in a relevant manner.
			$1.268...t^2 - 8t - 12.989... = 0$	A1	Signs must be correct
			$t = \frac{8\pm\sqrt{64 - 4\times 1.268...\times(-12.989...)}}{2\times 1.268...}$	M1	Attempt to solve a relevant three-term quadratic equation
			$t = -1.339...$ or $7.647...$, so 7.65 seconds	A1	
				[6]	

Question			Answer	Marks	Guidance
8	(iv)		**Alternative 2-stage motion**		
			Let a be acceleration up the slope		
			$-40\times 9.8\times\sin 15° = 40a$	M1	Use of Newton's 2nd Law parallel to the slope
			$a = -2.536...$, ie 2.536 m s^{-2} down the slope	A1	Condone sign error
			Motion to highest point		
			$v = u + at \;\Rightarrow\; 0 = 8 - 2.536...t$	M1	Dependent on previous M1. Use of a suitable constant acceleration formula, for either t or s, in a relevant manner.
			$t = 3.154...$	A1	For either t or s
			$s = ut + \frac{1}{2}at^2 \Rightarrow s = 8\times 3.154... - \frac{1}{2}\times 2.536...\times 3.154...^2$		
			$s = 12.616...$		
			Distance to bottom = 12.989.. + 12. 616... = 25.605...		
			$s = ut + \frac{1}{2}at^2 \Rightarrow 25.605... = \frac{1}{2}\times 2.536...\times t^2$	M1	Use of a suitable constant acceleration formula
			$t = 4.493...$		
			Total time = 3.154... + 4.493... = 7.647 ... s	A1	