OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4761

Mechanics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4761
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 An object C is moving along a vertical straight line. Fig. 1 shows the velocity-time graph for part of its motion. Initially C is moving upwards at $14\,\text{m s}^{-1}$ and after 10 s it is moving downwards at $6\,\text{m s}^{-1}$.

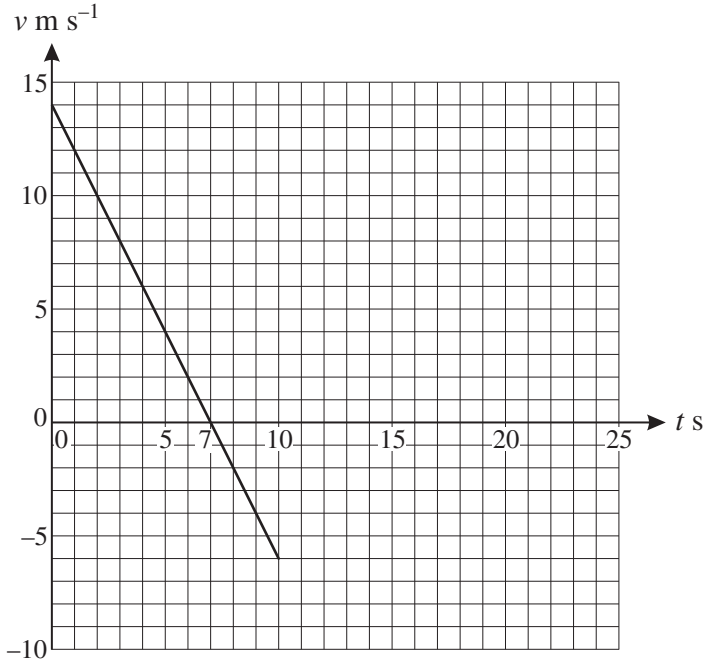

Fig. 1

C then moves as follows.

- In the interval $10 \leqslant t \leqslant 15$, the velocity of C is constant at $6\,\text{m s}^{-1}$ downwards.
- In the interval $15 \leqslant t \leqslant 20$, the velocity of C increases uniformly so that C has zero velocity at $t = 20$.

(i) Complete the velocity-time graph for the motion of C in the time interval $0 \leqslant t \leqslant 20$. **[2]**

(ii) Calculate the acceleration of C in the time interval $0 < t < 10$. **[2]**

(iii) Calculate the displacement of C from $t = 0$ to $t = 20$. **[4]**

2 Fig. 2 shows two forces acting at A. The figure also shows the perpendicular unit vectors **i** and **j** which are respectively horizontal and vertically upwards.

The resultant of the two forces is **F** N.

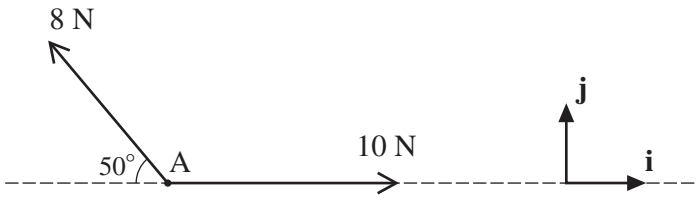

Fig. 2

(i) Find **F** in terms of **i** and **j**, giving your answer correct to three significant figures. **[3]**

(ii) Calculate the magnitude of **F** and the angle that **F** makes with the upward vertical. **[3]**

3 Two cars, P and Q, are being crashed as part of a film 'stunt'.

At the start

- P is travelling directly towards Q with a speed of $8\,\text{m}\,\text{s}^{-1}$,
- Q is instantaneously at rest and has an acceleration of $4\,\text{m}\,\text{s}^{-2}$ directly towards P.

P continues with the same velocity and Q continues with the same acceleration. The cars collide T seconds after the start.

(i) Find expressions in terms of T for how far each of the cars has travelled since the start. **[2]**

At the start, P is 90 m from Q.

(ii) Show that $T^2 + 4T - 45 = 0$ and hence find T. **[5]**

4 At time t seconds, a particle has position with respect to an origin O given by the vector

$$\mathbf{r} = \begin{pmatrix} 8t \\ 10t^2 - 2t^3 \end{pmatrix},$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are perpendicular unit vectors east and north respectively and distances are in metres.

(i) When $t = 1$, the particle is at P. Find the bearing of P from O. **[2]**

(ii) Find the velocity of the particle at time t and show that it is never zero. **[3]**

(iii) Determine the time(s), if any, when the acceleration of the particle is zero. **[3]**

5 Fig. 5 shows two boxes, A of mass 12 kg and B of mass 6 kg, sliding in a straight line on a rough horizontal plane. The boxes are connected by a light rigid rod which is parallel to the line of motion. The only forces acting on the boxes in the line of motion are those due to the rod and a constant force of F N on each box.

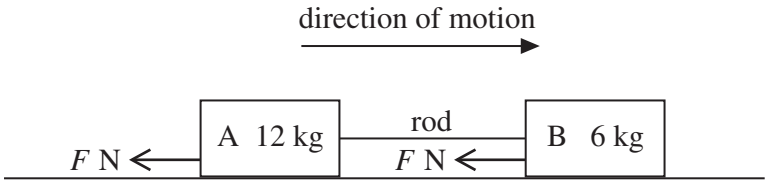

Fig. 5

The boxes have an initial speed of 1.5 m s^{-1} and come to rest after sliding a distance of 0.375 m.

(i) Calculate the deceleration of the boxes and the value of F. **[4]**

(ii) Calculate the magnitude of the force in the rod and state, with a reason, whether it is a tension or a thrust (compression). **[3]**

Section B (36 marks)

6 A toy sledge of mass 4 kg is being pulled in a straight line by a light string. The resistance to its motion is 6 N.

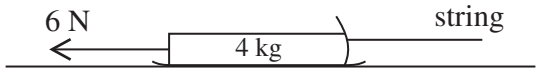

Fig. 6.1

At one time, the string is horizontal and the sledge is on horizontal ground, as shown in Fig. 6.1. The acceleration of the sledge is 3 m s^{-2} forwards.

(i) Calculate the tension in the string. **[3]**

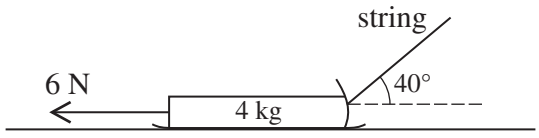

Fig. 6.2

At another time, the sledge is again on horizontal ground but the string is now at 40° to the horizontal, as shown in Fig. 6.2. The tension in the string is 25 N.

(ii) Calculate the acceleration of the sledge. **[3]**

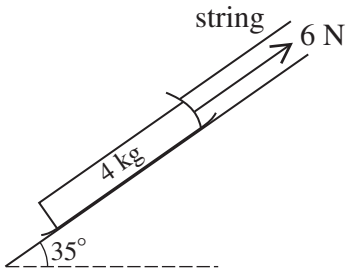

Fig. 6.3

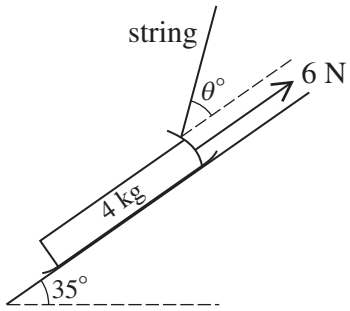

Fig. 6.4

In another situation the sledge is on a slope inclined at 35° to the horizontal, as shown in Fig. 6.3. It is held in equilibrium by the light string parallel to the slope. The resistance to motion of 6 N acts up the slope.

(iii) Calculate the tension in the string. **[3]**

The sledge is now held in equilibrium with the light string inclined at θ° to the slope, as shown in Fig. 6.4. The tension in the string is 25 N and the resistance to motion remains 6 N acting up the slope.

(iv) (*A*) Show all the forces acting on the sledge. **[2]**

(*B*) Calculate the angle θ. **[3]**

(*C*) Calculate the normal reaction of the slope on the sledge. **[3]**

7

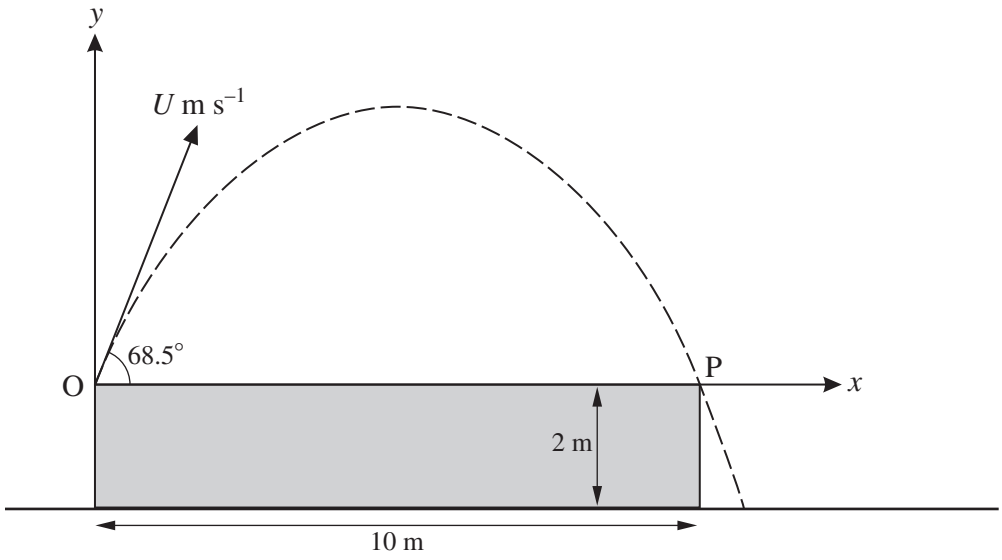

Fig. 7

Fig. 7 shows a platform 10 m long and 2 m high standing on horizontal ground. A small ball projected from the surface of the platform at one end, O, just misses the other end, P. The ball is projected at 68.5° to the horizontal with a speed of U m s^{-1}. Air resistance may be neglected.

At time t seconds after projection, the horizontal and vertical displacements of the ball from O are x m and y m.

(i) Obtain expressions, in terms of U and t, for

(A) x,

(B) y. **[3]**

(ii) The ball takes T s to travel from O to P.

Show that $T = \dfrac{U \sin 68.5^\circ}{4.9}$ and write down a second equation connecting U and T. **[4]**

(iii) Hence show that $U = 12.0$ (correct to three significant figures). **[3]**

(iv) Calculate the horizontal distance of the ball from the platform when the ball lands on the ground. **[5]**

(v) Use the expressions you found in part **(i)** to show that the cartesian equation of the trajectory of the ball in terms of U is

$$y = x \tan 68.5^\circ - \frac{4.9x^2}{U^2(\cos 68.5^\circ)^2}.$$

Use this equation to show again that $U = 12.0$ (correct to three significant figures). **[4]**

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GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4761:** Mechanics 1

Mark Scheme for January 2011

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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comment

You should expect to follow through from one part to another unless the scheme says otherwise but not follow through within a part unless the scheme specifies this.
Each script must be viewed as a whole at some stage so that
(i) a candidate's writing of letters, digits, symbols on diagrams etc can be better interpreted;
(ii) repeated mistakes can be recognised (e.g. calculator in wrong angle mode throughout – penalty 1 in the script and FT except given answers).

You are advised to 'set width' for most questions but to 'set height' for the following:

Q 1		mark	note
(i)		B1 B1 2	Section from $t = 10$ to $t = 15$ Section from $t = 15$ to $t = 20$. FT connecting from their point when $t = 15$. Ignore graph outside $0 \le t \le 20$.
(ii)	$\frac{-6-14}{10} = -2$ so -2 m s^{-2}	M1 A1 2	Attempt at $\frac{\Delta v}{\Delta t}$
(iii)	**either** Displacement is $\frac{14}{2}\times 7 - \frac{13+5}{2}\times 6$ or $\frac{14}{2}\times 7 - \frac{3\times 6}{2} - 5\times 6 - \frac{5\times 6}{2}$ $= -5$ so 5 m downwards	M1 B1 B1 A1	FT misread from graph or graphing error to all but final A1 cao Attempt at whole area. Condone 'overlap' but not 'gaps'. 'Positive' area expression correct. Condone sign error. 'Negative' area expression correct. Condone overall sign error. Accept -5 m cao

	or Displacement is $14\times10+\frac{1}{2}\times(-2)\times10^2-\ 5\times 6+\ \frac{-6+0}{2}\times5$ $= 140 – 100 – 30 –15 = –5$ so 5 m downwards	 M1 A1 B1 A1 4	 Using *suvat* from 0 to 10 or 15 to 20. Condone 'overlap' but not 'gaps' Subtracting 30 or 15 or 45 Accept –5 m cao
		8	

Q 2		mark	notes
(i)	$\mathbf{F} = (10 – 8\cos50)\mathbf{i} + 8\sin50\mathbf{j}$ $= 4.85769…\mathbf{i} + 6.128355…\ \mathbf{j}$ so $4.86\mathbf{i} + 6.13\mathbf{j}$ (3 s. f.)	M1 A1 A1 3	Resolution. Accept $s \leftrightarrow c$. Condone resolution in only one direction. Award for a vector with either component correct or consistent $s \leftrightarrow c$ error is only mistake in the vector. Need not be evaluated. cao. Must be in $a\mathbf{i} + b\mathbf{j}$ or column format. Must be correct to 3 s. f.
(ii)	$\lvert\mathbf{F}\rvert = \sqrt{4.85769...^2 + 6.12835...^2} = 7.820101...$ so 7.82 (3 s. f.) angle is $\arctan\frac{4.857...}{6.128...}$ $= 38.40243…$ so 38.4° (3 s. f.)	 B1 M1 F1 3	 FT **their F** Or equivalent. FT **their F.** Accept $\arctan\frac{6.128...}{4.857...}$. Accept complementary angle and ± signs FT only **their** *F*.
		6	

Q 3		mark	notes
(i)	For P: the distance is $8T$	B1	Allow – ve. Allow any form.
	For Q: the distance is $\frac{1}{2}\times 4\times T^2$	B1 2	Allow – ve. Allow any form.
(ii)	Require $8T+\frac{1}{2}\times 4\times T^2=90$	M1	For linking correct expressions or **their** expressions from (i) with 90. Condone sign errors and use of displacement instead of distance. Condone '= 0'implied.
		A1	The expression is correct or correctly derived from **their** (i). Reason not required.
	so $8T + 2T^2 – 90 = 0$ so $T^2 + 4T – 45 = 0$	E1	Must be established. Do not award if **their** 'correct expression' comes from incorrect manipulation.
	This gives $(T – 5)(T + 9) = 0$	M1	Solving to find +ve root. Accept $(T + 5)(T – 9)$.
	so $T = 5$ since $T > 0$	A1 5	Condone 2nd root not found/discussed but not both roots given.
		7	

Q 4		mark	notes
(i)	When $t = 1$, $\mathbf{r} = \begin{pmatrix} 8 \\ 10-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$		Accept column or $a\mathbf{i} + b\mathbf{j}$ notation
	$[8\mathbf{i} + (10-2)\mathbf{j} = 8\mathbf{i} + 8\mathbf{j}]$	B1	May be implied
	Bearing OP is 045°	F1 2	Accept 45°. Accept NE and northeast. Condone $\lvert\mathbf{r}\rvert$ given as well.
(ii)	$\mathbf{v} = \begin{pmatrix} 8 \\ 20t - 6t^2 \end{pmatrix}$ $[8\mathbf{i} + (20t - 6t^2)\mathbf{j}]$	M1	Differentiating both components. Condone 1 error if clearly attempting differentiation.
		A1	Must be a vector answer.
	The **i** cpt is always 8 so $\mathbf{v} \neq \mathbf{0}$ for any t	E1 3	Accept any correct argument e.g. based on **i** cpt never 0.
(iii)	$\mathbf{a} = \begin{pmatrix} 0 \\ 20 - 12t \end{pmatrix}$ $[(20 - 12t)\mathbf{j}]$	M1	Differentiating as a vector. Condone 1 error if clearly attempting differentiation of **their v**.
		F1	FT **their v**.
	$\mathbf{a} = \mathbf{0}$ when $t = \frac{20}{12} = \frac{5}{3}$		
	so $\frac{5}{3}$ s (1.67 s (3 s. f.))	B1 3	cao. Condone obtained from scalar equation.
		8	

Q5		mark	notes
(i)	In direction $\rightarrow$ $0^2 = 1.5^2 + 2 \times a \times 0.375$ so $a = -3$ and deceleration is 3 m s^{-2} N2L on both boxes $\rightarrow$ $-2F = (12+6) \times (-3)$ so $F = 27$	M1 A1 M1 A1 4	Use of $v^2 = u^2 + 2as$ or complete sequence of *suvat.* CWO. Accept ± 3 and ignore accel or decal. N2L. Correct mass. Condone $F = mga$. Allow F on LHS. FT **their** a. Accept sign errors. No extra terms. cao Condone this obtained from an equation with consistent signs not justified.
(ii)	Suppose the force in the rod is a tension T N2L gives box A $\rightarrow$ $T - 27 = 12 \times (-3)$ [box B $\rightarrow$ $-T - 27 = 6 \times (-3)$] so $T = -9$ and the force has magnitude 9 N It is a thrust (tension is +ve).	 M1 F1 E1 3	 N2L. $F = ma$. Correct mass. The '27' and the '3' must have the same sign. Ignore the sign of 'T'. FT only for mod(their 27) in place of '27' and/or mod(their 3) in place of '3' in this sign pattern. No extra terms. Accept $T = \pm 9$. FT only for mod(their 27) in place of '27' and/or mod(their 3) in place of '3'. cao Only accept thrust with $T = \pm 9$ and a sound argument.
		7	

Q 6		mark	notes
(i)	Let tension be T N N2L $\rightarrow$ $T - 6 = 4 \times 3$ T = 18 so 18 N	 M1 A1 A1 3	 Condone $F = mga$. Condone resistance omitted or an extra force. Allow only sign error(s). cao
(ii)	Let acceleration be a m s^{-2} $25\cos 40 - 6 = 4a$ a = 3.28777.. so 3.29 m s^{-2} (3 s. f.)	 M1 M1 A1 3	 Attempt at resolution of 25 N. Allow $s \leftrightarrow c$. Allow $F = mga$ and sign error(s). No extra forces. Both forces present. cao
(iii)	Let tension be T N up the slope $T + 6 - 4 \times 9.8 \times \sin 35 = 0$ T = 16.48419… so 16.5 N (3 s. f.)	 M1 B1 A1 3	 Resolving along slope. Allow 6 N omitted. If different direction used all required forces present (except 6 N). Allow $s \leftrightarrow c$. No extra forces. Allow sign errors. Condone g omitted. If resolution is along plane, weight term correct. If resolution in another direction, one resolution correct.
(iv) (A)		 B1 B1 2	 At least two of tension, weight and NR marked correctly with arrows and labels (accept mg, W, T and words etc). All correct. No extra forces. Accept mg, W, T and words etc. Condone resolved parts as well only if clearly indicated as such by e.g. using dotted lines.
(B)	**continued**		

Q6 (iv) (B)	up the slope $25\cos\theta + 6 - 4g\sin 35 = 0$ so $25\cos\theta = 16.48414...$ so $\theta = 48.7483....$ so 48.7° (3 s. f.)	M1 A1 A1 3	No extra forces. Allow $s \leftrightarrow c$. All forces present and required resolutions attempted. Allow sign errors. Condone g omitted. Condone g omitted. cao [If they use **their** (iii): M1 Equating **their** (iii) to an attempt at resolving 25. Allow $s \leftrightarrow c$. No extra forces. A1 FT **their** T from (iii) A1 cao]
(C)	Resolve perp to slope $R + 25\sin\theta - 4\times 9.8\times\cos 35 = 0$ R = 13.315248.. so 13.3 N (3 s. f.)	M1 A1 A1 3	All forces present and resolutions attempted. No extra forces. Allow $s \leftrightarrow c$. FT their angle. Condone g omitted. FT their angle. Condone g omitted. cao
		17	

Q7		**mark**	**notes**
(i) (A)	$x = Ut\cos 68.5°$	B1 1	
(i) (B)	$y = Ut\sin 68.5° - 4.9\times t^2$	M1 A1 2	Allow 'u' = U. Allow $s \leftrightarrow c$. Allow g as g, ±9.8, ±9.81, ±10. Allow +2. Accept not 'shown'. Do not allow +2. Allow e.g + 0.5 × (- 9.8) × t^2 instead of – 4.9t^2. Accept g not evaluated
	continued		

Q7 (ii)	**either** At D, $y = 0$ so $U\sin 68.5°T - 4.9\times T^2 = 0$ $\Rightarrow T\left(U\sin 68.5° - 4.9T\right) = 0$ so $T = 0$ (at C) or $T = \dfrac{U\sin 68.5°}{4.9}$ (at D) **or** Use (i)(A) and put $x = 10$ with $t = T$ to get $UT\cos 68.5° = 10$	 M1 M1 E1 M1 M1 E1 B1 4	 Equating correct y to 0 or **their** y to correct value. Attempting to factorise (or solve). Allow ÷ T without comment. Properly shown. Accept no ref to $T = 0$. Accept $T = 0$ given as well without comment. Find time to top Double time to the top Use of $x = 10$ with $t = T$
(iii)	Eliminating T from the results in (ii) gives $U\cos 68.5°\times\dfrac{U\sin 68.5°}{4.9} = 10$ so $U = 11.98729\ldots$ so 12.0 (3 s. f.)	 M1 M1 E1 3	 Substituting, using correct expressions or **their** expressions from (ii). Attempt to solve for U^2 or U. Some evidence seen. e.g. $142.8025.. < U^2 < 145.2025...$ with clear statement, or 11.9… seen with clear statement or 11.98… seen. Accept 11.98… seen for full marks.
(iv)	**continued**		

(iv)	Require $Ut\sin 68.5° - 4.9t^2 = -2$	M1	Equating correct y to -2 or **their** y to correct value. Allow use of U, 11.987… or 12. Allow implicit '= 0'
	Solving $4.9t^2 - Ut\sin 68.5° - 2 = 0$	M1	Dep on 1st M1. Attempt to solve a 3 term quadratic to find at least the +ve root. Allow if two correct roots seen WW.
	$t = -$ 0.1670594541… , 2.4431591… (Using 12: – 0.1669052502.. , 2.445478886..)	A1	Accept only + ve root given
			Alternative method of e.g. finding time to highest point and then time to the ground. M1 all times attempted, at least one by a sound method. M1 both methods sound and complete. A1.
	Require $U\cos 68.5° \times 2.44… - 10$ = 0.7336…. so 0.734 m (3 s. f.) (Using 12 consistently, 0.7552… so 0.755 (3 s. f.))	M1 A1	Dep on first M1. Allow **their** expression for x. Allow ' – 10' omitted. cao. Accept $0.73 \le x \le 0.76$
		5	
(v)	Eliminate t from (i) (B)		
	using $t = \dfrac{x}{U\cos 68.5°}$ from (i)(A)	M1	May be implied. FT their (i).
	so $y = x\tan 68.5° - \dfrac{4.9x^2}{U^2(\cos 68.5°)^2}$	E1	Clearly shown.
	We require $y = 0$ when $x = 10$	M1	Must see attempt to solve. Or use $x = 10.73…$ when $y = -2$.
	so $U = 11.98729…$ so 12.0 (3 s. f.)	E1	Must see evidence of fresh calculation or statement that they have now got the same expression for evaluation.
		4	
		19	

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