OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4761

Mechanics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1 A ring is moving up and down a vertical pole. The displacement, s m, of the ring above a mark on the pole is modelled by the displacement-time graph shown in Fig. 1. The three sections of the graph are straight lines.

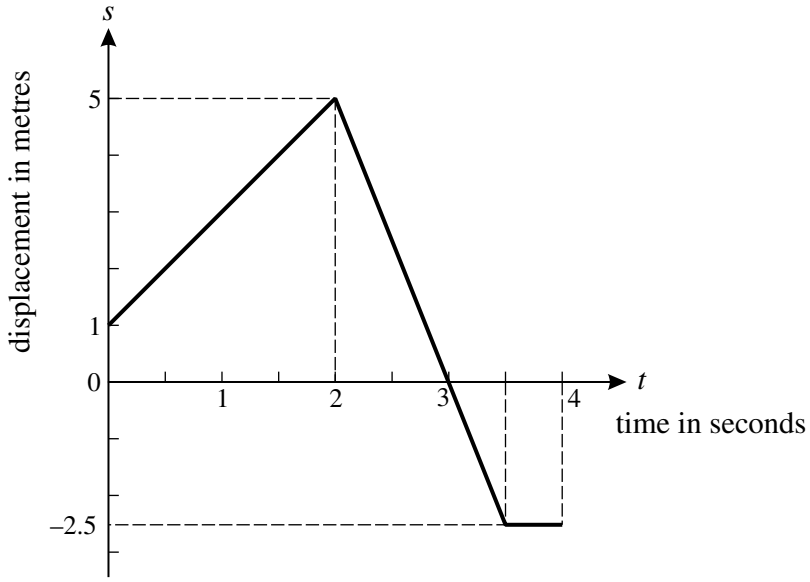

Fig. 1

(i) Calculate the velocity of the ring in the interval $0 < t < 2$ and in the interval $2 < t < 3.5$. **[2]**

(ii) Sketch a velocity-time graph for the motion of the ring during the 4 seconds. **[2]**

(iii) State the direction of motion of the ring when

(A) $t = 1$,

(B) $t = 2.75$,

(C) $t = 3.25$. **[1]**

2 A particle of mass 5 kg has constant acceleration. Initially, the particle is at $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m with velocity $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ m s^{-1}; after 4 seconds the particle has velocity $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m s^{-1}.

(i) Calculate the acceleration of the particle. **[2]**

(ii) Calculate the position of the particle at the end of the 4 seconds. **[3]**

(iii) Calculate the force acting on the particle. **[2]**

3 In this question, **i** is a horizontal unit vector and **j** is a unit vector pointing vertically upwards.

A force **F** is $-\mathbf{i} + 5\mathbf{j}$.

(i) Calculate the magnitude of **F**.

Calculate also the angle between **F** and the upward vertical. **[4]**

Force **G** is $2a\mathbf{i} + a\mathbf{j}$ and force **H** is $-2\mathbf{i} + 3b\mathbf{j}$, where a and b are constants. The force **H** is the resultant of forces $4\mathbf{F}$ and **G**.

(ii) Find **G** and **H**. **[4]**

4 A box of mass 2.5 kg is on a smooth horizontal table, as shown in Fig. 4. A light string AB is attached to the table at A and the box at B. AB is at an angle of 50° to the vertical. Another light string is attached to the box at C; this string is inclined at 15° above the horizontal and the tension in it is 20 N. The box is in equilibrium.

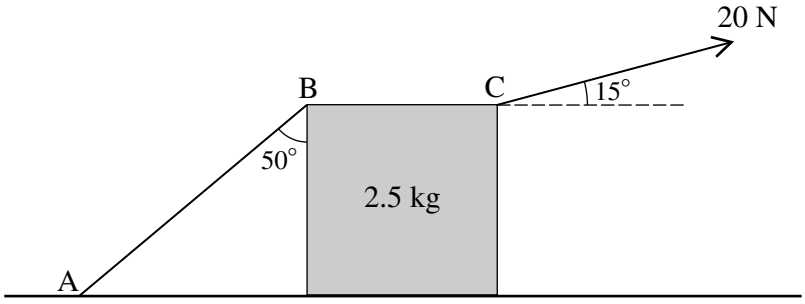

Fig. 4

(i) Calculate the horizontal component of the force exerted on the box by the string at C. **[1]**

(ii) Calculate the tension in the string AB. **[2]**

(iii) Calculate the normal reaction of the table on the box. **[4]**

The string at C is replaced by one inclined at 15° below the horizontal with the same tension of 20 N.

(iv) Explain why this has no effect on the tension in string AB. **[1]**

5 The velocity, v m s^{-1}, of a particle moving along a straight line is given by

$$v = 3t^2 - 12t + 14,$$

where t is the time in seconds.

(i) Find an expression for the acceleration of the particle at time t. **[2]**

(ii) Find the displacement of the particle from its position when $t = 1$ to its position when $t = 3$. **[4]**

(iii) You are *given* that v is always positive. Explain how this tells you that the distance travelled by the particle between $t = 1$ and $t = 3$ has the same value as the displacement between these times. **[2]**

Section B (36 marks)

6

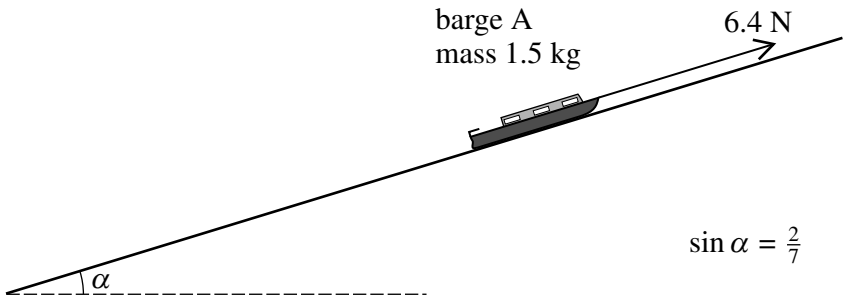

Fig. 6.1

Fig. 6.1 shows a toy barge A of mass 1.5 kg on a rough plane. The plane is at an angle α to the horizontal where $\sin\alpha = \frac{2}{7}$.

(i) Show that the component of the weight of the barge down the slope is 4.2 N. **[2]**

The barge is held in equilibrium by a force of 6.4 N acting up and parallel to the plane.

(ii) Determine the frictional force on the barge and state whether it acts up or down the plane. **[2]**

The force of 6.4 N is removed and the barge now slides down the plane with acceleration $1.2\,\text{m s}^{-2}$.

(iii) Calculate the new frictional force on the barge. **[4]**

(iv) Determine how far the barge travels while its speed increases from $0.8\,\text{m s}^{-1}$ to $2\,\text{m s}^{-1}$. **[3]**

Fig. 6.2 shows barge A on the same slope with a second barge B of mass 2 kg attached to it by means of a light rigid coupling parallel to the plane. The frictional force on barge B is 0.7 N and the frictional force on barge A is now 2.3 N. At one stage of the motion the two barges are being pulled up the plane by a force of 10 N parallel to the plane.

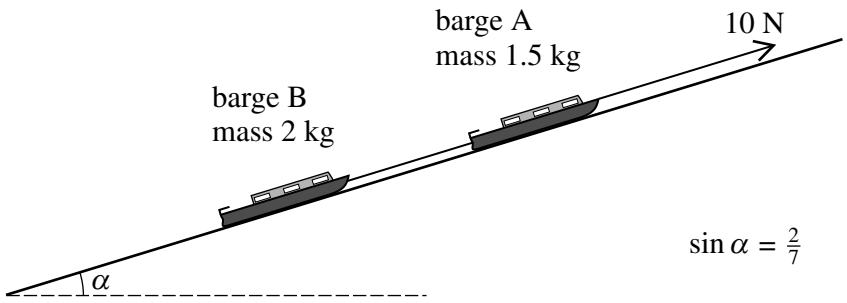

Fig. 6.2

(v) Draw diagrams showing the forces acting on each barge.

Calculate the acceleration of the barges and clearly indicate its direction.

Find the force in the coupling, stating whether this is a tension or a thrust (compression). **[7]**

7

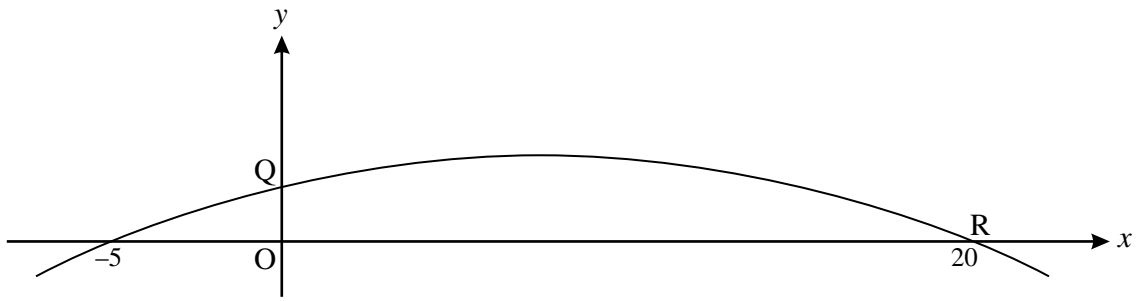

Fig. 7

Fig. 7 shows the graph of $y = \frac{1}{100}(100 + 15x - x^2)$.

For $0 \leqslant x \leqslant 20$, this graph shows the trajectory of a small stone projected from the point Q where y m is the height of the stone above horizontal ground and x m is the horizontal displacement of the stone from O. The stone hits the ground at the point R.

(i) Write down the height of Q above the ground. **[1]**

(ii) Find the horizontal distance from O of the highest point of the trajectory and show that this point is 1.5625 m above the ground. **[5]**

(iii) Show that the time taken for the stone to fall from its highest point to the ground is 0.565 seconds, correct to 3 significant figures. **[3]**

(iv) Show that the horizontal component of the velocity of the stone is $22.1\,\text{m}\,\text{s}^{-1}$, correct to 3 significant figures. Deduce the time of flight from Q to R. **[5]**

(v) Calculate the speed at which the stone hits the ground. **[4]**

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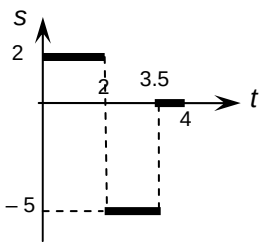
1 (i)	$0 < t < 2$, $v = 2$ $2 < t < 3.5$ $v = -5$	B1 B1	Condone '5 downwards' and ' – 5 downwards'	2
(ii)		 B1 B1	Condone intent – e.g. straight lines free-hand and scales not labelled; accept non-vertical sections at $t = 2$ & 3.5. Only horizontal lines used and 1st two parts present. BOD t-axis section. One of 1st 2 sections correct. FT (i) and allow if answer correct with (i) wrong All correct. Accept correct answer with (i) wrong. FT (i) only if 2nd section –ve in (i)	2
(iii)	(A) upwards; (B) and (C) downwards	E1	All correct. Accept +/- ve but not towards/away from O Accept forwards/backwards. Condone additional wrong statements about position.	1
				5
2 (i)	$\begin{pmatrix}12\\9\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix} + 4\mathbf{a}$ so $\mathbf{a} = \begin{pmatrix}2.5\\3\end{pmatrix}$	M1 A1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ If vector **a** seen, isw.	2
(ii)	**either** $\mathbf{r} = \begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} \times 4 + \frac{1}{2}\mathbf{a} \times 4^2$ $\mathbf{r} = \begin{pmatrix}27\\14\end{pmatrix}$ so $\begin{pmatrix}27\\14\end{pmatrix}$ m **or**	M1 A1 A1 M1 A1 A1 SC2 for $\begin{pmatrix}28\\12\end{pmatrix}$	For use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **their a.** Initial position may be omitted. FT **their a.** Initial position may be omitted. cao. Do not condone magnitude as final answer. Use of $\mathbf{s} = 0.5t(\mathbf{u} + \mathbf{v})$ Initial position may be omitted. Correct substitution. Initial position may be omitted. cao Do not condone mag as final answer.	3

(iii)	Using N2L $\mathbf{F} = 5\mathbf{a} = \begin{pmatrix}12.5\\15\end{pmatrix}$ so $\begin{pmatrix}12.5\\15\end{pmatrix}$ N	M1 F1	Use of $\mathbf{F} = m\mathbf{a}$ or $\mathbf{F} = mg\mathbf{a}$. FT **their a** only. Do not accept magnitude as final ans.	2
				7
3 (i)	$\lvert\mathbf{F}\rvert = \sqrt{(-1)^2 + 5^2}$ $= \sqrt{26} = 5.0990... = 5.10$ (3 s. f.) Angle with **j** is arctan(0.2) so 11.309… so 11.3° (3 s. f.)	M1 A1 M1 A1	Accept $\sqrt{-1^2 + 5^2}$ even if taken to be $\sqrt{24}$ accept arctan(p) where $p = \pm 0.2$ or ± 5 o.e. cao	4
(ii)	$\begin{pmatrix}-2\\3b\end{pmatrix} = 4\begin{pmatrix}-1\\5\end{pmatrix} + \begin{pmatrix}2a\\a\end{pmatrix}$ $a = 1$, $b = 7$ so $\mathbf{G} = \begin{pmatrix}2\\1\end{pmatrix}$ and $\mathbf{H} = \begin{pmatrix}-2\\21\end{pmatrix}$ or $\mathbf{G} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{H} = -2\mathbf{i} + 21\mathbf{j}$	M1 M1 A1 A1	$\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ soi Formulating at least 1 scalar equation from **their** vector equation soi a correct **or G** follows from their wrong a **H** cao	4
				8
4(i)	20cos 15 = 19.3185…. so 19.3 N (3 s. f.) in direction BC	B1	Accept no direction. Must be evaluated	1
(ii)	Let the tension be T T sin 50 = 19.3185…. so T = 25.2185… so 25.2 N (3 s. f.)	M1 F1	Accept $\sin \leftrightarrow \cos$ but not (i) × sin 50 FT **their** 19.3… only. cwo	2
(iii)	$R + 20 \sin 15 - 2.5g - 25.2185... \times \cos 50 = 0$ R = 35.5337… so 35.5 N (3 s. f.)	M1 B1 A1 A1	Allow 1 force missing or 1 tension not resolved. FT T. No extra forces. Accept mass used. Accept $\sin \leftrightarrow \cos$. Weight correct All correct except sign errors. FT **their** T cao. Accept 35 or 36 for 2. s.f.	4
(iv)	The horizontal resolved part of the 20 N force is not changed.	E1	Accept no reference to vertical component but do not accept 'no change' to both components. No need to be explicit that value of tension in AB depends only on horizontal component of force at C	
				1
				8

5(i)	$a = 6t - 12$	M1 A1	Differentiating cao	2
(ii)	We need $\int_1^3 (3t^2 - 12t + 14)\mathrm{d}t$ $= \left[t^3 - 6t^2 + 14t\right]_1^3$ **either** $= (27 - 54 + 42) - (1 - 6 + 14)$ $= 15 - 9 = 6$ so 6 m **or** $s = t^3 - 6t^2 + 14t + C$ $s = 0$ when $t = 1$ gives $0 = 1 - 6 + 14 + C$ so $C = -9$ Put $t = 3$ to give $s = 27 - 54 + 42 - 9 = 6$ so 6 m.	M1 A1 M1 A1 M1 A1	Integrating. Neglect limits. At least two terms correct. Neglect limits. Dep on 1st M1. Use of limits with attempt at subtraction seen. cao Dep on 1st M1. An attempt to find C using $s(1) = 0$ and then evaluating $s(3)$. cao	4
(iii)	$v > 0$ so the particle always travels in the same (+ve) direction As the particle never changes direction, the final distance from the starting point is the displacement.	E1 E1	Only award if explicit Complete argument	
				2
				8
6 (i)	Component of weight down the plane is $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ N	M1 E1	Use of mgk where k involves an attempt at resolution Accept $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ or $14.7 \times \frac{2}{7} = 4.2$ seen	2
(ii)	Down the plane. Take F down the plane. $4.2 - 6.4 + F = 0$ so $F = 2.2$. Friction is 2.2 N down the plane	M1 A1	Allow sign errors. All forces present. No extra forces. Must have direction. [Award 1 for 2.2 N seen and 2 for 2.2 N down plane seen]	2
(iii)	F up the plane N2L down the plane $4.2 - F = 1.5 \times 1.2$ so $F = 4.2 - 1.8 = 2.4$ Friction is 2.4 N up the plane	M1 A1 A1 A1	N2L. $F = ma$. No extra forces. Allow weight term missing or wrong Allow only sign errors ± 2.4 cao. Accept no reference to direction if $F = 2.4$.	4
(iv)	$2^2 = 0.8^2 + 2 \times 1.2 \times s$ $s = 1.4$ so 1.4 m	M1 A1 A1	Use of $v^2 = u^2 + 2as$ or sequence All correct in 1 or 2-step method	3

(v)	Diagrams	B1	Frictions and coupling force correctly labelled with arrows.	
		B1	All forces present and properly labelled with arrows.	
	either			
	Up the plane	M1	N2L. $F = ma$. No extra forces. Condone sign errors.	
	$10 - 3.5\times 9.8\times \frac{2}{7} - (2.3+0.7) = 3.5a$		Allow total/part weight **or** total/part friction omitted (but not both). Allow mass instead of weight and mass/weight not or wrongly resolved.	
		B1	Correct overall mass and friction	
	$a = -0.8$ so 0.8 m s^{-2}.			
	down the plane	A1	Clear description or diagram	
	For barge B up the plane			
	$T - 2\times 9.8\times \frac{2}{7} - 0.7 = 2\times(-0.8)$	M1	N2L on one barge with **their** $\pm a$ ($\neq 1.2$ or 0). All forces present and weight component attempted. No extra forces. Condone sign errors.	
	$T = 4.7$ so 4.7 N. Tension	A1	cao	
	or (separate equations of motion)		In eom for A **or** B allow weight or friction missing and also allow mass used instead of weight and wt not or wrongly resolved. In other equn weight component attempted and friction term present.	
	Barge A	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
	Barge B	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
		M1	Solving a pair of equns in a and T	
	$a = -0.8$ so 0.8 m s^{-2}.			
	down the plane	A1	Clear description or diagram	
	$T = 4.7$ so 4.7 N. Tension	A1	cao cwo	
				7
				18
7 (i)	$y(0) = 1$	B1		1
	Either			
(ii)	$\frac{1}{2}(20+5) - 5 = 7.5$	M1	Use of symmetry e.g. use of $\frac{1}{2}(20+5)$	
		A1	12.5 o.e. seen	
		A1	7.5 cao	
	or	M1	Attempt at y' and to solve $y' = 0$	
		A1	$k(15 - 2x)$ where $k = 1$ or $\frac{1}{100}$	
		A1	7.5 cao, seen as final answer	
	$y(7.5) = \frac{1}{100}(100 + 15\times 7.5 - 7.5^2)$	M1	FT **their** 7.5	
	$= \frac{25}{16}$ (1.5625) so 1.5625 m	E1	AG	
			[SC2 only showing 1.5625 leads to $x = 7.5$]	5

(iii)	$4.9t^2 = \frac{25}{16}$ (1.5625)	M1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of $\pm 10,\ \pm 9.8,\ \pm 9.81$. If sequence of *suvat* used, complete method required.	
	$t^2 = 0.31887\ldots$ so $t = \pm\ 0.56469\ldots$	A1	In any method only error accepted is sign error	
	Hence 0.565 s (3 s. f.)	E1	AG. Condone no reference to –ve value. www. 0.565 must be justified as answer to 3 s. f.	3
(iv)	$\dot{x} = \dfrac{12.5}{0.56469...} = 22.1359...$	M1	or 25 / (2×0.56469..)	
		B1	Use of 12.5 or equivalent	
	so 22.1 m s^{-1} (3 s. f.))	E1	22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii).	
	Either			
	Time is $\dfrac{20}{12.5} \times 0.56469...$ s	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or			
	Time is $\dfrac{20}{22.1359...}$ s	M1		
	= 0.903507… so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or			
	(iii) $+ \dfrac{7.5}{\textbf{their } \dot{x}}$	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$	M1	Must have attempts at both components	
	$\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or	M1	Or equiv. $u = 0$. Condone use of $\pm 10,\ \pm 9.8,\ \pm 9.81$.	
	$\dot{y} = 0 + 9.8 \times 0.5646...$		Accept wrong s (or t in alternative method)	
	$= \frac{245}{8}$ (30.625) or $\dot{y} = \pm 5.539...$	A1	Or equivalent. May be implied. Could come from (iii) if $v^2 = u^2 + 2as$ used there. Award marks again.	
	so $v = \sqrt{490 + 30.625} = 22.8172\ldots$ m s^{-1}			
	so 22.8 m s^{-1} (3 s. f.)	A1	cao. www	4
				18