OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4761

Mechanics 1

QUESTION PAPER

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

• The question paper will be found in the centre of the printed answer book.
• Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
• **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
• Use black ink. Pencil may be used for graphs and diagrams only.
• Read each question carefully. Make sure you know what you have to do before starting your answer.
• Answer **all** the questions.
• Do **not** write in the bar codes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Final answers should be given to a degree of accuracy appropriate to the context.
• The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

• The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
• You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
• The total number of marks for this paper is **72**.
• The printed answer book consists of **16** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

OCR is an exempt Charity

Turn over

Section A (36 marks)

1   A pellet is fired vertically upwards at a speed of $11\,\mathrm{m\,s^{-1}}$. Assuming that air resistance may be neglected, calculate the speed at which the pellet hits a ceiling 2.4 m above its point of projection.

   [3]

2   A particle travels with constant acceleration along a straight line. A and B are points on this line 8 m apart.

   The motion of the particle is as follows.

   - Initially it is at A.

   - After 32 s it is at B.

   - When it is at B its speed is $2.25\,\mathrm{m\,s^{-1}}$ and it is moving away from A.

   In either order, calculate the acceleration and the initial velocity of the particle, making the directions clear.   **[5]**

3   Force **F** is $\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$ N, force **G** is $\begin{pmatrix} -6 \\ y \\ z \end{pmatrix}$ N and force **H** is $\begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$ N.

   (i) Given that **F** and **G** act in parallel lines, find y and z.   **[2]**

   Forces **F** and **H** are the only forces acting on an object of mass 5 kg.

   (ii) Calculate the acceleration of the object. Calculate also the magnitude of this acceleration.   **[5]**

4   Fig. 4 shows a block of mass 15 kg on a smooth plane inclined at $20°$ to the horizontal. The block is held in equilibrium by a horizontal force of magnitude P N.
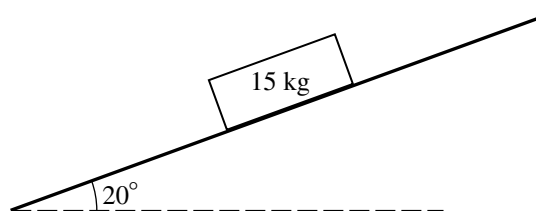

Fig. 4

   (i) Show all the forces acting on the block.   **[2]**

   (ii) Calculate P.   **[3]**

5   A small object is projected over horizontal ground from a point O at ground level and makes a loud noise on landing. It has an initial speed of $30\,\mathrm{m\,s^{-1}}$ at $35°$ to the horizontal.

   Assuming that air resistance on the object may be neglected and that the speed of sound in air is $343\,\mathrm{m\,s^{-1}}$, calculate how long after projection the noise is heard at O.   **[8]**

6   In this question, **i** and **j** are unit vectors east and north respectively. Position vectors are with respect to an origin O. Time t is in seconds.

A skater has a constant acceleration of $-2\mathbf{j}\,\mathrm{m\,s}^{-2}$. At $t = 0$, his velocity is $4\mathbf{i}\,\mathrm{m\,s}^{-1}$ and his position vector is $3\mathbf{j}\,\mathrm{m}$.

  (i) Find expressions in terms of t for the velocity and the position vector of the skater at time t.   **[5]**

 (ii) Calculate as a bearing the direction of motion of the skater when $t = 2.5$.   **[3]**

Section B (36 marks)

7   A ring is moving on a straight wire. Its velocity is v m s^{-1} at time t seconds after passing a point Q.

Model A for the motion of the ring gives the velocity-time graph for $0 \leqslant t \leqslant 6$ shown in Fig. 7.
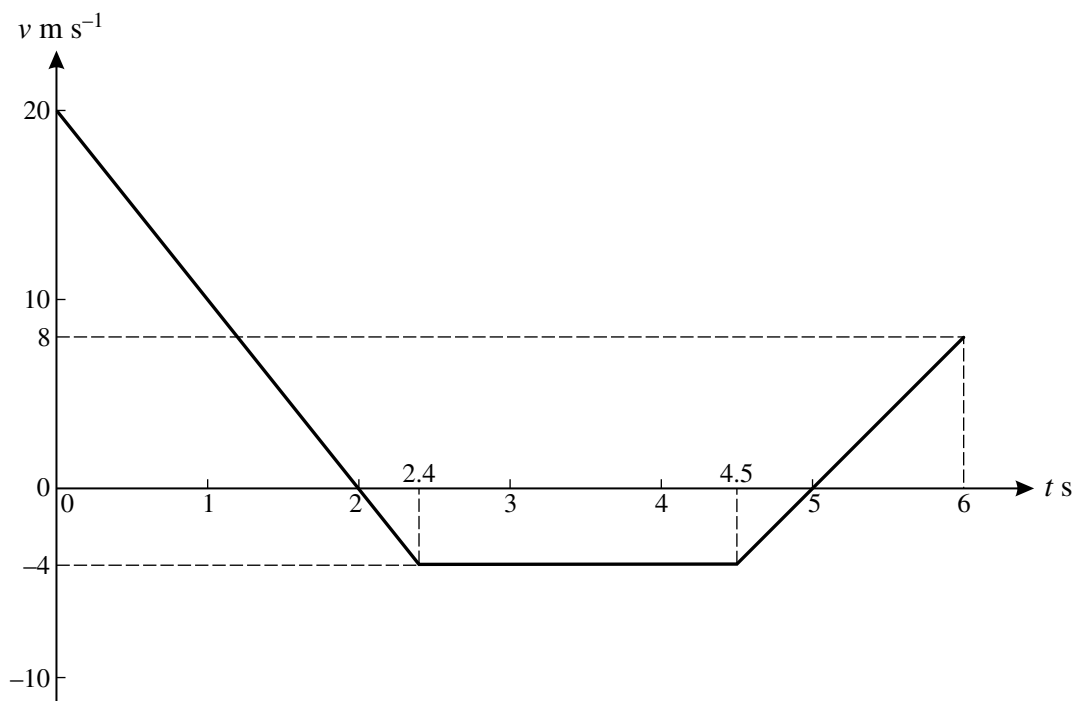

Fig. 7

Use model A to calculate the following.

   (i) The acceleration of the ring when $t = 0.5$. **[2]**

  (ii) The displacement of the ring from Q when

     (*A*) $t = 2$,

     (*B*) $t = 6$. **[5]**

In an alternative model B, the velocity of the ring is given by $v = 2t^2 - 14t + 20$ for $0 \leqslant t \leqslant 6$.

 (iii) Calculate the acceleration of the ring at $t = 0.5$ as given by model B. **[3]**

  (iv) Calculate by how much the models differ in their values for the least v in the time interval $0 \leqslant t \leqslant 6$. **[4]**

   (v) Calculate the displacement of the ring from Q when $t = 6$ as given by model B. **[4]**

8 A trolley C of mass 8 kg with rusty axle bearings is initially at rest on a horizontal floor.

The trolley stays at rest when it is pulled by a horizontal string with tension 25 N, as shown in Fig. 8.1.
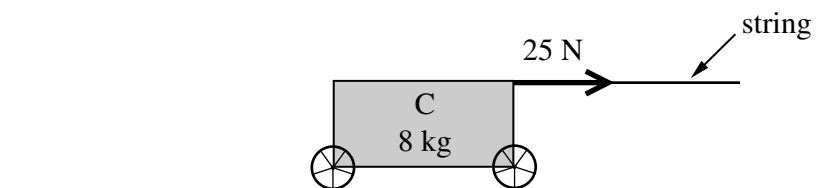

Fig. 8.1

  (i) State the magnitude of the horizontal resistance opposing the pull. **[1]**

A second trolley D of mass 10 kg is connected to trolley C by means of a light, horizontal rod.

The string now has tension 50 N, and is at an angle of 25° to the horizontal, as shown in Fig. 8.2. The two trolleys stay at rest.
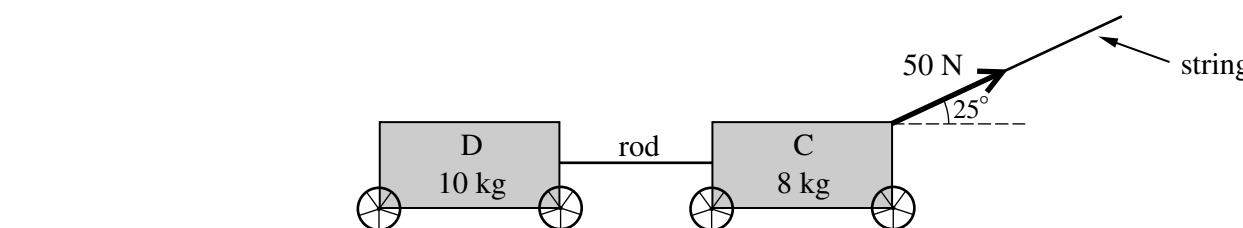

Fig. 8.2

 (ii) Calculate the magnitude of the total horizontal resistance acting on the two trolleys opposing the pull. **[2]**

(iii) Calculate the normal reaction of the floor on trolley C. **[3]**

The axle bearings of the trolleys are oiled and the total horizontal resistance to the motion of the two trolleys is now 20 N. The two trolleys are still pulled by the string with tension 50 N, as shown in Fig. 8.2.

(iv) Calculate the acceleration of the trolleys. **[3]**

In a new situation, the trolleys are on a slope at 5° to the horizontal and are initially travelling down the slope at $3\,\text{m s}^{-1}$. The resistances are 15 N to the motion of D and 5 N to the motion of C. There is no string attached. The rod connecting the trolleys is parallel to the slope. This situation is shown in Fig. 8.3.

Fig. 8.3
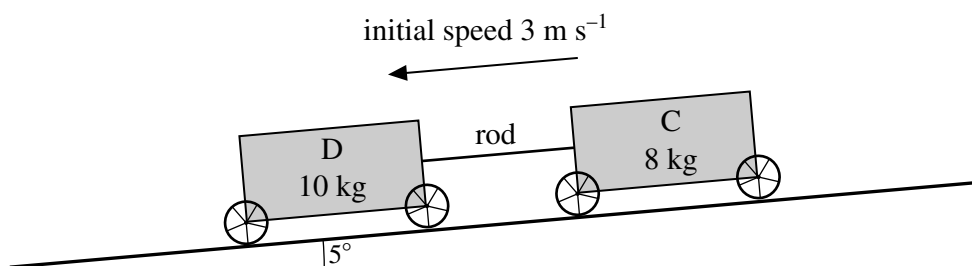
  (v) Calculate the speed of the trolleys after 2 seconds and also the force in the rod connecting the trolleys, stating whether this rod is in tension or thrust (compression). **[9]**
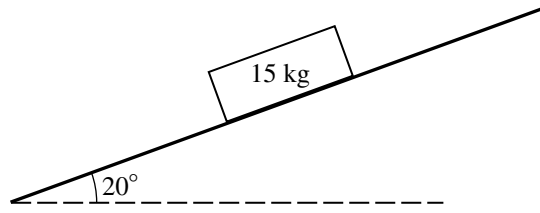
4 (i)

Fig. 4

4 (ii)

Turn over

comment
You should expect to follow through from one part to another unless the scheme says otherwise but not follow through within a part unless the scheme specifies this.
Each script must be viewed as a whole at some stage so that
(i) a candidate's writing of letters, digits, symbols on diagrams etc can be better interpreted;
(ii) repeated mistakes can be recognised (e.g. calculator in wrong angle mode throughout – penalty 1 in the script and FT except given answers).

You are advised to 'set height' in *scoris*, particularly for question 7(ii). Questions 5 and 8(v) also spread onto two pages.

Q 1		m a r k	notes
	$v^2 = 11^2 + 2\times(-9.8)\times 2.4$	M1	Use of $v^2 = u^2 + 2as$ or complete sequence of correct *suvat*. Accept sign errors in substitution.
		A1	All correct
	$v = 8.6$ so 8.6 m s^{-1}.	A1	cao [Award all marks if 8.6 seen WWW] Do not condone ±8.6.
		3	

Q 2		m a r k	comment
	either for u first: $8 = \frac{1}{2}(u + 2.25) \times 32$ $u = -1.75$ so 1.75m s^{-1} $2.25 = -1.75 + 32a$ $a = 0.125$ so 0.125 m s^{-2} Directions of u and a are defined	M1 A1 M1 F1 F1 5	Using $s = \frac{1}{2}(u + v)t$ Use of any appropriate *suvat* with **their** values and correct signs Sign must be consistent with their u, FT from their value of u Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
	Or for a first: $8 = 2.25 \times 32 - \frac{1}{2} \times a \times 32^2$ $a = 0.125$ so 0.125 m s^{-2} $2.25 = u + 32 \times 0.125$ $u = -1.75$ so 1.75m s^{-1} Directions of u and a are defined	M1 A1 M1 F1 F1 5	Using $s = vt - \frac{1}{2}at^2$ Use of any appropriate *suvat* with **their** values and correct signs Sign must be consistent with their a, FT from their value of a Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
	Or *using simultaneous equations* Set up one relevant equation with a and u. Set up second relevant equation with a and u. Solving to find $u = -1.75$ so 1.75m s^{-1} Solving to find $a = 0.125$ so 0.125 m s^{-2} Directions of u and a are defined	M1 M1 A1 F1 F1 5	Using **one** of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ Using **another** of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ FT from their value of u or a, whichever found first Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
		5	

Q 3		mark	Notes
(i)	$-6 = -2 \times 3$ so $y = 3 \times 3 = 9$ and $z = -4 \times 3 = -12$	M1 A1 2	May be implied Both correct [Award 2 for both correct answers seen WW]
(ii)	$\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} = 5\mathbf{a}$ $\mathbf{a} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ so accn is $\begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ m s^{-2} Magnitude is $\sqrt{0.2^2 + (-0.4)^2 + (-1)^2}$ $= 1.09544\ldots$ so 1.10 m s^{-2}, (3 s. f.)	M1 B1 A1 M1 F1 5	Use of Newton's 2nd Law in vector form for all 3 cpts of attempted resultant Treat use of wrong vectors as MR. Correct LHS The acceleration may be written as a magnitude in a given direction. FT **their** values. Condone missing brackets. Condone no – signs. Accept 1.1. Accept surd form. Must come from a vector with 3 non-zero components for **a**
		7	

Q 4		mark	Comment
(i)		B1	Any one force in correct direction correctly labelled with arrow **or** all forces with correct directions and arrows. A force may be replaced by its components if labelled correctly eg $m$gcos20°, $m$gsin20°.
		B1	All correct (Accept words for labels and weight as W, mg, 147 (N)) No extra or duplicate forces. Do not allow force **and** its components unless components are clearly distinguished, eg by broken lines.
		2	
(ii)	**Either** Up the plane $P\cos20 - 15 \times 9.8 \times \sin20 = 0$	M1	Attempt to resolve at least one force up plane. Accept mass not weight. No extra forces. If other directions used, all forces must be present but see below for resolving vertically and horizontally.
		A1	Accept only error as consistent $s \leftrightarrow c$.
	$P = 53.50362\ldots$ so 53.5 (3 s. f.)	A1	Cao
		3	
	Or Vertically and horizontally $R\cos 20° = 15g$, $R\sin 20° = P$ Eliminate R	M1	Attempt to resolve all forces both horizontally and vertically and attempt to combine into a single equation. No extra forces. Accept $s \leftrightarrow c$. Accept mass not weight.
	$P = \dfrac{15g}{\cos 20°} \times \sin 20°$	A1	Accept only error as consistent $s \leftrightarrow c$.
	$P = 53.5$ (3.s.f.)	A1	Cao
		3	
	Or Triangle of forces Triangle drawn and labelled	M1	All sides must be labelled and in correct orientation; three forces only; condone no arrows
	$\dfrac{P}{15g} = \tan 20°$	A1	Oe
	$P = 53.5$ (3.s.f.)	A1	Cao
		3	
		5	

Q 5		mark	notes
	Usual notation **either** consider height: Attempt to substitute for *u* and *a* in $s = ut + \frac{1}{2}at^2$	M1	Accept: *g* as *g*, ±9.8, ±9.81, ±10; *u* = 30; s ↔ c .
	$y = 30\sin 35\ t - 4.9t^2$	A1	Derivation need not be shown
	Need *y* = 0 for time of flight *T*	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ (= 3.511692…)	A1	cao. Any form. May not be explicit.
	Or Consider time to top Attempt to substitute for *u* and *a* in $v = u + at$	M1	Accept: *g* as *g*, ±9.8, ±9.81, ±10; *u* = 30; s ↔ c .
	$v = 30\sin 35 - 9.8t$	A1	Derivation need not be shown
	Need *v* = 0 and to double for time of flight *T*	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ (= 3.511692…)	A1	cao. Any form. May not be explicit.
	then $x = 30\cos 35\ T$	M1	Accept s ↔ c if consistent with above
	so $x = 30\cos 35 \times \dfrac{30\sin 35}{4.9}$ (= 86.29830…)	F1	FT for their time Condone consistent s ↔ c error (which could lead to correct answer here).
	Required time for sound is *x*/343	M1	FT from their *x*
	Total time is 3.511692… + 0.251598… = 3.76329… so 3.76 s (3 s. f.)	A1	cao following fully correct working throughout question.
		8	

Q6		mark	notes
(i)	**Either** using *suvat*: Use of $\mathbf{v} = \mathbf{u} + t\mathbf{a}$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ Use of $\mathbf{r} = (\mathbf{r}_0 +) t\mathbf{u} + \tfrac{1}{2} t^2\mathbf{a}$ $+ 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$	 M1 A1 M1 B1 A1 5	Column vectors may be used throughout; lose 1 mark once if **j** components put at top or if fraction line included. . Notation used must be clear. substitution required. Must be vectors. substitution required. $\mathbf{r}_0$ not required. Must be vectors. May be seen on either side of a meaningful equation for **r** Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \tfrac{1}{2} \times 2 \times t^2 \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12c in marking instructions)
	Or using integration: $\mathbf{v} = \int \mathbf{a}\,dt$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ $\mathbf{r} = \int \mathbf{v}\,dt$ $+ 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$	 M1 A1 M1 B1 A1 5	 Attempt at integration. Condone no '**+c**'. Must be vectors. cao Integrate their **v** but must contain 2 components. Must be vectors. May be seen on either side of a meaningful equation for **r** Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \tfrac{1}{2} \times 2 \times t^2 \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12e in marking instructions)
		5	
(ii)	$\mathbf{v}(2.5) = 4\mathbf{i} - 5\mathbf{j}$ Angle is $(90+)\arctan\dfrac{5}{4}$ $= 141.34019\ldots$ so $141°$ (3 s. f.)	B1 M1 A1 3	FT **their v** Award for arctan attempted oe. FT **their** values. Allow argument to be $\pm$ (**their i** cpt)/(**their j** cpt) or $\pm$ (**their j** cpt)/(**their i** cpt). Allow this mark if bearing of position vector attempted. cao
		8	

Q7		mark	notes
(i)	$\dfrac{-20}{2} = -10$ -10 m s^{-2}	M1 A1 2	Use of a suitable triangle to attempt at $\Delta v / \Delta t$ for suitable interval. Accept wrong sign. cao. Allow both marks if correct answer seen.
(ii) (A) **(B)**	Signed area under graph $\frac{1}{2} \times 2 \times 20 = 20$	M1 A1	Using the relevant area or other complete method
	either using areas Signed area $2 \le t \le 5$ is $\frac{1}{2} \times \left((5-2) + (4.5 - 2.4) \right) \times (-4) = -10.2$ Signed area $5 \le t \le 6$ is $\frac{1}{2} \times 1 \times 8 = 4$ Total displacement is 13.8 m	B1 B1 B1	Allow $+10.2$. cao but FT from their 20 in part (A)
	or using *suvat* From $t = 0$ to $t = 2.4$: 19.2 From $t = 4.5$ to $t = 6$: 3.0 From $t = 2.4$ to $t = 4.5$: -8.4 Total : 13.8	B1 B1 B1 5	 Both required and both must be correct.
(iii)	$a = 4t - 14$ $a(0.5) = -12$ so -12 m s^{-2}	M1 A1 A1 3	Differentiate. Do not award for division by *t*.
(iv)	Model A gives -4 m s^{-1} For model B we need *v* when $a = 0$ $v\left(\frac{7}{2}\right) = -4.5$ so model B is 0.5 m s^{-1} less	B1 M1 A1 F1 4	May be implied by other working Using (iii) or an argument based on symmetry or sketch graph that $a = 0$ when $t = 3.5$ Accept values without more or less

(v)			Do not penalise poor notation
	Displacement is $\displaystyle\int_0^6 \left(2t^2 - 14t + 20\right)\,dt$	M1	Limits not required.
	$= \left[\dfrac{2t^3}{3} - 7t^2 + 20t\right]_0^6$	A1	Limits not required. Accept 2 terms correct.
	$= 12$ so 12 m.	M1 A1	Substitute limits cao. Accept bottom limit not substituted.
		4	
		18	

Q 8		m a r k	notes
(i)	25 N	B1 1	Condone no units. Do not accept -25 N.
(ii)	50 cos25 = 45.31538… so 45.3 N (3 s. f.)	M1 A1 2	Attempt to resolve 50 N. Accept s ↔ c . No extra forces. cao but accept – 45.3.
(iii)	Resolving vertically $R + 50 \sin 25 - 8 \times 9.8 = 0$ $R = 57.26908…$ so 57.3 N (3 s. f.)	M1 A1 A1 3	All relevant forces with resolution of 50 N. No extras. Accept s ↔ c . All correct.
(iv)	Newton's 2nd Law in direction DC $50\cos 25 - 20 = 18a$ $a = 1.4064105…$ so 1.41 m s^{-2} (3 s. f.)	M1 A1 A1 3	Newton's 2nd Law with $m = 18$. Accept $F = mga$. Attempt at resolving 50 N. Allow 20 N omitted and s ↔ c . No extra forces. Allow only sign error and s ↔ c . cao
Q8	continued		
(v)	Resolution of weight down the slope	B1	$mg\sin5°$ where $m = 8$ or 10 or 18, wherever first seen
	either Newton's 2nd Law down slope overall $18 \times 9.8 \times \sin5 - 20 = 18a$ $a = -0.2569…$ Newton's 2nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives For D: $10\times9.8\times\sin 5 - 15 - T = 10a$ (For C: $8\times9.8\times\sin 5 - 5 + T = 8a$) $T = -3.888… = -3.89$ N (3 s. f.) The force is a thrust	M1 A1 M1 F1 A1 A1	$F = ma$. Must have 20 N and $m = 18$. Allow weight not resolved and use of mass. Accept s ↔ c and sign errors (including inconsistency between the 15 N and the 5 N). cao $F = ma$. Must consider the motion of either C or D and include: component of weight, resistance and T. No extra forces. Condone sign errors and s ↔ c . Do not condone inconsistent value of mass. FT only applies to a, and only if direction is consistent. '+T' if T taken as a thrust '-T' if T taken as a thrust If T taken as thrust, then $T = +3.89$. Dependent on T correct

or Newton's 2nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives	M1	$F = ma$. Must consider the motion of C and include: component of weight, resistance and T. No extra forces. Condone sign errors and s $\leftrightarrow$ c . Do not condone inconsistent value of mass.	
	M1	$F = ma$. Must consider the motion of D and include: component of weight, resistance and T. No extra forces. Condone sign errors and s $\leftrightarrow$ c . Do not condone inconsistent value of mass.	
For C: $8 \times 9.8 \times \sin 5 - 5 + T = 8a$ For D: $10 \times 9.8 \times \sin 5 - 15 - T = 10a$	A1	Award for either the equation for C or the equation for D correct. '-T' if T taken as a thrust '+T' if T taken as a thrust	
$a = -0.2569\ldots$ $T = -3.888\ldots = -3.89$ N (3s.f.)	A1	First of a and T found is correct. If T taken as thrust, then $T = +3.89$.	
	F1	The second of a and T found is FT	
The force is a thrust	A1	Dependent on T correct	
then After 2 s: $v = 3 + 2 \times a$	M1	Allow sign of a not followed. FT their value of a. Allow change to correct sign of a at this stage.	
$v = 2.4860303..$ so 2.49 m s^{-1} (3 s. f.)	F1	FT from magnitude of **their** a but must be consistent with its direction.	
	9		
	18		