

# Wednesday 25 January 2012 – Afternoon

## AS GCE MATHEMATICS (MEI)

4761 Mechanics 1

### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4761
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

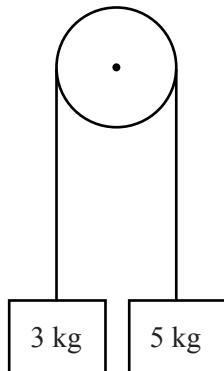
### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**Section A (36 marks)**

- 1** Fig. 1 shows two blocks of masses 3 kg and 5 kg connected by a light string which passes over a smooth, fixed pulley.

Initially the blocks are held at rest but then they are released.



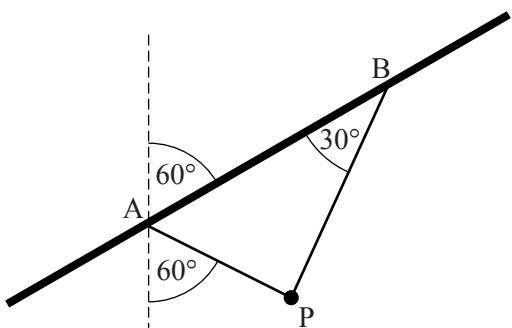
**Fig. 1**

Find the acceleration of the blocks when they start to move, and the tension in the string.

[5]

- 2** Fig. 2 shows a small object, P, of weight 20 N, suspended by two light strings. The strings are tied to points A and B on a sloping ceiling which is at an angle of  $60^\circ$  to the upward vertical. The string AP is at  $60^\circ$  to the downward vertical and the string BP makes an angle of  $30^\circ$  with the ceiling.

The object is in equilibrium.



**Fig. 2**

- (i)** Show that  $\angle APB = 90^\circ$ .

[1]

- (ii)** Draw a labelled triangle of forces to represent the three forces acting on P.

[3]

- (iii)** Hence, or otherwise, find the tensions in the two strings.

[3]

- 3 Two girls, Marie and Nina, are members of an Olympic hockey team. They are doing fitness training.

Marie runs along a straight line at a constant speed of  $6 \text{ ms}^{-1}$ .

Nina is stationary at a point O on the line until Marie passes her. Nina immediately runs after Marie until she catches up with her.

The time,  $t$  s, is measured from the moment when Nina starts running. So when  $t = 0$ , both girls are at O.

Nina's acceleration,  $a \text{ ms}^{-2}$ , is given by

$$\begin{aligned} a &= 4 - t && \text{for } 0 \leq t \leq 4, \\ a &= 0 && \text{for } t > 4. \end{aligned}$$

- (i) Show that Nina's speed,  $v \text{ m s}^{-1}$ , is given by

$$\begin{aligned} v &= 4t - \frac{1}{2}t^2 && \text{for } 0 \leq t \leq 4, \\ v &= 8 && \text{for } t > 4. \end{aligned}$$

[3]

- (ii) Find an expression for the distance Nina has run at time  $t$ , for  $0 \leq t \leq 4$ .

Find how far Nina has run when  $t = 4$  and when  $t = 5\frac{1}{3}$ .

[4]

- (iii) Show that Nina catches up with Marie when  $t = 5\frac{1}{3}$ .

[1]

- 4 A projectile P travels in a vertical plane over level ground. Its position vector  $\mathbf{r}$  at time  $t$  seconds after projection is modelled by

$$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} t - \begin{pmatrix} 0 \\ 5 \end{pmatrix} t^2,$$

where distances are in metres and the origin is a point on the level ground.

- (i) Write down

(A) the height from which P is projected,

(B) the value of  $g$  in this model.

[2]

- (ii) Find the displacement of P from  $t = 3$  to  $t = 5$ .

[2]

- (iii) Show that the equation of the trajectory is

$$y = 5 + \frac{4}{3}x - \frac{x^2}{180}.$$

[4]

- 5 The vectors  $\mathbf{p}$  and  $\mathbf{q}$  are given by

$$\mathbf{p} = 8\mathbf{i} + \mathbf{j} \text{ and } \mathbf{q} = 4\mathbf{i} - 7\mathbf{j}.$$

(i) Show that  $\mathbf{p}$  and  $\mathbf{q}$  are equal in magnitude. [3]

(ii) Show that  $\mathbf{p} + \mathbf{q}$  is parallel to  $2\mathbf{i} - \mathbf{j}$ . [2]

(iii) Draw  $\mathbf{p} + \mathbf{q}$  and  $\mathbf{p} - \mathbf{q}$  on the grid.

Write down the angle between these two vectors. [3]

### Section B (36 marks)

- 6 Robin is driving a car of mass 800 kg along a straight horizontal road at a speed of  $40\text{ ms}^{-1}$ .

Robin applies the brakes and the car decelerates uniformly; it comes to rest after travelling a distance of 125 m.

(i) Show that the resistance force on the car when the brakes are applied is 5120 N. [4]

(ii) Find the time the car takes to come to rest. [2]

For the rest of this question, assume that when Robin applies the brakes there is a constant resistance force of 5120 N on the car.

The car returns to its speed of  $40\text{ ms}^{-1}$  and the road remains straight and horizontal.

Robin sees a red light 155 m ahead, takes a short time to react and then applies the brakes.

The car comes to rest before it reaches the red light.

(iii) Show that Robin's reaction time is less than 0.75 s. [2]

The 'stopping distance' is the total distance travelled while a driver reacts and then applies the brakes to bring the car to rest. For the rest of this question, assume that Robin is still driving the car described above and has a reaction time of 0.675 s. (This is the figure used in calculating the stopping distances given in the Highway Code.)

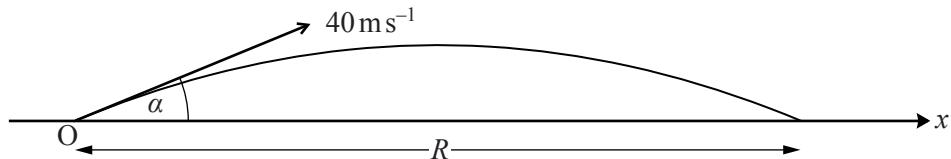
(iv) Calculate the stopping distance when Robin is driving at  $20\text{ ms}^{-1}$  on a horizontal road. [3]

The car then travels down a hill which has a slope of  $5^\circ$  to the horizontal.

(v) Find the stopping distance when Robin is driving at  $20\text{ ms}^{-1}$  down this hill. [6]

(vi) By what percentage is the stopping distance increased by the fact that the car is going down the hill?  
Give your answer to the nearest 1%. [1]

- 7 Fig. 7 shows the trajectory of an object which is projected from a point O on horizontal ground. Its initial velocity is  $40\text{ ms}^{-1}$  at an angle of  $\alpha$  to the horizontal.



**Fig. 7**

- (i) Show that, according to the standard projectile model in which air resistance is neglected, the flight time,  $T$  s, and the range,  $R$  m, are given by

$$T = \frac{80 \sin \alpha}{g} \text{ and } R = \frac{3200 \sin \alpha \cos \alpha}{g}. \quad [6]$$

A company is designing a new type of ball and wants to model its flight.

- (ii) Initially the company uses the standard projectile model.

Use this model to show that when  $\alpha = 30^\circ$  and the initial speed is  $40\text{ ms}^{-1}$ ,  $T$  is approximately 4.08 and  $R$  is approximately 141.4.

Find the values of  $T$  and  $R$  when  $\alpha = 45^\circ$ . [3]

The company tests the ball using a machine that projects it from ground level across horizontal ground. The speed of projection is set at  $40\text{ ms}^{-1}$ .

When the angle of projection is set at  $30^\circ$ , the range is found to be 125 m.

- (iii) Comment briefly on the accuracy of the standard projectile model in this situation. [1]

The company refines the model by assuming that the ball has a constant deceleration of  $2\text{ ms}^{-2}$  in the horizontal direction.

In this new model, the resistance to the vertical motion is still neglected and so the flight time is still 4.08 s when the angle of projection is  $30^\circ$ .

- (iv) Using the new model, with  $\alpha = 30^\circ$ , show that the horizontal displacement from the point of projection,  $x$  m at time  $t$  s, is given by

$$x = 40t \cos 30^\circ - t^2.$$

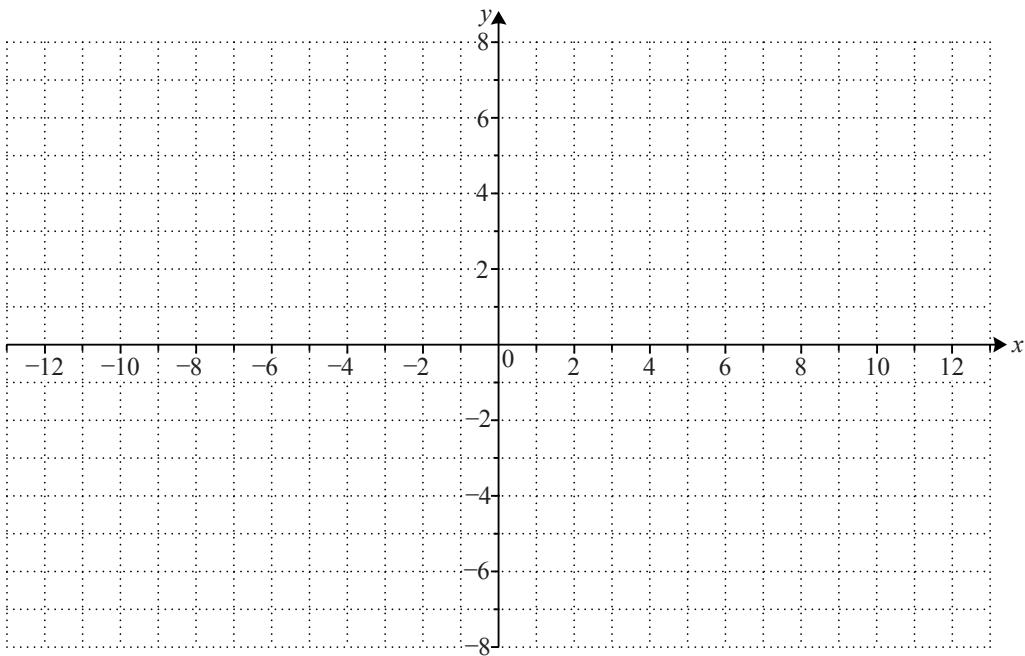
Find the range and hence show that this new model is reasonably accurate in this case. [4]

The company then sets the angle of projection to  $45^\circ$  while retaining a projection speed of  $40\text{ ms}^{-1}$ . With this setting the range of the ball is found to be 135 m.

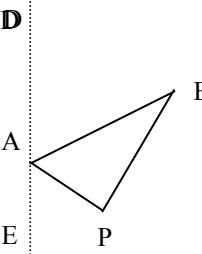
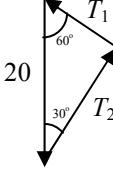
- (v) Investigate whether the new model is also accurate for this angle of projection. [3]

- (vi) Make one suggestion as to how the model could be further refined. [1]

5 (iii)



Question		Answer	Marks	Guidance	
1		<p>Let the tension in the string be <math>T</math> N and the acceleration be <math>a</math> m s<sup>-2</sup></p> <p>3 kg block: <math>T - 3g = 3a</math></p> <p>5kg block: <math>5g - T = 5a</math></p> $2g = 8a$ $a = g/4 = 2.45 \text{ (m s}^{-2}\text{)}$ $T = 36.75 \text{ (N)}$	M1 M1 M1 A1 A1 [5]	<p>One correct equation which must involve <math>a</math> and a weight</p> <p>A second correct equation; this one must involve <math>a</math>, <math>T</math> and a weight</p> <p>Elimination of both <math>T</math> and <math>a</math> from their equations or substitution of their <math>a</math> in their 3-term equation</p> <p>Cao dependent on at least one of the first two M marks</p> <p>Cao dependent on at least one of the first two M marks</p>	

2	(i)	 <p><math>\angle BAD = 60^\circ</math> and <math>\angle PAE = 60^\circ \Rightarrow \angle PAB = 60^\circ</math>          (Angles on straight line)</p> <p><math>\angle PAB = 60^\circ</math> and <math>\angle ABP = 30^\circ \Rightarrow \angle APB = 90^\circ</math>          (Angles in triangle)</p>	B1 [1]	Any valid argument Allow an “argument” containing only numbers and no words	
2	(ii)		B1 B1 B1	<p><b>Diagram</b>          A triangle with angles of approximately <math>90^\circ</math>, <math>60^\circ</math> and <math>30^\circ</math>. No mark for an isosceles or equilateral triangle. No extra forces.          Do not award this mark to candidates who draw a force diagram, ie showing 3 concurrent forces.</p> <p><b>Arrows</b>          Triangle: the sides are marked with arrows following a cycle round the triangle          Force diagram: the directions of the three forces are indicated by arrows. No extra forces.</p> <p><b>Labels</b>          Triangle: all sides are labelled consistently and unambiguously with the angles, ie 20 opposite <math>90^\circ</math>, <math>T_{AP}</math> opposite <math>30^\circ</math> and <math>T_{BP}</math> opposite <math>60^\circ</math>.          Force diagram: all forces labelled          Condone 20g or <math>W</math></p>	

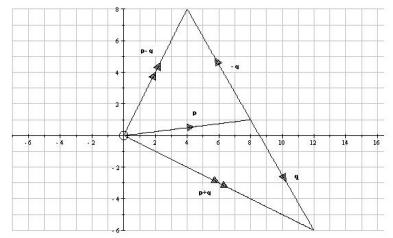
		$T_1$ = Tension in AP. $T_2$ = Tension in BP	[3]	
(iii)		$T_2 = 20 \cos 30^\circ$ Tension in string BP is 17.3 N Tension in string AP is 10 N	M1 A1 A1 [3]	An attempt to apply trigonometry to a triangle of forces to find the tension in either string No mark if derived from “weight = 20g” (giving 169.7) FT for “weight = 20g” (giving 98)

2.	(iii)	<b>Alternative</b> Horizontal equilibrium: $T_2 \cos 60^\circ = T_1 \cos 30^\circ$  Vertical equilibrium: $T_2 \sin 60^\circ + T_1 \sin 30^\circ = 20$  Tension in BP is 17.3 N Tension in AP is 10 N	(M1)	Award up to all 3 marks for using horizontal and vertical equilibrium  An attempt at both horizontal and vertical equilibrium equations
			(A1) (A1) ([3])	No mark if derived from “weight = 20g” (giving 169.7) FT from “weight = 20g” (giving 98)

2.	(iii)	<b>Alternative</b> $T_1 = 20 \cos 60^\circ, T_2 = 20 \cos 30^\circ$  Tension in BP is 17.3 N Tension in AP is 10 N	(M1)	Award up to all 3 marks for using equilibrium in directions PA and PB  An attempt to resolve the weight in the directions PA and PB. This method may be implied by subsequent work
			(A1) (A1) ([3])	No mark if derived from “weight = 20g” (giving 169.7) FT from “weight = 20g” (giving 98)

3	(i)	$v = \int (4-t)dt$ $v = 4t - \frac{1}{2}t^2 + c \quad (t=0, v=0 \Rightarrow c=0)$ $v = 4t - \frac{1}{2}t^2 \text{ for } 0 \leq t \leq 4$ When $t=4, v=8$ and for $t > 4, a=0$ so $v=8$ for $t > 4$	M1	Attempt to integrate	
			A1	Condone no mention of arbitrary constant	
			B1	$a=0$ must be seen or implied	
			[3]		
	(ii)	$s = \int (4t - \frac{1}{2}t^2)dt$ $s = 2t^2 - \frac{1}{6}t^3$ When $t=4$ , Nina has travelled $2 \times 4^2 - \frac{1}{6} \times 4^3 = 21\frac{1}{3}$ m When $t=5\frac{1}{3}$ , Nina has travelled $21\frac{1}{3} + 8 \times 1\frac{1}{3} = 32$ m	M1		
			A1	Again condone no mention of arbitrary constant	
			A1		
			F1	Allow follow through from their $21\frac{1}{3}$ Exact answer required; if rounded to 32, award 0	
			[4]		
	(iii)	When $t=5\frac{1}{3}$ , Marie has run $6 \times 5\frac{1}{3} = 32$ m. Nina has also run 32 m so caught up Marie	B1	Allow an equivalent argument that when Marie has run 32 m, $t=5\frac{1}{3}$ , as for Nina This mark is dependent on an answer 32 in part (ii) but allow this where it is a rounded answer and in this particular case the rounding can be in part (iii)	
			[1]		

<b>4</b>	(i)	(A) (B)	Height 5 m  $g$ has been taken to be $10 \text{ m s}^{-2}$	B1 B1 [2]	No units required; apply ISW if incorrect units given  Allow +10 or -10. No units required; apply ISW if incorrect units given	
<b>4</b>	(ii)		Displacement is $\begin{pmatrix} 150 \\ 80 \end{pmatrix} - \begin{pmatrix} 90 \\ 80 \end{pmatrix}$  $= \begin{pmatrix} 60 \\ 0 \end{pmatrix}$	M1 A1 [2]	Displacement must be given as a vector. Allow a description of a vector in words. Attempts at substitution for $t$ and subtraction of vectors must be seen  Cao If the candidate then goes on to give a non-vector answer of "60 m", apply ISW.	
<b>4</b>	(iii)		$x = 30t$ $y = 5 + 40t - 5t^2$  $y = 5 + 40 \times \left(\frac{x}{30}\right) - 5 \times \left(\frac{x}{30}\right)^2$  $y = 5 + \frac{4}{3}x - \frac{x^2}{180}$	B1 B1 M1 A1 [4]	Attempt to eliminate $t$  No errors	

<b>5</b>	(i)	$ p  = \sqrt{8^2 + 1^2}$ $ p  = \sqrt{65}$ $ q  = \sqrt{4^2 + (-7)^2} = \sqrt{65}$ They are equal	M1  A1  A1 [3]	For applying Pythagoras theorem  Condone no explicit statement that they are equal
<b>5</b>	(ii)	$p + q = 12\mathbf{i} - 6\mathbf{j}$ $p + q = 6(2\mathbf{i} - \mathbf{j})$ so $p + q$ is parallel to $2\mathbf{i} - \mathbf{j}$	M1  E1 [2]	Accept argument based on gradients being equal. “Parallel” may be implied
<b>5</b>	(iii)	 The angle is $90^\circ$	B1  B1  B1 [3]	One mark for each of $p + q$ and $p - q$ drawn correctly SC1 if arrows missing or incorrect from otherwise correct vectors  Cao

<b>6</b>	(i)	$v^2 - u^2 = 2as$ $0^2 - 40^2 = 2 \times a \times 125$ $\Rightarrow a = -6.4$ $F = ma$ $F = 800 \times (-)6.4 = (-)5120$	M1  A1  M1  E1  [4]	Substitution required. For $u$ $v$ interchange award up to M1 A0  Condone no – sign  Allow +5120 or -5120
<b>6</b>	(ii)	$v = u + at$ $0 = 40 - 6.4 \times t$ $t = 6.25$ It takes 6.25 seconds to stop	M1  A1  [2]	FT for $a$
		<b>Alternative</b> $s = \frac{1}{2}(u + v)t$ $125 = \frac{1}{2}(40 + 0) \times t$ $t = 6.25$ it takes 6.25 seconds to stop	(M1)  (A1)  [2]	
		<b>Alternative</b> $s = ut + \frac{1}{2}at^2$ $125 = 40t + \frac{1}{2} \times (-6.4)t^2$ $3.2t^2 - 40t + 125 = 0$ $t = 6.25$	(M1)  (A1)  ([2])	

<b>6</b>	(iii)	Reaction distance $< 155 - 125 = 30 \text{ m}$ Time taken to travel 30 m at $40 \text{ m s}^{-1}$ is $0.75 \text{ s}$	M1 E1 [2]	30 must be seen and used	
<b>6</b>	(iv)	Distance travelled before braking $= 20 \times 0.675 = 13.5 \text{ m}$ Distance travelled while braking $= \frac{20^2}{2 \times 6.4} = 31.25$ Stopping distance $= 13.5 + 31.25 = 44.75 \text{ m}$	B1 B1 B1 [3]	Cao	

6	(v)	<p>The distance travelled during the reaction time is not affected by the slope. It is <math>20 \times 0.675 = 13.5 \text{ m}</math></p> <p>Component of the car's weight down the slope</p> $= mg \sin \alpha = 800 \times 9.8 \times \sin 5^\circ (= 683.3 \text{ N})$ <p>Force opposing motion when the brakes are applied <math>= 5120 - 683.3 = 4436.9</math></p> <p>Acceleration <math>= (-) \frac{4436.7}{800} = (-)5.546 \text{ ms}^{-2}</math></p> <p>Distance travelled while braking</p> $= -\frac{u^2}{2a} = -\frac{400}{2 \times (-)5.546} = 36.06 \text{ m}$ <p>Stopping distance <math>= 13.5 + 36.06 = 49.56 \text{ m}</math></p>		<p>M1 13.5 is rewarded later</p> <p>A1 Allow cos for sin for M1 Allow omission of <math>g</math> for this mark only</p> <p>Cao</p> <p>M1 The resistance (5120) and their weight component (683.3) must have opposite signs.</p> <p>A1</p> <p>A1</p> <p>F1 Allow FT for 36.06 from previous answer. Allow FT of 13.5 from part (iv)</p> <p>[6]</p>	
6	(vi)	<p>Increase in stopping distance on account of slope</p> $= 49.56 - 44.75 = 4.81 \text{ m}$ <p>Percentage increase <math>= \frac{4.81}{44.75} \times 100 = 11\%</math></p>	B1	<p>Cao This mark is dependent on a correct final answer to part (v)</p> <p>[1]</p>	

7	(i)	Vertical motion: initial speed $40 \sin \alpha$ $h = (40 \sin \alpha)t - \frac{1}{2}gt^2$ $h = 0 \Rightarrow t = 0 \text{ or } \frac{2 \times 40 \times \sin \alpha}{g}$ $\Rightarrow T = \frac{80 \sin \alpha}{g}$	B1	
			M1	Correct expression for $h$ must be seen. Condone omission of the case $t = 0$
		<b>Alternative</b> Vertical motion: initial speed $40 \sin \alpha$ $v = 40 \sin \alpha - gt$ When $v = 0$ , $t = \frac{T}{2}$ $\Rightarrow T = \frac{80 \sin \alpha}{g}$	(B1)	
			(M1)	Correct expression for $v$ must be seen
			(E1)	Perfect argument
			[6]	
		Horizontal motion: initial speed $40 \cos \alpha$ $R = 40 \cos \alpha \times T$ $\Rightarrow R = \frac{3200 \sin \alpha \cos \alpha}{g}$	B1	
			M1	There must be evidence of intention to use $T$
			E1	Perfect argument
			[6]	
7	(ii)	$\alpha = 30^\circ$ : $T = \frac{80 \sin 30^\circ}{9.8} \approx 4.08$ $\Rightarrow R = \frac{3200 \times \sin 30^\circ \times \cos 30^\circ}{9.8} = 141.4$ $\alpha = 45^\circ$ : $T = 5.77$	B1	Both answers required for the mark. Evidence of substitution required
			B1	

		$\alpha = 45^\circ; R = 163.3$	B1 [3]	Accept 3 significant figures	
7	(iii)	The standard model is not accurate; 125 is much less than 141.4	B1 [1]	The comment must be based on the figures given in the question	
7	(iv)	Horizontal motion: $s = ut + \frac{1}{2}at^2$ $x = 40 \cos 30^\circ \times t - \frac{1}{2} \times 2 \times t^2$ $x = 40t \cos 30^\circ - t^2$ Flight time = 4.08 s $R = 40 \times \cos 30^\circ \times 4.08 - \frac{1}{2} \times 2 \times 4.08^2$ $R = 124.7$ This is close to the experimental result of 125 m	M1  A1  M1  E1 [4]	Use of correct formula  A comparison with 125 m is required	

7	(v)	<p>When <math>\alpha = 45^\circ</math>, <math>T = 5.77</math></p> $R = 40 \times \cos 45^\circ \times 5.77 - \frac{1}{2} \times 2 \times 5.77^2$ $R = 129.9$ <p>129.9 m is not very close to 135 m so the model is not very accurate for this angle.</p>	M1  A1  B1  [3]	<p>Use of correct formula, with substitution for <math>\alpha</math> and <math>T</math>. FT their <math>T</math> from (ii) but not 4.08.</p> <p>SC1 for substituting for <math>T</math> but using <math>30^\circ</math> for <math>\alpha</math></p> <p>Comparison of their 129.9 with 135</p> <p>If 4.08 used for <math>T</math> and answer 98.8 obtained for <math>R</math> allow FT for this mark</p> <p>Allow argument that to get to 135m takes 6.07 s which is greater than 5.77 s</p>	
7	(vi)	Allow for resistance in the vertical direction as well	B1  [1]	Any sensible comment, but do not award a mark for "Allow for air resistance" without mention of the vertical direction.	