Section A (36 marks)

1 The velocity-time graph shown in Fig. 1 represents the straight line motion of a toy car. All the lines on the graph are straight.

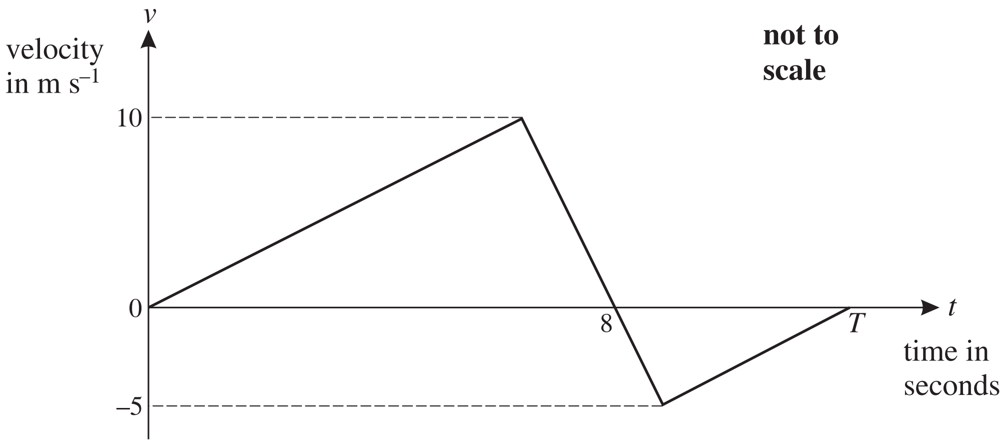

Fig. 1

The car starts at the point A at $t = 0$ and in the next 8 seconds moves to a point B.

(i) Find the distance from A to B. **[2]**

T seconds after leaving A, the car is at a point C which is a distance of 10 m from B.

(ii) Find the value of T. **[3]**

(iii) Find the displacement from A to C. **[1]**

2 A small box has weight $\mathbf{W}$ N and is held in equilibrium by two strings with tensions $\mathbf{T}_1$ N and $\mathbf{T}_2$ N. This situation is shown in Fig. 2 which also shows the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$ that are horizontal and vertically upwards, respectively.

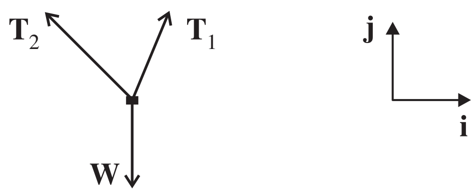

Fig. 2

The tension $\mathbf{T}_1$ is $10\mathbf{i} + 24\mathbf{j}$.

(i) Calculate the magnitude of $\mathbf{T}_1$ and the angle between $\mathbf{T}_1$ and the vertical. **[3]**

The magnitude of the weight is w N.

(ii) Write down the vector $\mathbf{W}$ in terms of w and $\mathbf{j}$. **[1]**

The tension $\mathbf{T}_2$ is $k\mathbf{i} + 10\mathbf{j}$, where k is a scalar.

(iii) Find the values of k and of w. **[3]**

3 Fig. 3 is a sketch of the velocity-time graph modelling the velocity of a sprinter at the start of a race.

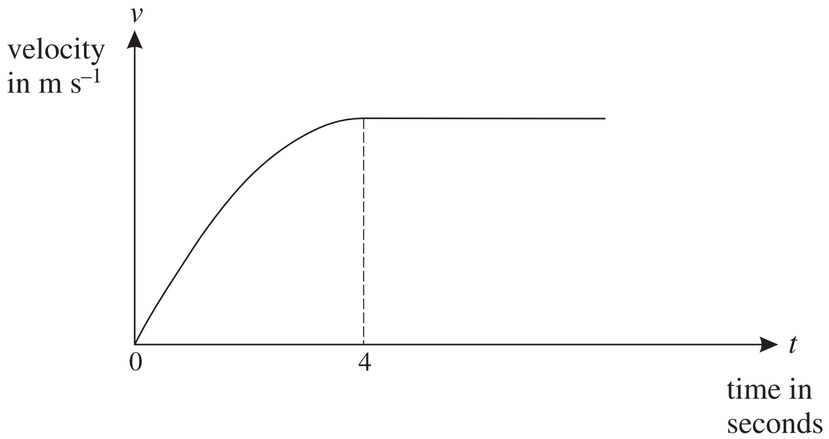

Fig. 3

(i) How can you tell from the sketch that the acceleration is not modelled as being constant for $0 \leqslant t \leqslant 4$? **[1]**

The velocity of the sprinter, $v\,\mathrm{m\,s^{-1}}$, for the time interval $0 \leqslant t \leqslant 4$ is modelled by the expression

$$v = 3t - \tfrac{3}{8}t^2.$$

(ii) Find the acceleration that the model predicts for $t = 4$ and comment on what this suggests about the running of the sprinter. **[3]**

(iii) Calculate the distance run by the sprinter from $t = 1$ to $t = 4$. **[4]**

4 Fig. 4 shows a particle projected over horizontal ground from a point O at ground level. The particle initially has a speed of $32\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal. The particle is a horizontal distance of 44.8 m from O after 5 seconds. Air resistance should be neglected.

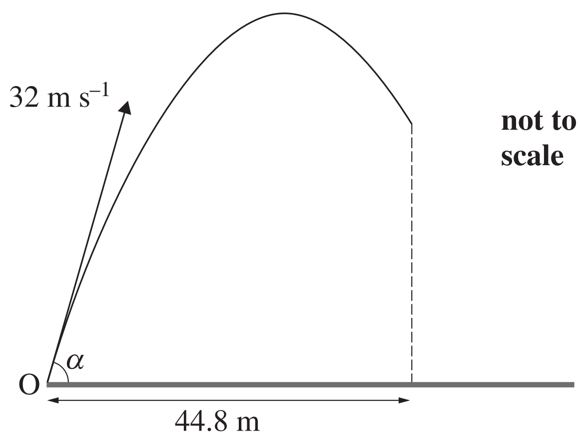

Fig. 4

(i) Write down an expression, in terms of α and t, for the horizontal distance of the particle from O at time t seconds after it is projected. **[1]**

(ii) Show that $\cos\alpha = 0.28$. **[2]**

(iii) Calculate the greatest height reached by the particle. **[4]**

5 The position vector of a toy boat of mass 1.5 kg is modelled as $\mathbf{r} = (2 + t)\mathbf{i} + (3t - t^2)\mathbf{j}$ where lengths are in metres, t is the time in seconds, $\mathbf{i}$ and $\mathbf{j}$ are horizontal, perpendicular unit vectors and the origin is O.

(i) Find the velocity of the boat when $t = 4$. **[3]**

(ii) Find the acceleration of the boat and the horizontal force acting on the boat. **[3]**

(iii) Find the cartesian equation of the path of the boat referred to x- and y-axes in the directions of $\mathbf{i}$ and $\mathbf{j}$, respectively, with origin O. You are not required to simplify your answer. **[2]**

Section B (36 marks)

6 An empty open box of mass 4 kg is on a plane that is inclined at 25° to the horizontal.

In one model the plane is taken to be smooth.

The box is held in equilibrium by a string with tension T N parallel to the plane, as shown in Fig. 6.1.

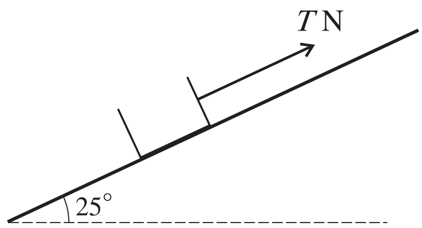

Fig. 6.1

(i) Calculate T. **[2]**

A rock of mass m kg is now put in the box. The system is in equilibrium when the tension in the string, still parallel to the plane, is 50 N.

(ii) Find m. **[3]**

In a refined model the plane is rough.

The empty box, of mass 4 kg, is in equilibrium when a frictional force of 20 N acts down the plane and the string has a tension of P N inclined at 15° to the plane, as shown in Fig. 6.2.

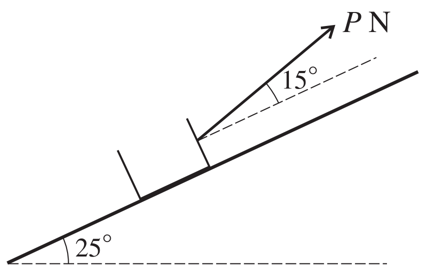

Fig. 6.2

(iii) Draw a diagram showing all the forces acting on the box. **[2]**

(iv) Calculate P. **[5]**

(v) Calculate the normal reaction of the plane on the box. **[4]**

7 A box of mass 8 kg slides on a horizontal table against a constant resistance of 11.2 N.

(i) What horizontal force is applied to the box if it is sliding with acceleration of magnitude $2\,\mathrm{m\,s^{-2}}$? **[3]**

Fig. 7 shows the box of mass 8 kg on a long, rough, horizontal table. A sphere of mass 6 kg is attached to the box by means of a light inextensible string that passes over a smooth pulley. The section of the string between the pulley and the box is parallel to the table. The constant frictional force of 11.2 N opposes the motion of the box. A force of 105 N parallel to the table acts on the box in the direction shown, and the acceleration of the system is in that direction.

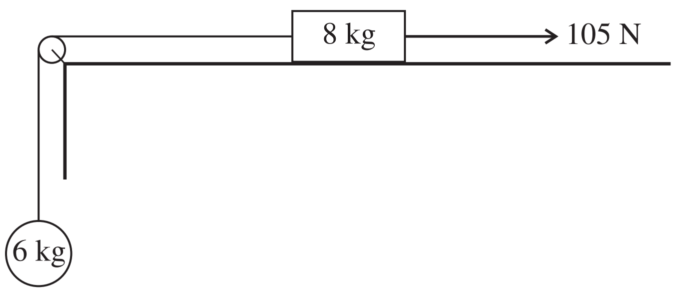

Fig. 7

(ii) What information in the question indicates that while the string is taut the box and sphere have the same acceleration? **[1]**

(iii) Draw two separate diagrams, one showing all the horizontal forces acting on the box and the other showing all the forces acting on the sphere. **[2]**

(iv) Show that the magnitude of the acceleration of the system is $2.5\,\mathrm{m\,s^{-2}}$ and find the tension in the string. **[7]**

The system is stationary when the sphere is at point P. When the sphere is 1.8 m above P the string breaks, leaving the sphere moving upwards at a speed of $3\,\mathrm{m\,s^{-1}}$.

(v) (*A*) Write down the value of the acceleration of the sphere after the string breaks. **[1]**

(*B*) The sphere passes through P again at time T seconds after the string breaks. Show that T is the positive root of the equation $4.9T^2 - 3T - 1.8 = 0$. **[2]**

(*C*) Using part (*B*), or otherwise, calculate the total time that elapses after the sphere moves from P before the sphere again passes through P. **[4]**

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Q 1		mark	comment	sub
(i)	$0.5\times 8\times 10 = 40$ m	M1	Attempt to find whole area or … If *suvat* used in 2 parts, accept any *t* value $0 \le t \le 8$ for max.	
		A1	cao	2
(ii)	$0.5\times 5(T-8) = 10$	M1	$0.5\times 5\times k = 10$ seen. Accept ±5 and ±10 only. If *suvat* used need whole area; if in 2 parts, accept any *t* value $8 \le t \le T$ for min.	
		B1	Attempt to use *k* = *T* – 8.	
	T = 12	A1	cao. [Award 3 if *T* = 12 seen]	3
(iii)	40 – 10 = 30 m	B1	FT **their** 40.	1
		6		

Q 2		mark	comment	sub
(i)	$\sqrt{10^2+24^2} = 26$ so 26 N	B1		
	arctan ($^{10}/_{24}$)	M1	Using arctan or equiv. Accept arctan ($^{24}/_{10}$) or equiv.	
	= 22.619… so 22.6° (3 s. f.)	A1	Accept 157.4°.	3
(ii)	**W** = – *w* **j**	B1	Accept $\begin{pmatrix} 0 \\ -w \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -w\mathrm{j} \end{pmatrix}$	1
(iii)	$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = \mathbf{0}$	M1	Accept in any form and recovery from **W** = *w* **j**. Award if not explicit and part (ii) and **both** *k* and *w* correct.	
	k = – 10	B1	Accept from wrong working.	
	w = 34	B1	Accept from wrong working but not – 34. [Accept – 10 **i** *or* 34 **j** but not both]	3
		7		

Q 3		mark	comment	sub
(i)	The line is not straight	B1	Any valid comment	1
(ii)	$a = 3 - {}^{6t}\!/_{8}$	M1	Attempt to differentiate. Accept 1 term correct but not $3 - \frac{3t}{8}$.	
	$a(4) = 0$	F1		
	The sprinter has reached a steady speed	E1	Accept 'stopped accelerating' but not just *a* = 0. Do not FT $a(4) \ne 0$.	3

		mark	comment	sub
(iii)	We require $\int_1^4 \left(3t - \frac{3t^2}{8}\right) dt$	M1	Integrating. Neglect limits.	
	$= \left[\frac{3t^2}{2} - \frac{t^3}{8}\right]_1^4$	A1	One term correct. Neglect limits.	
	$= (24 - 8) - \left(\frac{3}{2} - \frac{1}{8}\right)$	M1	Correct limits subst in integral. Subtraction seen. If arb constant used, evaluated to give s = 0 when t = 1 and then sub t = 4.	
	= $14\frac{5}{8}$ m (14.625 m)	A1	cao. Any form. [If trapezium rule used M1 use of rule (must be clear method and at least two regions) A1 correctly applied M1 At least 6 regions used A1 Answer correct to at least 2 s.f.)]	4
		8		

Q 4		mark	comment	sub
(i)	$32\cos\alpha t$	B1		1
(ii)	$32\cos\alpha \times 5 = 44.8$	M1	FT **their** x.	
	so $160\cos\alpha = 44.8$ and $\cos\alpha = 0.28$	E1	Shown. Must see some working e.g cosα = 44.8/160 or 160 cosα = 44.8. If 32 × 0.28 × 5 = 44.8 seen then this needs a statement that 'hence cosα = 0.28'.	2
(iii)	$\sin\alpha = 0.96$	B1	Need not be explicit e.g. accept sin(73.73…) seen.	
	either			
	$0 = (32 \times 0.96)^2 - 2 \times 9.8 \times s$	M1	Allow use of 'u' = 32, g = ± (10, 9.8, 9.81).	
		A1	Correct substitution.	
	s = 48.1488… so 48.1 m (3 s. f.)	A1	cao	
	or			
	Time to max height is given by 32 × 0.96 -9.8 T = 0 so T = 3.1349….	B1	Could use ½ total time of flight to the horizontal.	
	y = 32 × 0.96 t – 4.9 t^2	M1	Allow use of 'u' = 32, g = ± (10, 9.8, 9.81) May use $s = \frac{(u+v)}{2}t$.	
	putting t = T, y = 48.1488 so 48.1 m (3 s. f.)	A1	cao	4
		7		

Q 5		mark	comment	sub
(i)	$\mathbf{v} = \mathbf{i} + (3-2t)\mathbf{j}$	M1	Differentiating **r.** Allow 1 error. Could use const accn.	
		A1		
	v(4) = **i** – 5**j**	F1	Do not award if $\sqrt{26}$ is given as vel (accept if **v** given and *v* given as well called speed or magnitude).	
				3
(ii)	**a** = – 2**j**	B1	Diff **v.** FT **their v.** Award if – 2**j** seen & isw.	
	Using N2L **F** = $1.5\times$(– 2**j)**	M1	Award for $1.5\times(\pm\ \textbf{their a} \text{ or } a)$ seen.	
	so – 3**j** N	A1	cao Do not award if final answer is not correct. [Award M1 A1 for – 3**j** WW]	
				3
(iii)	$x = 2+t$ and $y = 3t - t^2$	B1	Must have both but may be implied.	
	Substitute $t = x-2$			
	so $y = 3(x-2)-(x-2)^2$	B1	cao. isw. Must see the form *y* = ….	
	$[= (x-2)(5-x)]$			
				2
		8		

Q 6		mark	comment	sub
(i)	Up the plane $T - 4g\sin 25 = 0$	M1	Resolving parallel to the plane. If any other direction used, all forces must be present. Accept $s \leftrightarrow c$. Allow use of *m*. No extra forces.	
	T = 16.5666… so 16.6 N (3 s. f.)	A1		
				2
(ii)	Down the plane, $(4+m)g\sin 25 - 50 = 0$	M1	No extra forces. Must attempt resolution in at least 1 term. Accept $s \leftrightarrow c$. Accept *Mg*sin25. Accept use of mass.	
		A1	Accept *Mg*sin25	
	m = 8.0724… so 8.07 (3 s. f.)	A1		
				3
(iii)	Diagram	B1	Any 3 of weight, friction normal reaction and *P* present	

		B1	in approx correct directions with arrows. All forces present with suitable directions, labels and arrows. Accept *W*, *mg*, 4*g* and 39.2.	2
(iv)	Resolving up the plane	M1	Resolving parallel to the plane or All forces must be present . Accept $s \leftrightarrow c$. Allow use of *m*. At least one resolution attempted and accept wrong angles. Allow sign errors.	
		B1	$P\cos 15$ term correct. Allow sign error.	
	$P\cos 15 - 20 - 4g\sin 25 = 0$	B1	Both resolutions correct. Weight used. Allow sign errors. FT use of *P*sin 15.	
		A1	All correct but FT use of *P*sin 15.	
	P = 37.8565…. so 37.9 N (3 s. f.)	A1		5
(v)	Resolving perpendicular to the plane	M1	May use other directions. All forces present. No extras. Allow $s \leftrightarrow c$. Weight not mass used. Both resolutions attempted. Allow sign errors.	
	$R + P\sin 15 - 4g\cos 25 = 0$	B1	Both resolutions correct. Allow sign errors. Allow use of *P*cos15 if *P*sin15 used in (iv).	
		F1	All correct. Only FT **their** *P* and their use of *P*cos15.	
	R = 25.729… so 25.7 N	A1	cao	4
		16		

If there is a consistent $s \leftrightarrow c$ error in the weight term throughout the question, penalise only two marks for this error. In the absence of other errors this gives
(i) 35.52… (ii) 1.6294… (iv) 57.486… (v) 1.688…

For use of mass instead of weight lose maximum of 2.

Q 7		mark	comment	sub
(i)	With the 11.2 N resistance acting to the left			
	N2L $F - 11.2 = 8 \times 2$	M1	Use of N2L (allow $F = mga$). Allow 11.2 omitted; no extra forces.	
		A1	All correct	
	$F = 27.2$ so 27.2 N	A1	cao	3
(ii)	The string is inextensible	E1	Allow 'light inextensible' but not other irrelevant reasons given as well (e.g. smooth pulley).	1
(iii)		B1	One diagram with all forces present; no extras; correct arrows and labels accept use of words.	
		B1	Both diagrams correct with a common label.	2
(iv)	**method (1)**	M1	For either box or sphere, $F = ma$. Allow omitted force and sign errors but not extra forces. Need correct mass. Allow use of mass not weight.	
	box N2L $\rightarrow 105 - T - 11.2 = 8a$	A1	Correct and in any form.	
	sphere N2L $\uparrow$ $T - 58.8 = 6a$	A1	Correct and in any form. [box and sphere equns with consistent signs]	
	Adding $35 = 14a$	M1	Eliminating 1 variable from 2 equns in 2 variables.	
	$a = 2.5$ so 2.5 m s^{-2}	E1		
	Substitute $a = 2.5$ giving $T = 58.8 + 15$	M1	Attempt to substitute in either box or sphere equn.	
	$T = 73.8$ so 73.8 N	A1		
	method (2)			
	$105 - 11.2 - 58.8 = 14a$	M1	For box and sphere, $F = ma$. Must be correct mass. Allow use of mass not weight.	
	$a = 2.5$	A1		
		E1	Method made clear.	
		M1	For **either** box **or** sphere, $F = ma$. Allow omitted force and sign errors but not extra forces. Need correct mass. Allow use of mass not weight.	
	either: box N2L $\rightarrow 105 - T - 11.2 = 8a$			
	or: sphere N2L $\uparrow$	A1	Correct and in any form.	

	$T - 58.8 = 6a$			
	Substitute a = 2.5 in either equn	M1	Attempt to substitute in either box or sphere equn.	
	T = 73.8 so 73.8 N	A1	[If AG used in either equn award M1 A1 for that equn as above and M1 A1 for finding T. For full marks, both values must be shown to satisfy the second equation.]	7
(v) **(A)**	g downwards	B1	Accept $\pm g,\ \pm 9.8,\ \pm 10,\ \pm 9.81$	1
(B)	Taking ↑ + ve, $s = -1.8$, u = 3 and $a = -9.8$	M1	Some attempt to use $s = ut + 0.5at^2$ with $a = \pm 9.8$ etc $s = \pm 1.8$ and $u = \pm 3$. Award for a = g even if answer to (A) wrong.	
	so $-1.8 = 3T - 4.9T^2$			
	and so $4.9T^2 - 3T - 1.8 = 0$	E1	Clearly shown. No need to show +ve required.	2
(C)	See over			
(C)	Time to reach 3 m s $^{-1}$ is given by			
	$3 = 0 + 2.5t$ so t = 1.2	B1		
	remaining time is root of quad	M1	Quadratic solved and + ve root added to time to break.	
	time is 0.98513… s	B1	Allow 0.98. [Award for answer seen WW]	
	Total 2.1851…so 2.19 s (3 s. f.)	A1	cao	
	With the 11.2 N resistance acting to the right			
(i)	F + 11.2 = 8 × 2 so F = 4.8		The same scheme as above	
(iii)			The 11.2 N force may be in either direction, otherwise the same scheme	
(iv)	The same scheme with + 11.2 N instead of – 11.2 N acting on the box method (1) box N2L $\rightarrow 105 - T + 11.2 = 8a$ sphere as before			
	method (2) 105 + 11.2 – 58.8 = 14a These give a = 4.1 and T = 83.4		Allow 2.5 substituted in box equation to give T = 96.2 If the sign convention gives as positive the direction of the sphere descending, a = – 4.1. Allow substituting a = 2.5 in the equations to give T = 43.8 (sphere) or 136.2 (box).	
(v)			In (C) allow use of a = 4.1 to give time to break as 0.73117..s. and total time as 1.716…s	4