OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

AS GCE MATHEMATICS (MEI)

4761/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4761/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

This paper has been pre modified for carrier language

Section A (36 marks)

1 Fig. 1 shows a block of mass 3 kg on a plane which is inclined at an angle of 30° to the horizontal.

A force P N is applied to the block parallel to the plane in the upwards direction.

The plane is rough so that a frictional force of 10 N opposes the motion.

The block is moving at constant speed up the plane.

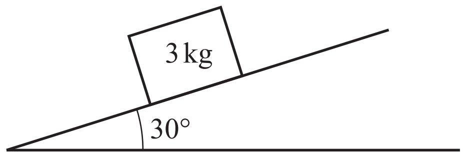


Fig. 1

(i) Mark and label all the forces acting on the block. **[3]**

(ii) Calculate the magnitude of the normal reaction of the plane on the block. **[1]**

(iii) Calculate the magnitude of the force P. **[2]**

2 In this question, the unit vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are in the directions east and north.

Distance is measured in metres and time, t, in seconds.

A radio-controlled toy car moves on a flat horizontal surface. A child is standing at the origin and controlling the car.

When $t = 0$, the displacement of the car from the origin is $\begin{pmatrix}0\\-2\end{pmatrix}$ m, and the car has velocity $\begin{pmatrix}2\\0\end{pmatrix}$ m s^{-1}.

The acceleration of the car is constant and is $\begin{pmatrix}-1\\1\end{pmatrix}$ m s^{-2}.

(i) Find the velocity of the car at time t and its speed when $t = 8$. **[4]**

(ii) Find the distance of the car from the child when $t = 8$. **[4]**

3 Fig. 3 shows two people, Sam and Tom, pushing a car of mass 1000 kg along a straight line l on level ground.

Sam pushes with a constant horizontal force of 300 N at an angle of 30° to the line l.

Tom pushes with a constant horizontal force of 175 N at an angle of 15° to the line l.

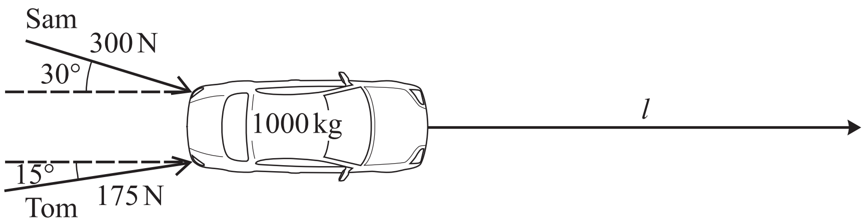


Fig. 3

(i) The car starts at rest and moves with constant acceleration. After 6 seconds it has travelled 7.2 m.

Find its acceleration. **[3]**

(ii) Find the resistance force acting on the car along the line l. **[4]**

(iii) The resultant of the forces exerted by Sam and Tom is not in the direction of the car's acceleration. Explain briefly why. **[1]**

4 A particle is travelling along a straight line with constant acceleration. P, O and Q are points on the line, as illustrated in Fig. 4. The distance from P to O is 5 m and the distance from O to Q is 30 m.

Fig. 4

Initially the particle is at O. After 10 s, it is at Q and its velocity is $9\,\mathrm{m\,s^{-1}}$ in the direction $\overrightarrow{OQ}$.

(i) Find the initial velocity and the acceleration of the particle. **[4]**

(ii) Prove that the particle is never at P. **[3]**

5 Ali is throwing flat stones onto water, hoping that they will bounce, as illustrated in Fig. 5.

Ali throws one stone from a height of 1.225 m above the water with initial speed $20\,\text{m}\,\text{s}^{-1}$ in a horizontal direction. Air resistance should be neglected.

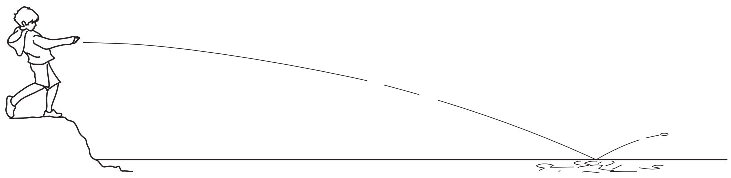


Fig. 5

(i) Find the time it takes for the stone to reach the water. **[2]**

(ii) Find the speed of the stone when it reaches the water and the angle its trajectory makes with the horizontal at this time. **[5]**

Section B (36 marks)

6 The speed of a 100 metre runner in $\mathrm{m\,s^{-1}}$ is measured electronically every 4 seconds.

The measurements are plotted as points on the speed-time graph in Fig. 6. The vertical dotted line is drawn through the runner's finishing time.

Fig. 6 also illustrates Model P in which the points are joined by straight lines.

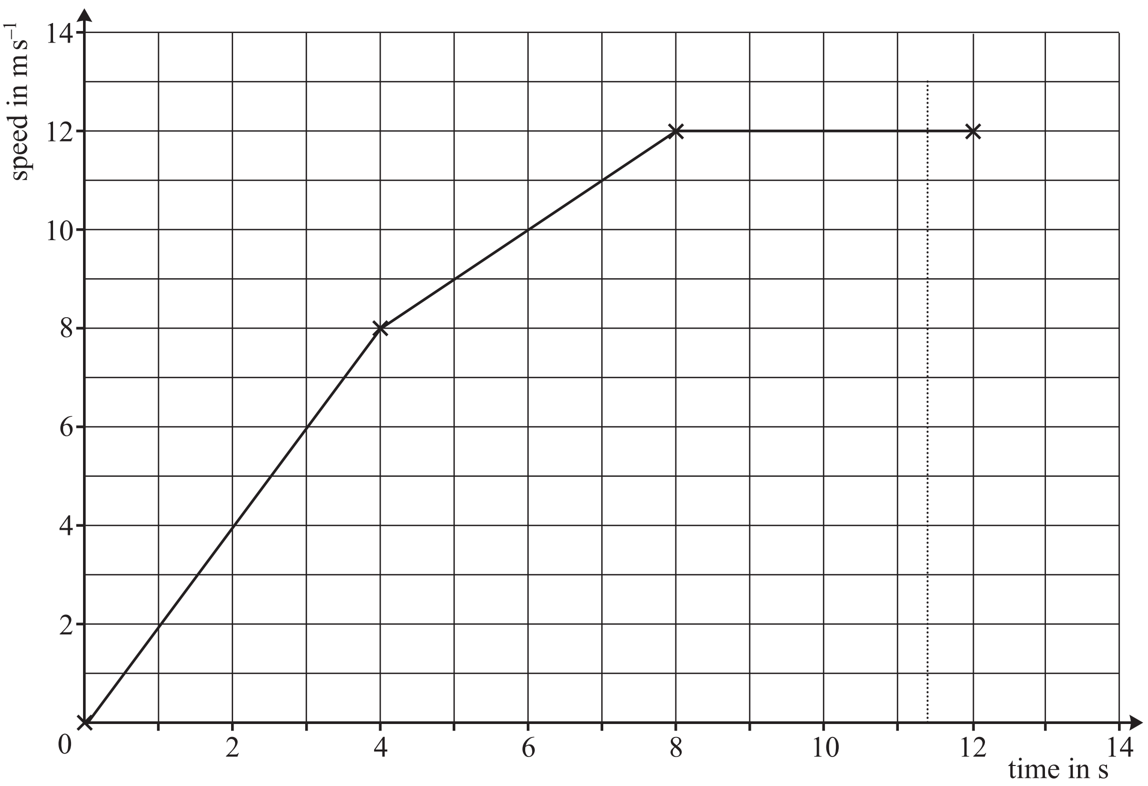


Fig. 6

(i) Use Model P to estimate

(A) the distance the runner has gone at the end of 12 seconds,

(B) how long the runner took to complete 100 m. **[6]**

A mathematician proposes Model Q in which the runner's speed, $v\ \mathrm{m\,s^{-1}}$ at time t s, is given by

$$v = \frac{5}{2}t - \frac{1}{8}t^2.$$

(ii) Verify that Model Q gives the correct speed for $t = 8$. **[1]**

(iii) Use Model Q to estimate the distance the runner has gone at the end of 12 seconds. **[4]**

(iv) The runner was timed at 11.35 seconds for the 100 m.

Which model places the runner closer to the finishing line at this time? **[3]**

(v) Find the greatest acceleration of the runner according to each model. **[4]**

7 A block of weight 50 N is in equilibrium, suspended from fixed points A and B which are 2 m apart on a horizontal ceiling.

Fig. 7.1 illustrates one way of doing this. A light, inextensible string of length 2.8 m is passed round a small smooth light pulley attached to a point C on the block. The parts of the string from C to A and from C to B should be treated as straight lines making angles θ and ϕ with the vertical.

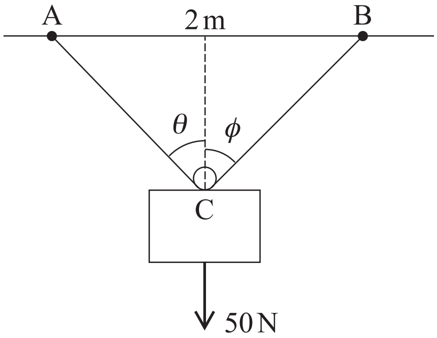


Fig. 7.1

(i) (*A*) State which piece of the information that you have been given tells you that the tension in the string is the same on each side of the pulley. **[1]**

(*B*) Hence show that $\theta = \phi$. **[2]**

(ii) Show that $\cos\theta = \dfrac{\sqrt{24}}{7}$. **[2]**

(iii) Find the tension in the string. **[3]**

Fig. 7.2 illustrates another way of suspending the block from the same two points, A and B, with the string now cut into two parts, AC and BC. The length of AC is 1.2 m and BC is 1.6 m. The angles the strings make with the horizontal are α and β. The tension in the string AC is T_1 N and the tension in the string BC is T_2 N.

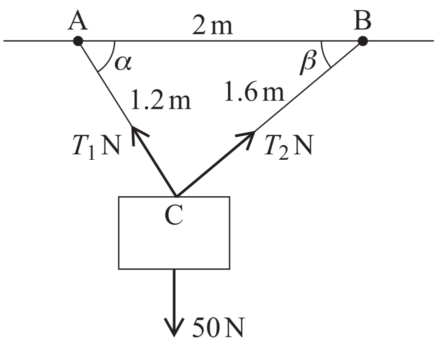


Fig. 7.2

(iv) Show that $\angle ACB = 90°$.

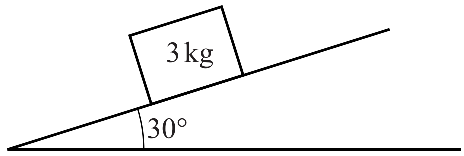
Write down the values of $\cos\alpha$ and $\cos\beta$. **[2]**

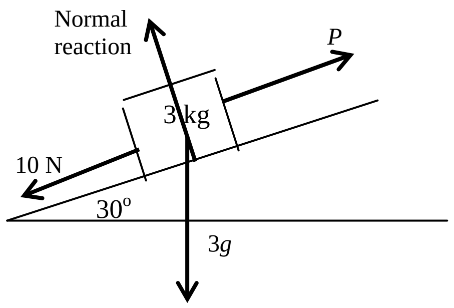
(v) Find T_1 and T_2. **[5]**

In a different arrangement, the string is cut so that the lengths of the two parts are 0.5 m and 2.3 m.

(vi) Describe how the block hangs in equilibrium in this case and state the tensions in the two strings. **[3]**

Section A (36 marks)

1 (i)	
1 (ii)	
1 (iii)	

Question			Answer	Marks	Guidance
1	(i)			B1 B1 B1 **[3]**	3 marks –1 / error or omission Forces must have arrows and labels Accept "weight" and "friction"
1	(ii)		$R = 3g\cos 30° = 25.46... = 25.5$ (to 3 significant figures)	B1 **[1]**	Accept 25 or 26
1	(iii)		$P = 10 + 3g\sin 30°$ $P = 24.7$	M1 A1 **[2]**	Correct elements must be present Cao
2	(i)		$\mathbf{v} = \mathbf{u} + \mathbf{a}t$ Velocity $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ **(** $= \begin{pmatrix} 2-t \\ t \end{pmatrix}$ **)** When $t = 8$, $\mathbf{v} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ speed $\sqrt{(-6)^2 + 8^2}$ =10 m s^{-1}	M1 A1 A1 A1 **[4]**	May be implied by either of the next two answers but not the final answer. Evidence of use of vectors in question necessary. May be implied by the final answer Cao but condone no units Give SC2 for 10 without working

Question			Answer	Marks	Guidance
2	(ii)		$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	Use of correct equation with substitution. Condone omission of $\mathbf{r}_0$. Or equivalent equation
			$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \times 8 + \frac{1}{2} \times \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times 8^2$	A1	Condone omission of $\mathbf{r}_0$. Follow through for their value of $\mathbf{v}$
			$\mathbf{r} = \begin{pmatrix} -16 \\ 30 \end{pmatrix}$	A1	Cao but may be implied by a correct final answer.
			Distance = 34 m	A1	Allow for 35.77... from $\mathbf{r} = \begin{pmatrix} -16 \\ 32 \end{pmatrix}$ and 37.57... from $\mathbf{r} = \begin{pmatrix} -16 \\ 34 \end{pmatrix}$
				[4]	
3	(i)		$s = ut + \frac{1}{2}at^2$	M1	Substitution required
			$7.2 = \frac{1}{2} \times a \times 6^2$	A1	
			$a = 0.4 \text{ ms}^{-2}$	A1	Cao
				[3]	
3	(ii)		$F = ma$	M1	Attempt at Newton's second law
				M1	Attempt at resolving both S and T
			$300\cos 30° + 175\cos 15° - R = 1000 \times 0.4$	A1	(Correct elements present and no extras); follow through for a
			$R = 28.8\text{ N}$	A1	Cao
				[4]	
3	(iii)		The resistance perpendicular to the line of motion has been ignored.	B1	Allow There is also a sideways resistance force
				[1]	

Question			Answer	Marks	Guidance
4	(i)		**Either** $s=\frac{1}{2}(u+v)t$ Take O as the origin.	M1	Use of one relevant equation, including substitution
			$30=\frac{1}{2}\times(u+9)\times 10$		
			$u=-3$	A1	
			$v=u+at$	M1	Use of a second relevant equation including substitution
			$9=-3+10a$		
			$a=1.2$	A1	
			or $v=u+at \Rightarrow u+10a=9$	M1	Use of one relevant equation, including substitution
			$s=ut+\frac{1}{2}at^2 \Rightarrow u+5a=3$	M1	Use of a second relevant equation including substitution
			Solving simultaneously: $a=1.2$	A1	
			$u=-3$	A1	
			or $s=vt-\frac{1}{2}at^2$	M1	Use of one relevant equation, including substitution
			$\Rightarrow a=1.2$	A1	
			$v=u+at$	M1	Use of a second relevant equation including substitution
			$\Rightarrow u=-3$	A1	
				[4]	
4	(ii)		**Either** $s=ut+\frac{1}{2}at^2$		
			Solving for P: $-5=-3t+\frac{1}{2}\times 1.2t^2$	M1	Quadratic equation with $s=-5$
			$0.6t^2-3t+5=0$		
			Discriminant $=3^2-4\times 0.6\times 5=-3$	M1	Considering the discriminant or equivalent
			No real roots for t ($\Rightarrow$ Particle is never at P)	E1	Cao without wrong working in the whole question.

Question			Answer	Marks	Guidance
			Or Find when $v = 0$ $v = u + at$, $v = 0 \Rightarrow t = 2.5$ $s = ut + \frac{1}{2}at^2$ and $t = 2.5$ $\Rightarrow s = -3.75 > -5$	M1 M1 E1	 Or use $v^2 = u^2 + 2as$ Cao without wrong working in the whole question. Comparison necessary
			Special cases when their *u* > 0 and their *a* > 0	SC1 SC1	"It is always going to the right" Demonstration that it is at –5 for two negative times.
				[3]	
5	(i)		Vertical motion: $s = ut + \frac{1}{2}at^2$ At water: $-1.225 = 0 \times t + \frac{1}{2} \times (-9.8) \times t^2$ $\Rightarrow t = 0.5$ s	 M1 A1 **[2]**	 Condone sign errors Signs must be consistent
5	(ii)		Horizontal component of velocity = 20 m s^{-1} Vertical component = $0.5 \times 9.8 = 4.9$ m s^{-1} Speed $= \sqrt{20^2 + 4.9^2} = 20.6$ $\tan\alpha = \frac{4.9}{20}$ $\alpha = 13.8°$	B1 B1 M1 M1 A1 **[5]**	 Follow through for "their *t* x 9.8" Use of Pythagoras on previous two answers Use of an appropriate trig ratio with their figures for **v**. Must be explicit if final answer is incorrect. Cao

Question			Answer	Marks	Guidance
6	**(i)**	(*A*)	Distance travelled = Area under the graph	M1	Attempt to find area
			$\frac{1}{2}\times 4\times 8+\frac{1}{2}\times 4\times(8+12)+4\times 12$	M1	Splitting into suitable parts
			104 m	A1	Cao
					Allow all 3 marks for 104 without any working
6	**(i)**	(*B*)	**Either**		
			Working backwards from distance when $t=12$	M1	
			$12-\frac{(104-100)}{12}$	M1	Allow this mark for 0.33… Follow through from their total distance
			11.67 s	A1	Cao
			Or		
			Working forwards from when $t=8$	M1	
			$8+\frac{(100-56)}{12}$	M1	Allow this mark for 3.67… Follow through from their distance at time 8s
			11.67 s	A1	Cao
				[6]	
6	**(ii)**		Substituting $t=8$ gives $v=\frac{5}{2}\times 8-\frac{1}{8}\times 8^2=12$	B1	
				[1]	

Question			Answer	Marks	Guidance
6	(iii)		Distance $=\int_0^{12}\left(\frac{5t}{2}-\frac{t^2}{8}\right)\mathrm{d}t$	M1	Integrating v. Condone no limits.
			$\left[\frac{5t^2}{4}-\frac{t^3}{24}\right]_0^{12}$	A1	Condone no limits
			$[180-72]\quad(-[0]\ )$	M1	Substituting $t=12$
			108 m	A1	
				[4]	
6	(iv)		Model P: distance at $t=11.35$ is 96.2	B1	Cao
			Model Q: distance at $t=11.35$ is		
			$\left[\frac{5t^2}{4}-\frac{t^3}{24}\right]_0^{11.35}=100.1$	M1	Substituting 11.35 in their expression from part **(iii)**
			Model Q places the runner closer	E1	Cao from correct previous working for both models
				[3]	
6	(v)		Model P: Greatest acceleration $\frac{8}{4}=2$ m s^{-2}	B1	
			Model Q: $a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{5}{2}-\frac{t}{4}$	M1	Differentiating v
				A1	
			Model Q: Greatest acceleration is 2.5 m s^{-2}	B1	Award if correct answer seen
				[4]	

Question			Answer	Marks	Guidance
7	(i)	(*A*)	The pulley is smooth	B1 **[1]**	Award for "smooth" seen.
7	(i)	(*B*)	Horizontal equilibrium: $T\sin\theta = T\sin\phi$ $\Rightarrow \theta = \phi$	M1 E1 **[2]**	Attempt at horizontal equilibrium. Allow sin-cos interchange. The argument must be based on forces. Do not allow if sin-cos interchange
7	(ii)		Call M the mid point of AB. AM = 1, AC=1.4, $\angle\mathbf{AMC} = 90°$ Pythagoras $\Rightarrow$ $\mathbf{MC} = \sqrt{1.4^2 - 1^2} = \sqrt{0.96}$ $\cos\theta = \frac{\sqrt{0.96}}{1.4} = \frac{\sqrt{24}}{7}$	M1 E1 **[2]**	Setting up triangle and use of trigonometry If decimals are matched, at least 3 figures must be given
7	(iii)		Vertical equilibrium $2T\cos\theta = 50$ $T = 35.7$ N	M1 A1 A1 **[3]**	Use of vertical equilibrium Accept $T\cos\theta = 25$ as an equivalent statement Cao
7	(iv)		$1.2^2 + 1.6^2 = 2^2$ $\Rightarrow \angle\text{ACB} = 90°$ $\cos\alpha = 0.6,\ \cos\beta = 0.8$	B1 B1 **[2]**	Use of Pythagoras, or equivalent Both No marks for sin-cos interchange

Question			Answer	Marks	Guidance
7	(v)		**Either resolving horizontally and vertically**		
			$T_1 \cos\alpha = T_2 \cos\beta$	M1	Attempt at horizontal equation. Allow consistent sin-cos interchange
			$T_1 \sin\alpha + T_2 \sin\beta = 50$	M1	Attempt at vertical equation. Allow consistent sin-cos interchange
			$0.6T_1 = 0.8T_2$ $0.8T_1 + 0.6T_2 = 50$	A1	Substitution in both equations. Dependent on both M marks. Cao
			Solving simultaneously	M1	Dependent on both the previous M marks
			$T_1 = 40,\ T_2 = 30$	A1	Cao
			Or resolving in the direction of the strings		
			Resolving in both directions	M1	A serious attempt to use this method. Allow sin-cos interchange
			$T_1 = 50\sin\alpha$	M1	
			$\Rightarrow T_1 = 50 \times 0.8 = 40$	A1	
			$T_2 = 50 \times \sin\beta$	M1	
			$\Rightarrow T_2 = 50 \times 0.6 = 30$	A1	
			Or triangle of forces		
			Use of a triangle of forces	M1	The triangle must be closed and have a right angle opposite the weight
			Labels	M1	The sides must be correctly annotated
			Angles	M1	The angles must be correctly annotated
			$T_1 = 50 \times 0.8 = 40$	A1	Cao Dependent of first M mark
			$T_2 = 50 \times 0.6 = 30$	A1	Cao Dependent of first M mark
				[5]	

Question			Answer	Marks	Guidance
7	**(vi)**		Attempt to find $\angle CAB$	M1	May be implied by the remaining answers
			Tension in AC is 50 N (it takes all the weight)	B1	
			Tension in BC is zero (it is slack)	B1	
				[3]	